20789/C

N. III 1
19

88323

Mathematical Digeſts.

CONTAINING THE

ELEMENTS and APPLICATION of GEOMETRY,

AND

PLANE TRIGONOMETRY,

WHETHER BY

Inſtrumental Conſtruction, or by Calculation, to the Meaſuring of Heights and Diſtances, *&c.*

AND THE

Stereographic Projection of Spheric Trigonometry; with numerical Solutions, and the Application thereof, to ſeveral curious and important Problems in ASTRONOMY, NAVIGATION, and DIALLING.

WITH

TABLES for finding the Place, and Eclipſes of the SUN and MOON, according to the laſt Improvement of the *Newtonian* Theory; and many practical Problems in each Branch.

Deſign'd for a plain, methodical familiar Courſe of Inſtruction in the above-mentioned Parts of mathematical Science; very uſeful for all Lovers thereof.

AND PARTICULARLY

For all Teachers of Mathematics: Being a ſynthetical Method which the Author has found, by many Years Experience, to be moſt ſucceſsful and agreeable to his Pupils.

Inſcribed to all the

Schoolmaſters and Teachers of MATHEMATICS in *Great Britain* and *Ireland*

By Mr. *JOHN DOUGHARTY*,

Author of *The General Gauger*, and Teacher of the Mathematics at *Worceſter*.

LONDON:

Printed and Sold by E. OWEN, in *Hand-Court*, oppoſite *Great Turnſtile*, *Holborn*:

And R. MONTAGU, at the Book-Warehouſe in *Great Wild-Street*, near *Drury-Lane*; G. BICKHAN, Engraver, in *May*'s *Buildings*, *Covent-Garden*; H. SLATER, the Corner of *Clare-Court*, *Drury-Lane*; and W. REEVE, at *Shakeſpear*'s Head, in *Fleet-Street*.

WELLCOME
HISTORICAL
MEDICAL
LIBRARY

TO THE

SCHOOLMASTERS and TEACHERS of Mathematics in *Great Britain* and *Ireland.*

GENTLEMEN,

THE candid Reception which my former little Treatiſe, called *The General Gauger*, met with in the World, and an earneſt Deſire to be uſeful in my Generation, as far as my Abilities will admit, have induced me to appear once more in Print, and to beſpeak the Indulgence of the Public, to whom I bequeath this Legacy, for the Advancement of Education, as a Mark of my Gratitude for their former Favours; and ſince I have no Reaſon to doubt of finding their Candour for a Work of this Nature equal to my Sincerity and Well-meaning in the Publication thereof, eſpecially if it be fortunate enough to merit your Approbation; to whoſe Patronage could I then ſo properly recommend it, as to yours, for whoſe Uſe it was chiefly intended, and whoſe Emolument I had principally in view, in compoſing thereof?

IT is an old Obſervation, That a good Method is the Soul of Buſineſs: And it is equally true in the Buſineſs of the Schools, as in that of Commerce; I doubt not that your own Experience has often verified the Maxim: I think it was ſaid by that eminent Man of Buſineſs Cardinal RICHLIEU, That whatever Reputation he had acquired in his Negotiations was principally owing to his Purſuit of a good Method, and doing only one Thing at a Time. Since an uſeful Education is of the higheſt Importance to Mankind, whether conſidered as Individuals, or as Members of a Community; and ſince Art is long, tho' Life is ſhort; whoever therefore has any thing to offer, which has a Tendency either to ſhorten the Time uſually ſpent in a Courſe of Study, or to render the arduous Task of Education eaſier, both to Teachers and Learners, has certainly a juſt Claim to the Attention of all Tutors and Pupils in that Branch of Science whereof he treats. This is my Caſe at preſent, as the very Title does, in ſome meaſure, intimate: And I hope I may, without Vanity, ſay, that the following Sheets contain an eaſy, familiar, and perſpicuous Method of performing and demonſtrateing, what is moſt material and neceſſary for Learners to know, in the elemen-

tary

tary Parts of Geometry, Trigonometry, and Projection of the Sphere; and alſo the Application of that Doctrine to the Solution of a Variety of curious and intereſting Problems in Aſtronomy, Navigation and Dialling. In the former elementary Part, according to the Practice of ſeveral eminent Authors, I have ſelected a ſufficient Variety of ſuch Problems and Theorems as are indiſpenſibly neceſſary for underſtanding the reſt of this Work, which yet have a Connection and Dependence on one another; ſo that their Truth is evident by themſelves, without clogging the Learner's Memory with ſeveral trifling and ſuperfluous Things in *Euclid*; of whoſe firſt Six Books, mine may be called an Abridgment. In the Sequel of this Work, wherein the aforeſaid Theory is applied, I have endeavoured to exemplify it in ſuch Inſtances as are of conſiderable Importance in human Affairs, or uſeful in common Life: And tho' ſome of them have been thought ſufficiently abſtruſe, yet the Eaſineſs of the Manner, wherein they are here treated, makes them appear plain and familiar.

I AM very ſenſible, that tho' indeed the Subſtance of Science be moſt valuable in an Author, yet there is a Merit in, and a Regard ought to be paid to, its being exhibited in a commodious, eaſy and pleaſant Form: Of two Manners of doing the ſame Thing, one may be vaſtly preferable to the other. Experience is allowed to be a good Teacher; and I dare venture to ſay, that in above Forty Years Experience I never found any Method ſo well adapted for the Eaſe and Expectation both of Maſters and Scholars, nor any attended with ſo much Succeſs as is here laid down. I can aſſure them, neither Vanity nor Intereſt prompts me to be an Author now; but to the Motives before-mentioned. I may add that very common one of complying with the Importunity of Friends, of which ſeveral, who are competent Judges, having ſeen this Work in Manuſcript, were very ſolicitous for its being made public. Such as it is, Gentlemen, it is now ſubmitted to your Approbation or Cenſure: You well know, *Tenuia non ſpernenda*. If theſe my Endeavours ſhould be of any Utility in the Practice of Teaching, or in anywiſe contribute to your Succeſs, I ſhall reap ſome Satisfaction for my Labours, who am,

Your ſincere Wellwiſher,

and moſt humble Servant,

John Dougharty.

CHAP. I.

GEOMETRY.

DEFINITIONS.

GEOMETRY originally ſignified the Art of meaſuring the Earth; but is now a Science, which treats of Magnitudes, and is the principal Part of the *Mathematicks*.

MAGNITUDE is a continued Quantity, which conſiſts either in Lines, or Angles; in Surfaces, or Solids.

1. A Point is the Bound of a Line.
2. A Line is the Bound of a Surface.
3. A Surface the Bound of a Body.

THE firſt is ſaid to have neither Length nor Breadth; the ſecond, Length without Breadth; and the third, Length and Breadth without Depth. But that which hath Length, Breadth, and Thickneſs, is called a Body, or a Solid.

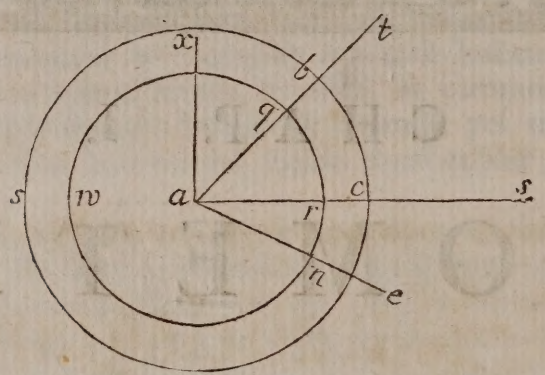
LINES are either ſtrait, or crooked, or curv'd ones. A ſtrait Line is what marks out the ſhorteſt Diſtance between any two Points. All Lines, that have not this Property, are crooked, or curv'd; ſome of which, as the Circle, Parabola, Hyperbola, Ellipſis, Quadratrix, Cycloid, Logarithmick Curve, *&c.* have very uſeful and ſurpriſing Properties.

A PLANE, or plane Surface, is that which lies evenly betwixt its bounding or extreme Lines, like the Head of a Drum, or Figure ſo extended in any other Shape.

An **Angle**, ſuppoſe *bac*, is made by the meeting of two Lines, as *ba* and *ca*, which is greater or leſs, according as the Lines that form it are inclin'd to each other: So the Angle *bae* is greater than the Angle *bac*; the Meaſure of the Angle *bae* being the Arch *qn*, and that of *bac* only the Arch *qr*.

NOW, as the Length of Lines is taken in Inches, Feet, Yards, Miles, *&c.* ſo the Meaſure of Angles is taken in Degrees, Minutes, Seconds, *&c.*

And every Circle being ſuppoſed to be divided into 360 equal Parts, called Degrees; a Degree into 60 Parts, called Minutes; and a Minute into 60 Parts, called Seconds, *&c.* if you deſcribe a Circle *q n w* (with any Extent of your Compaſſes) upon the angular Point *a*, ſo much of that Circle (*q r*) as is included between the two Lines *b a*, *c a*, is ſaid to be the Meaſure of that Angle. And here you muſt know, that if you produce the Line *ab* to *t*, and *a c* to *s*, or farther at Pleaſure, yet that cannot make the Angle *b a c*, or Arch *q r*, either more or leſs than before: Hence, if upon the Point *a* you deſcribe another Circle *s x b c*, the Arch *b c* will contain no more Degrees, Minutes, *&c.* than the Arch *q r*; only a Degree, Minute, *&c.* of the Arch *b c* will be greater than the like Parts of the Arch *q r*; *i. e.* if one Degree of the Arch *q r* be 40 Inches, then one Degree of the Arch *b c* will be more than 40 Inches.



Angles are either Right, Acute, or Obtuſe.

1. A Right Angle *x a c* is made by two Lines *x a*, *c a*, which ſtand perpendicular to each other.

2. An Acute Angle *q a r* is leſs than a Right Angle. And,

3. An Obtuſe Angle *x a n* is more than a Right Angle. Where *note*, that when an Angle is expreſs'd by three Letters, as *x a n*, 'tis meant that the angular Point is ſignified by the middle Letter. As the Angle above is made by the Lines *x a* and *n a*, ſo the Angle itſelf may be expreſſed by *x a n*, to diſtinguiſh it from any other Angle made about *a*.

Of Figures.

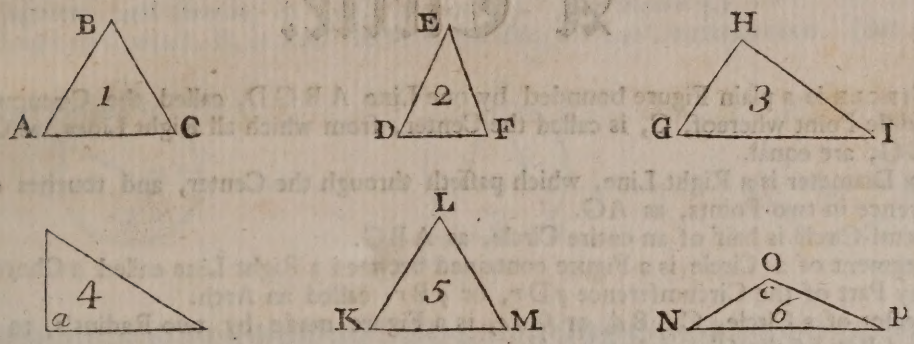
A Figure is a Magnitude bounded by one or more Lines, or Surfaces; of which ſome are plain, and ſome ſolid.

A Triangle.

1. A Triangle is a Figure bounded by three Lines, as the Triangle A B C; which, in reſpect to the Sides, is either (1) Equilateral, that has all its Sides equal, A B C. (2) Iſoſcele or equal-legg'd, which has two Sides equal, D E F. (3) Scalene, that has no Side equal, G H I. Or,

2. In

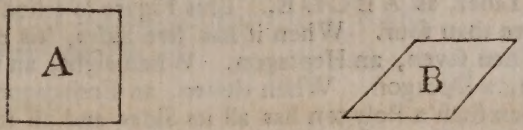
2. In reſpect to its Angles, is either (4) Right-angled, which has one Right Angle *a*. (5) Acute-angled, that has all its Angles Acute, as K L M; or, (6) Obtuſe, which has one Obtuſe Angle *c*, as N O P.



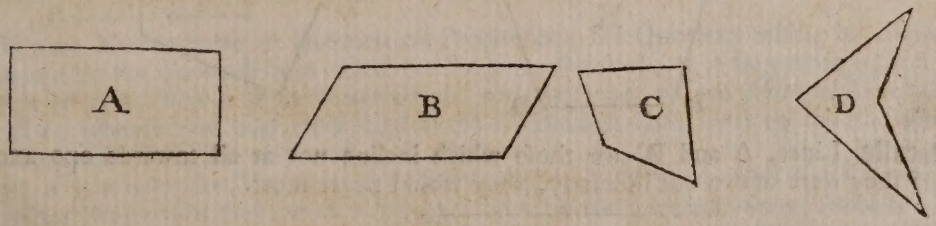
A Parallelogram.

A Parallelogram is a Figure that has four Sides, and as many Angles; and is either Equilateral, which has four equal Sides; or In-equilateral, whoſe Sides are unequal.

1. An Equilateral Parallelogram has either all its Angles right, as the Square A; or oblique Angles, as the Rhombus B.



2. An In-equilateral Parallelogram has either Right Angles, as the Right-angled Parallelogram A, called alſo the Oblong, or Long Square; or its Angles are Oblique, and then 'tis called a Rhomboides, as B: Every other four-ſided Figure is called a Trapezium, as C or D.



A Circle.

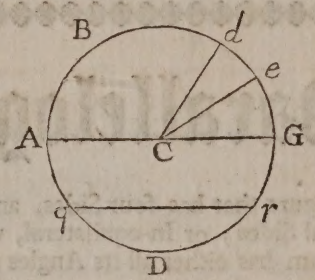
A CIRCLE is a plain Figure bounded by one Line A B G D, called the Circumference; the middle Point whereof, C, is called the Center, from which all Right Lines, as C A, C*d*, C*e*, C G, are equal.

THE Diameter is a Right Line, which paſſeth through the Center, and touches the Circumference in two Points, as A G.

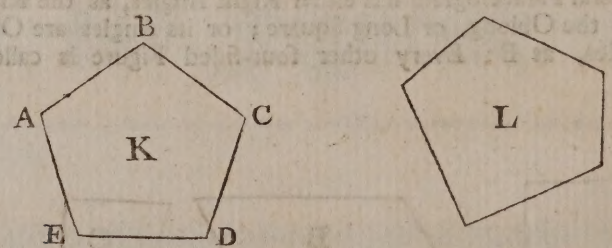
A Semi-Circle is half of an entire Circle, as A B G.

A Segment of a Circle is a Figure contained between a Right Line called a Chord (as *qr*) and any Part of the Circumference *q* D *r*, or *q* B *r*, called an Arch.

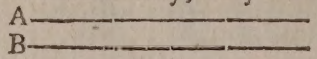
A Sector of a Circle, C A B*d*, or C*de*, is a Figure made by two Radius's, as C*d* and C*e*, and Part of the Circumference *de*.



A Multilateral (or many-ſided) Figure, called alſo a Polygon, is a Figure bounded by more than four Right Lines, as A B C D E. This Figure is called a Polygon, having ſeveral Sides and Angles, more than four. When it has five Sides, 'tis called a Pentagon. When ſix, an Hexagon. When ſeven, an Heptagon. When eight, an Octagon. When nine, a Nonagon. When ten, a Decagon. When eleven, an Endecagon: And, when twelve, a Dodecagon. And when ſuch a Polygon has all its Sides and all its Angles equal, it is called Regular, as K; but when there are any of them unequal, then Irregular, as L.



Parallel Lines, A and B, are thoſe which incline not at all towards one another: So that if they were drawn out infinitely, they would never meet.



A Right Line (A B) is ſaid to touch a Circle, when being continued it does not cut it: And ſuch a Line is called a Tangent, from the End of which (ſuppoſe A) if you draw a Line to the Center C, then the Line A C is called a Secant. (See *Figure the 1ſt*, next following.)

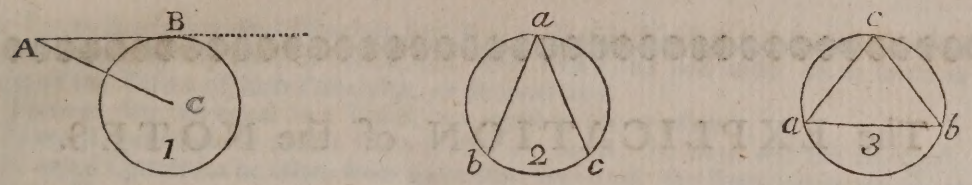
An

An Angle *a*, is ſaid to ſtand upon that Part of the Circumference *bc*, that is oppoſite to it. (See *Fig. the ſecond.*)

An Angle of the Segment, *acb* (ſee *Fig.* 2.), is made by the Circumference and a Right Line. or Chord, as *ac*.

An Angle in the Segment (*c*) is made by two Right Lines (*ac*, *bc*) riſing from the Angles of the Segment, and meeting in the Circumference. (See *Figure* 3.)

An Angle of Contact, B, is made between the Tangent and Circumference. (See *Fig.* 1.)



Of Proportion.

1. One Line or Number is ſaid to meaſure another, when the leſſer repeated a certain Number of Times is exactly equal to the greater; the greater of ſuch may be called a multiplied Magnitude, and the leſſer an aliquot Part thereof: Thus B being thrice repeated will exactly meaſure A, therefore is an aliquot Part of A; and ſuch a Part 4 is of 12, becauſe 4 being taken thrice will preciſely meaſure 12.

A ——— ——— ———
B ———

2. Like aliquot Parts, are thoſe which are an equal Number of Times contained in their Wholes. Thus B and D are like aliquot Parts of A and C.

A ——— ——— ———
B ———
C ——— ——— ———
D ———

3. Like Parts are thoſe that are equally contained in their reſpective Wholes; thus B and D are like Parts, becauſe B is contained once and a half in A, and D once and a half in C.

A ——— ——— ———
B ——— ———
C ——— ——— ———
D ——— ———

4. When Numbers are in Geometrical Proportion, the Quotient ariſing by dividing the Conſequent by the Antecedent is called the Ratio (or Reaſon); as if they were $4 : 8 :: 6 : 12$, then $\frac{8}{4} = \frac{12}{6} = 2$; here 2 is the Ratio which (with *Sturmius*, in his *Matheſis Enucleata*) you may call *e*; whence you may obſerve, that the Antecedent multiply'd by the Ratio produces the Conſequent: So $4 \times 2 = 8$ and $6 \times 2 = 12$.

5. So if you have the Diameter of a Circle $= 30 = b$ given to find the Circumference, the conſtant Proportion may be as $113 : 355 :: b :$ to the Circumference; here the Ratio is $\frac{355}{113} = 3,14152 = e$, then eb may expreſs the Circumference; and ſo of any other.

6. Let *b* be the firſt Term of three Quantities continually proportional, and *e* the Ratio; then they will ſtand thus; *b*, *eb*, *eeb*. Here you may ſee, that the Square of the middle Term *eb*, which is *eebb*, is equal to the Rectangle or Product of the Extremes *b* and *eeb*, which is alſo *eebb*.

7. If

7. If there are four such Terms as $b.\ eb.\ e^2b.\ e^3b.$ then the Rectangle of the Means will be equal to the Rectangle of the Extremes, $b \times e^3b = eb \times e^2b$.

The like, if four Quantities are discretely proportional, as $a : ea :: b : eb$; here $a \times eb = ea \times b$, *&c.* So if there were never so many Terms in continual Geometrical Proportion, the Product of the Extremes is equal to the Product of any two of the Means that are equally distant from the Extremes, as also to the Square of the middle Term, if the Terms are odd.

The EXPLICATION of the NOTES.

Signs or Characters that are used in this Work.

+	More	the Sign of Addition.
=	Less	the Sign of Subtraction.
×	Into	the Sign of Multiplication.
÷	By	the Sign of Division.
::	Proportion	the Sign of disjunct Geometrical Proportion.
÷	Continued	the Sign of continual Geometrical Proportion.
∴	Ergo	Therefore.
⊏		Greater than.
⊐		Lesser than.
=		Equal to.
⊛		Involve.
∪∩		Evolve.
√	Root	the Sign of the Square Root.
$\surd^2$		the Sign of the Square Root also.
$\surd^3$		the Sign of the Cube Root.
$\surd^4$		the Sign of the Biquadrate Root, *&c.*
∟		a Right Angle.

Postulates, or Petitions.

A Postulate is that which is manifest in itself, that it may easily be done, or conceived to be done. It is therefore required to be granted that we may,

1. From any Point given draw a Right Line unto any other given Point.
2. Draw a finite Right Line in Length still farther.
3. From any Center, at any Interval describe a Circle.

Axioms.

Axioms.

An Axiom is a ſelf-evident Principle, which is ſo clear in itſelf, that it is not capable of being made clearer by any kind of Proof, but what every one will aſſent to as ſoon as they underſtand the Terms of ſuch Principles, or Propoſitions.

1. Things that are equal to a Third, are equal to one another.
2. If equal Quantities be added to thoſe that are equal, the Sums will alſo be equal.
3. If equal Quantities be taken from thoſe that are equal, the Remainders will be equal.
4. If to Unequals you add Equals, the Sums or Wholes will be unequal.
5. If from unequal Quantities you take Equals, the Remainders will be unequal.
6. Things that are double, triple, quadruple, *&c.* of the ſame Thing, are equal to one another.
7. Thoſe Quantities are equal, which being apply'd one to the other neither exceeds.
8. If right Lines be equal, they will mutually agree with one another; and the ſame is true of Angles.
9. The Whole is greater than any of its Parts.
10. All right Angles are equal one to another.
11. If two right Lines be parallel, all the Perpendiculars contained between them will be equal.
12. Two right Lines cannot comprehend a Space; for that will require three at leaſt.

CHAP. II.

Geometrical Problems.

A PROBLEM is a Propoſition, which requireth ſomething to be done; as to make ſome Figure, to add or ſubſtract one Figure to or from another; to make a Circle paſs through three given Points not lying in a right Line, *&c.*

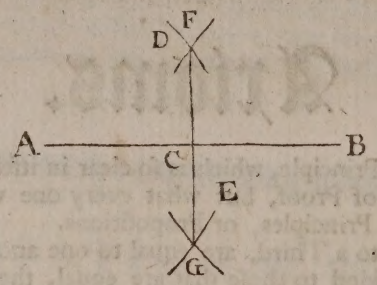
PROBLEM 1.

To divide a given Line (AB) *into two equal Parts.*

1. Set one Foot of your Compaſſes into the Point A, and opening the other Foot to any Diſtance above half the Length of the Line AB, make the Arches D and E above and below the Line.
2. With the ſame Extent, ſet one Foot at the other End B, and croſs thoſe Arches in F and G.

3. Draw

3. DRAW the Line F G, and this ſhall divide the Line A B into two equal Parts in C, as was required.



PROB. 2.

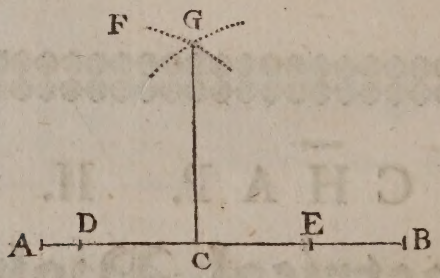
Upon a Point in a Line given to erect a Perpendicular.

LET A B be the Line given, and let C be the Point aſſign'd therein, upon which it is required to erect the Perpendicular G C.

1. SET one Foot in the Point C, and open the other to any convenient Diſtance, as to D, and make the Points D and E on each of the Point C.

2. SET one Foot in D, and open the other Foot at Pleaſure; then deſcribe the Arch F; and with the ſame Extent of your Compaſſes ſetting one Foot in E, croſs the former Arch in G.

3. DRAW the Line G, C, which ſhall be perpendicular to A, B, upon the Point C, as requir'd.



PROB. 3.

To erect a Perpendicular upon (or near) the End of a given Line.

TO do this, there are ſeveral Ways; but I ſhall only ſet down two, which are the moſt familiar and eaſy.

The Firſt Way.

LET A B be a given Line, and from the Point A, near the End thereof, let it be required to erect a Perpendicular A C.

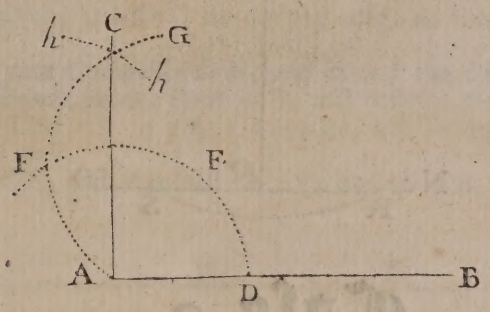
1. OPEN your Compaſſes to any Diſtance at Diſcretion, and ſetting one Foot in the given Point A, with the other deſcribe the Arch F E D.

2. THEN ſet one Foot in D (the Compaſſes being at the ſame Diſtance) croſs the ſaid Arch in E; and ſetting one Foot in E, with the other deſcribe the Arch A F G, croſſing the firſt Arch in F.

3. SET

3. SET one Foot in F, and with the other deſcribe the ſmall Arch *h h*, croſſing the former in the Point C.

4. DRAW the Line AC, and that ſhall be perpendicular to the Line AB, as was required.

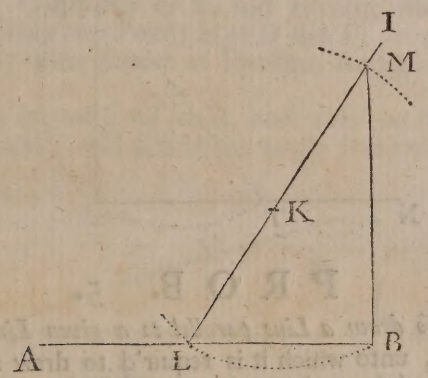


The Second Way.

LET B be the Point given, and from thence let it be required to draw the Line BM perpendicular to AB.

1. OPEN your Compaſſes to any convenient Diſtance, and ſetting one Foot in the Point B, put the other down any where (above the Line AB), as at K, and turn that which was in B, about upon the Center K, until it cuts the Line AB in the Point L.

2. DRAW the Line LI, and the Compaſſes remaining as before (at the Diſtance LK) ſet that from K to M ; ſo a Line drawn from M to B, will be perpendicular to AB, and from the given Point B, as was requir'd.



PROB. 4.

From a Point to let fall a Perpendicular to a Line given.

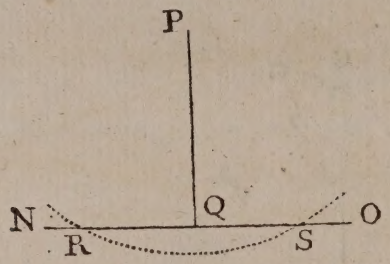
IN this there are two Caſes ; firſt, when the given Point is over or near the Middle of the Line ; and ſecondly, when it is near or almoſt over the End of the Line.

Caſe 1.

LET NO be a Right Line given, and from the Point P over it, let it be required to let fall the Perpendicular PQ.

1. OPEN your Compaſſes to any Diſtance greater than PQ, and ſetting one Foot in the given Point P, with the other deſcribe an Arch of a Circle cutting the given Line NO, in the Points R and S.

2. DIVIDE the Line RS into two equal Parts in Q (by *Prob.* 1.) ſo a Line drawn from the given Point P to Q, ſhall be perpendicular to the given Line NO.



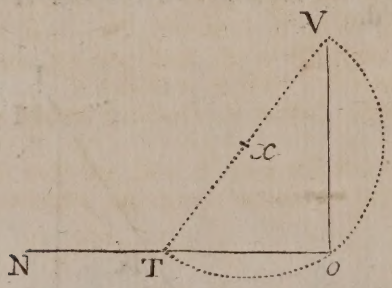
Caſe 2.

LET V be the Point given, from whence to let fall a Perpendicular to the Line NO.

1. FROM any Part of the given Line NO, as from T, draw a Right Line to the given Point V, which Line (by *Prob.* 1.) divide into two equal Parts in the Point *x*.

2. TAKE T*x* in your Compaſſes, and with that Extent, placing one Foot in *x* as a Center, deſcribe the Semicircle VOT, cutting the given Line NO in O.

3. DRAW a Line from V to O, and that will be perpendicular to the given Line NO, as was required.



PROB. 5.

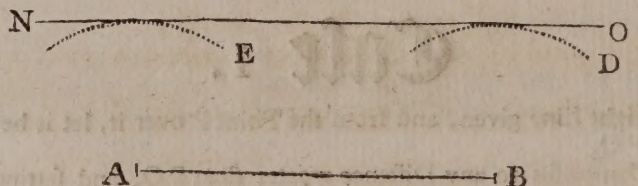
To draw a Line parallel to a given Line.

LET AB be a Line given, unto which it is requir'd to draw a Parallel.

1. SET one Foot in the End A, and opening to any Diſtance requir'd make the Arch E.

2. WITH the ſame Extent of your Compaſſes ſet one Foot in the other End B, and deſcribe the Arch D.

3. LAY your Ruler to the very Top of theſe two Arches, ſo that it does not croſs, but juſt touch them; then by the Side thereof draw the Line NO, and it ſhall be parallel to the Line AB, which was requir'd.



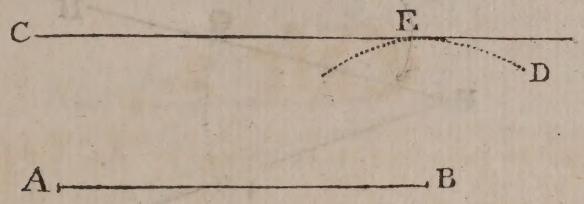
PROB.

PROB. 6.

To draw a Parallel to a Line given, from a Point assign'd.

Let AB be a Line given, and let C be a Point assign'd, from whence a Parallel to AB is to be drawn.

1. Set one Foot of your Compasses in A, and extend the other to the Point C.
2. With the same Extent set one Foot in B, and describe the Arch D.
3. Take the whole Line AB in your Compasses, and setting one Foot in C, with the other cross the Arch D in E.
4. Draw the Line CE, and this shall be a Parallel to AB, from the Point C, as was required.

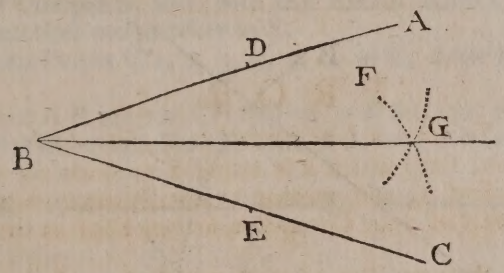


PROB. 7.

To divide an Angle given into two equal Parts.

Let ABC be an Angle given, to be divided into two equal Parts.

1. Set one Foot of your Compasses in B, and opening the other to any convenient Distance, then make in the Sides two Points as at D and E.
2. With the same, or any other Extent at Pleasure, setting one Foot in E, make the Arch F.
3. With the same Extent set one Foot in D, and cross the former Arch in G.
4. Draw the Line BG, and it will divide the Angle into two equal Parts.



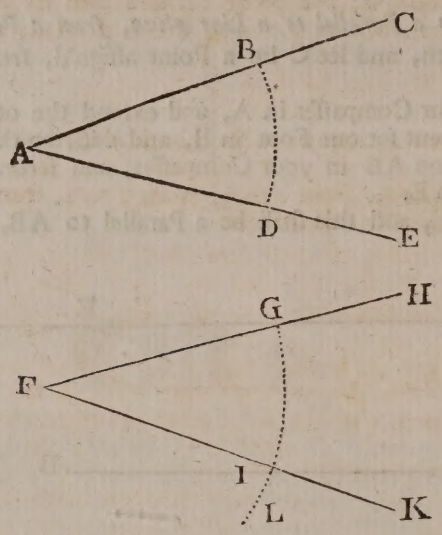
PROB. 8.

To make an Angle equal to an Angle given.

Let CAE be the given Angle, and let it be required to make another Angle equal thereto.

1. Upon the angular Point A, with any Opening of the Compasses describe the Arch BD.
2. Having drawn the Line FH, then upon the Center F (with the same Extent of your Compasses as before) describe the Arch GL.

3. TAKE the Arch BD in your Compaſſes, and ſet that from G to I, and draw the Line FK, and this will form an Angle HIK, equal to CAE, as was required.

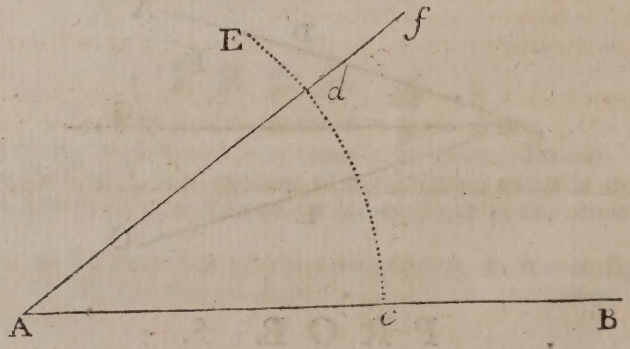


PROB 9.

WITH a given Line AB, to make an Angle of any given Number of Degrees, ſuppoſe 36°.

1. OPEN your Compaſſes to 60 (the Radius) of your Line of Chords (the Deſcription and Making of which you have in the following Part of this Book), and ſetting one Foot in A, with the other deſcribe the Arch *e*E.

2. TAKE in your Compaſſes the given Angle 36 Degrees out of the ſame Line, and ſet that Extent off from *e* to *d*, then lay a Ruler upon the Points A and *d*, and draw the Line A*f*, ſo will this Line A*f* make an Angle of 36 Degrees with the Line AB, as was required.



PROB. 10.

Two Lines, ſuppoſe A*f* and AB, being already drawn, and making a certain Angle BA*f*, to find the Quantity of that Angle.

1. TAKE 60°, out of your Line of Chords, and ſetting one Foot of your Compaſſes in A, with the other deſcribe the Arch *e*E.

2. TAKE *ed* in your Compaſſes, and ſet one Foot in the Beginning of the Line of Chords, and the other will reach to 36 upon the ſame Line; and that is the Meaſure of the Angle *f*AB requir'd. (*See the laſt Figure.*)

PROB.

PROB. 11.

To divide a given Line into any Number of equal Parts.

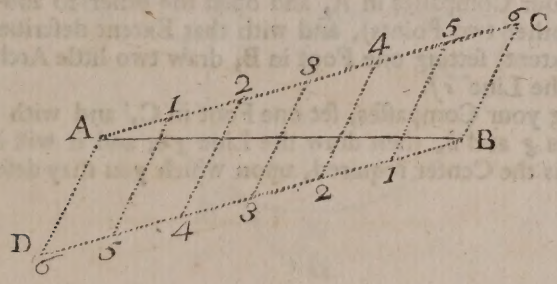
LET AB be the Line propos'd, and to be divided into ſix equal Parts.

1. FROM the End A, draw out at pleaſure, the Line AC, making any Angle with the Line AB.

2. FROM the other End B, draw the Line BD, parallel to AC, (by *Prob.* 5.) or making the Angle DBA equal to the Angle CAB (by *Prob.* 8.)

3. FROM the Points A and B, and upon the Lines AC, and BD, ſet off any ſix equal Parts, 1, 2, 3, 4 5, 6.

4. DRAW Lines from A to 6, from 1 to 5, from 2 to 4, from 3 to 3, *&c.* and thoſe will divide the given Line AB into ſix equal Parts.



Another Way.

IN a new Method of Fortification as practiced by Monſieur *Vauban*, you have a different Way (from that above) to divide a Line into any Number of equal Parts; and this I take to be preferable to the other, becauſe by it you divide any other Line, whether it is longer or ſhorter than that at firſt propos'd into the ſame Number of equal Parts.

Let AB *be a Line given, to be divided into ſix equal Parts.*

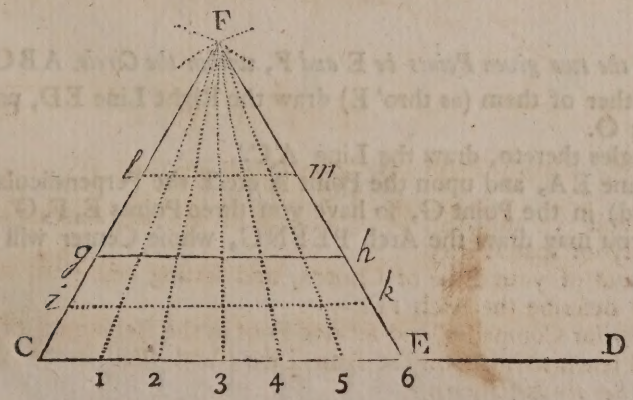
1. DRAW CD a Line of any convenient Length, and upon that from C ſet off as many equal Parts, as the given Line AB ought to have; ſix for Example, as from C to E which you may mark with 1, 2, 3, 4, 5, 6.

2. TAKE CE in your Compaſſes, and (with that Extent) upon C and E, as two Centers, deſcribe two Arches to interſect one another in F.

3. From F to the ſeveral Points C 1, 2, 3, 4, 5, 6, at E, draw Lines as FC. F1. F2, *&c.* FE.

4. TAKE the given Line AB in your Compaſſes, and ſet that from F to *g* and *h*, then draw the Line *g h*, which will be equal to the Line AB, and divided into ſix equal Parts, as was required.

I ——————K A——————B. G——————H



If the Line given had been longer than A B, ſuch as G H = F*i*, then *i k* is the ſame, and divided into ſix equal Parts; if it had been ſhorter, as I K = F*l*, then *l m* is the ſame, and divided into ſix equal Parts, ſo the Triangle being once made, ſerves to divide any Line, if not longer than C E, into the like Number of equal Parts, and this is oftner of great Uſe, eſpecially when a Sector is not at hand.

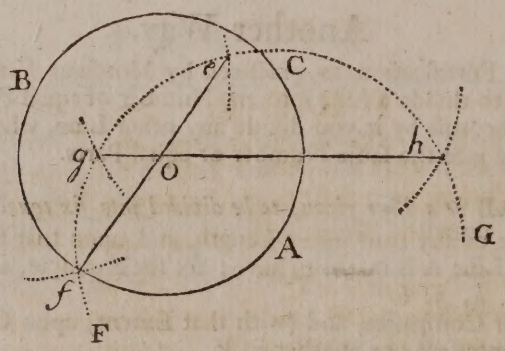
PROB. 12.

To find the Center of a Circle that ſhall paſs thro' any three given Points not lying in a Right Line, ſuppoſe ABC.

1. Set one Foot of your Compaſſes in A, and open the other to above half the Diſtance BA (the fartheſt of the other two Points), and with that Extent deſcribe the Arch F *g* G.

2. With the ſame Extent, ſetting one Foot in B, draw two little Arches to cut the former in *f* and *e*, then draw the Line *e f*.

3. Without altering your Compaſſes, ſet one Foot in C, and with the other croſs the Arch F *g* G in the Points *g* and *h*, then draw the Line *g h*, and it will interſect the Line *e f*, in the Point O; which is the Center required, upon which you may deſcribe the Circle ABC, which was to be done.



PROB. 13.

Two Points within a Circle being given, how to deſcribe the Arch of another Circle which ſhall paſs thro' thoſe two Points, and alſo divide the Circumference of the given Circle into two equal Parts.

Let the two given Points be E *and* F, *within the Circle* ABCD.

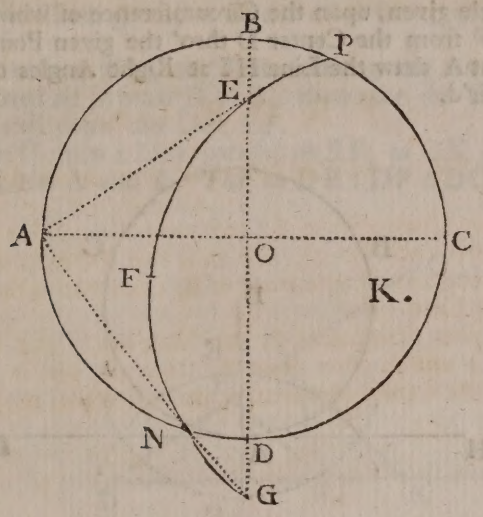
1. Through either of them (as thro' E) draw the Right Line ED, paſſing thro' the Center of the Circle at O.

2. At Right Angles thereto, draw the Line A C.

3. Draw the Line E A, and upon the Point A erect the Perpendicular AG, cutting the Line BD (produced) in the Point G, ſo have you three Points E, F, G, through which (by the laſt Problem) you may draw the Arch PEFNG, whoſe Center will be at K.

Now,

Now, if you lay a Rule upon the Points P and N, and it paſſes over the Center of the given Circle at O, then you may conclude that the Circle is truly drawn.



PROB. 14.

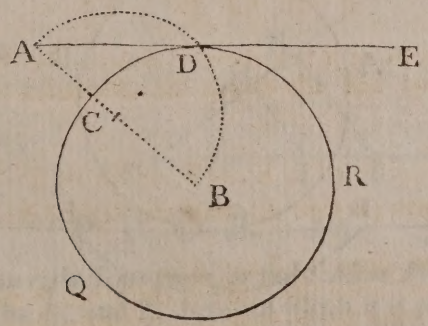
From a Point aſſign'd to draw a Tangent to a given Circle.

LET A be the Point, from which a Tangent to the Circle DRQ is to be drawn.

1. FROM the Point A, draw the Secant AB, which divide into two equal Parts in the Point C.

2. SET one Foot of your Compaſſes in C, and extend the other to A or B, and with that Extent deſcribe the Semicircle ADB, and it will interſect the given Circle in D.

3. FROM A, draw the Line ADE, which is the Tangent Line requir'd.



PROB.

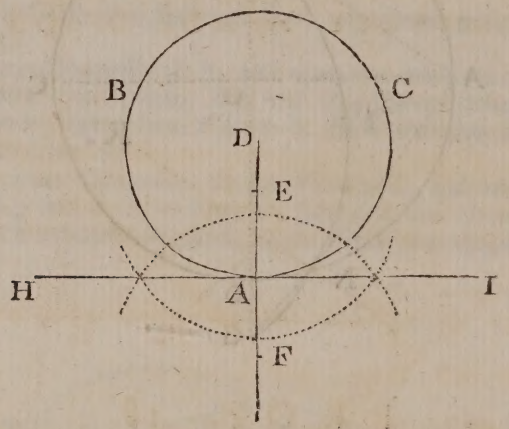
PROB. 15.

To draw a Tangent Line to a Point given in a Circle.

LET A B C be the Circle given, upon the Circumference of which is the Point A propos'd.

1. DRAW the Line D F from the Center D thro' the given Point A.

2. THROUGH the Point A draw the Line H I at Right Angles to the Line D F; ſo will H I be the Tangent requir'd.

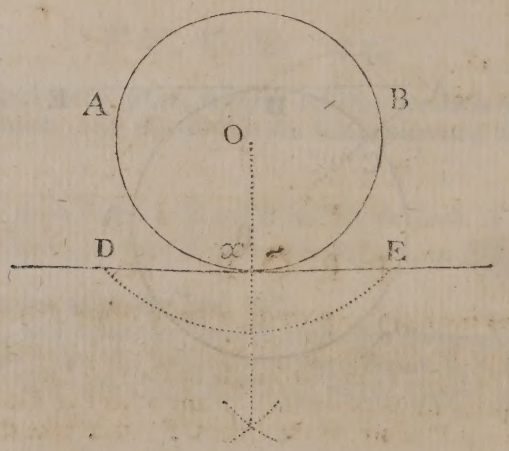


PROB. 16.

A CIRCLE being given together with a Tangent Line, to find the Point where it touches the Circle.

LET ABx be the Circle, and DE the Tangent, the Point where it toucheth the Circle is required.

FROM the Center O let fall a Perpendicular Ox to the Tangent DE; ſo is x the Point of Contact required.



PROB. 17

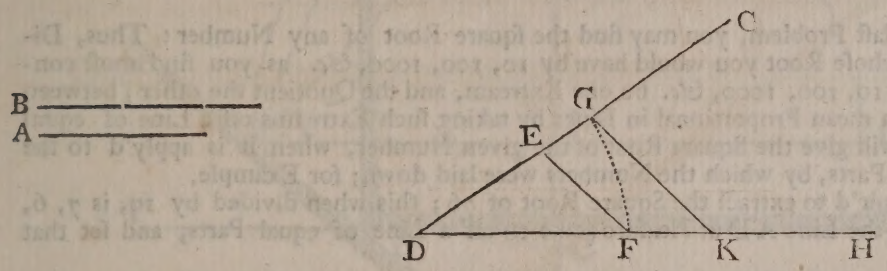
To find a third Line in Proportion to two Lines given.

LET A and B be the two Lines given, to find a third Line, which ſhall be in the ſame Proportion to them.

1. DRAW two Right Lines, as CD, and DH, making any Angle CDH.

2. TAKE the Line A, and ſet it from D to E; then take the Line B, and ſet it from D to F, and from D to G, and draw the Line EF.

3. THROUGH the Point G draw a Line parallel to EF, as GK; ſo ſhall DK be the third Proportional Line to the Lines A and B. For as DE : DF :: DG (= DF) : DK.



PROB. 18.

To find a fourth Line in Proportion to three Lines given.

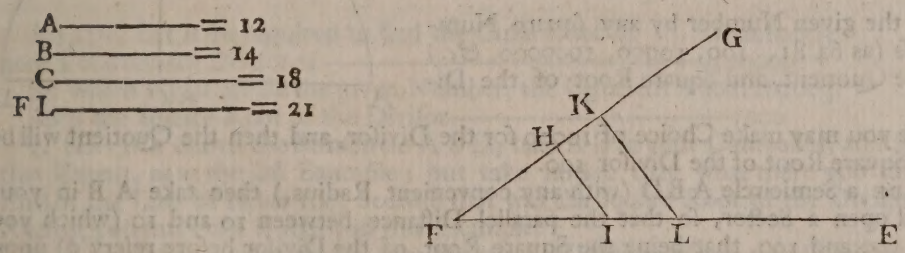
LET A, B, and C, be three Lines given, and let it be requir'd to find a fourth Line in Proportion to them.

1. DRAW F G, and F E to make any Angle at Pleaſure, as E F G.

2. TAKE the Line A, and ſet it from F to H; alſo take B, and ſet that from F to I, and draw the Line H I.

3. Take the third Line C, and ſet it from F to K, and thro' the Point K, draw K L, Parallel to H I, ſo ſhall F L be the fourth Line requir'd.

THESE two Problems do the Work of *The Rule of Three* in Lines.



PROB. 19.

Between two right Lines given, to find a Mean Proportional.

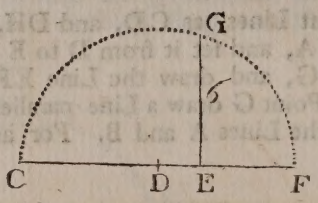
LET the two given Lines be A, and B, between which it is required to find a third Line G E, whoſe Square ſhall be equal to the Rectangle of the two Lines A, and B, (that being the Property of a mean Proportional between theſe or any other two Lines.)

1. DRAW a Right Line at Pleaſure, as the Line C F; then take the Line B in your Compaſs, and ſet it from C to E.

2. Take the Line A, and ſet that from E to F, and divide the Line C F, into two equal Parts in D; and upon D, as a Center, with the Diſtance D C, or D F, deſcribe the Semicircle C G F.

3. From the Point of joining of the two given Lines at E, erect a Perpendicular E G, cuting the Semicircle in G; ſo ſhall E G be the mean Proportional required.

A———=4
B————————=9



By Help of the laſt Problem, you may find the ſquare Root of any Number: Thus, Divide the Number whoſe Root you would have by 10, 100, 1000, &c. as you find moſt convenient; then will 10, 100, 1000, &c. be one Extream, and the Quotient the other; between which if you find a mean Proportional in Lines by taking ſuch Extreams off a Line of equal Parts, that Mean will give the Square Root of the given Number, when it is apply'd to the ſame Line of equal Parts, by which the Numbers were laid down; for Example,

1. Let it be requir'd to extract the Square Root of 76; this when divided by 10, is 7, 6, then having drawn the Line A D at Pleaſure (take 10 off a Line of equal Parts, and ſet that from A to C.

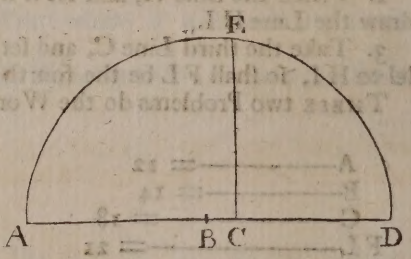
2. From the ſame Scale take 7,6 and ſet that from C to D; then add 10 and 7,6 together, and the Sum is 17, 6 the half of which is 8,8.

3. Take 8,8 in your Compaſſes, from the Scale, and ſet that from A to B, and upon B as a Center, at the Diſtance of B A, or B D, deſcribe the Semicircle A E D.

4. Upon the Point C, erect the Perpendicular C E, and take that in your Compaſſes to the Line of equal Parts, and it will give 8,71, &c. the Root requir'd.

Again, if the Square Root of 3136, was required, this when divided by 100, is 31,36, ſo a mean Proportional between 100, and 31, 36 will be 56, &c. for any other.

The preceding Method for extracting the Square Root brings to mind another ſomething like it for ex-extracting the Cube Root, and is as follows.



Suppose it was required to find the Cube Root of 175616.

1. Divide the given Number by any ſquare Number that is leſs (as 64,81, 100, 10000, 1000000, &c.) and reſerve the Quotient, and Square Root of the Diviſor.

In this Caſe you may make Choice of 10000 for the Diviſor, and then the Quotient will be 17,5616, and Square Root of the Diviſor 100.

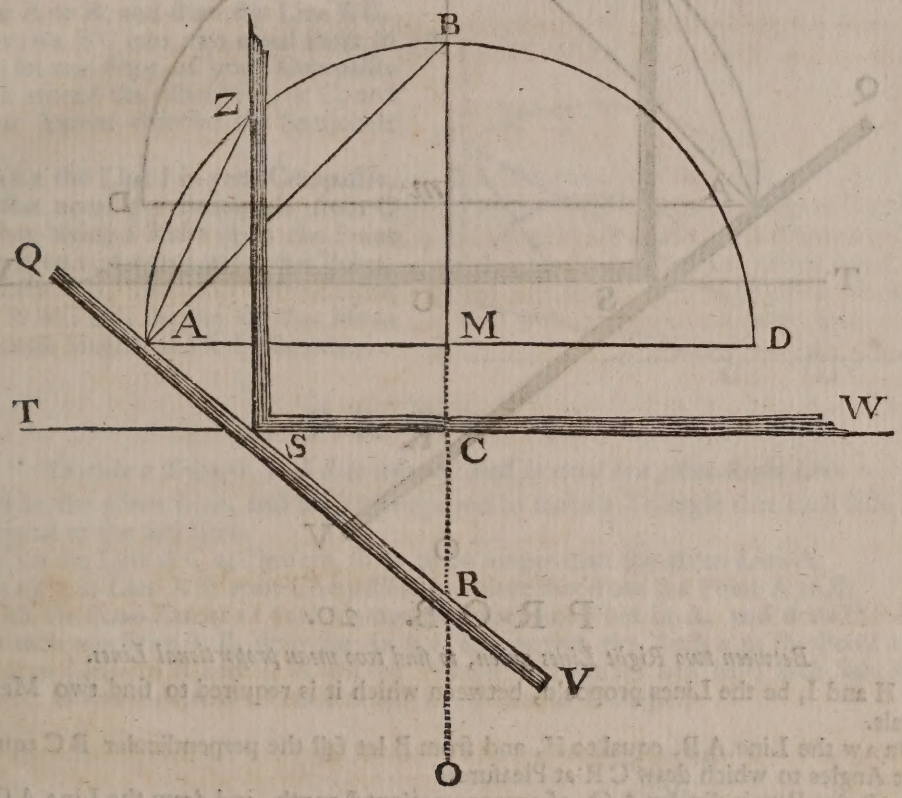
2. Describe a Semicircle A B D (with any convenient Radius,) then take A B in your Compaſſes and open a Sector, ſo that the parallel Diſtance between 10 and 10 (which you may now call 100 and 100, that being the Square Root of the Diviſor before reſerv'd) upon the Line of Lines, may be equal to that Extent of your Compaſſes.

3. The Sector being thus ſet, take the parallel Diſtance of 17,5616 (the reſerv'd Quotient) in your Compaſſes, and ſet that upon the Line B O from M to C and thro' the Point C draw T W parallel to A D.

4. Having done ſo far, take a Square and lay one Side of it upon the Line T W, ſo as that the other Side may cut the Semicircle A B D; and alſo lay a Ruler (as Q U) upon the Poiut A to cut the Line B O, then move the Square (along the Line T W) towards A, ſo that its angular Point at S, may move the Ruler Q V, (one Side remaining ſtill upon A) until the Line M R becomes equal to the Chord A Z (which you may know by trying two or three Times with your Compaſſes.)

5. Take

5. TAKE A Z, or M R in your Compaſſes to the Sector, and the Points will fall upon the parallel Diſtance of 56, which is the Cube Root of 175616.

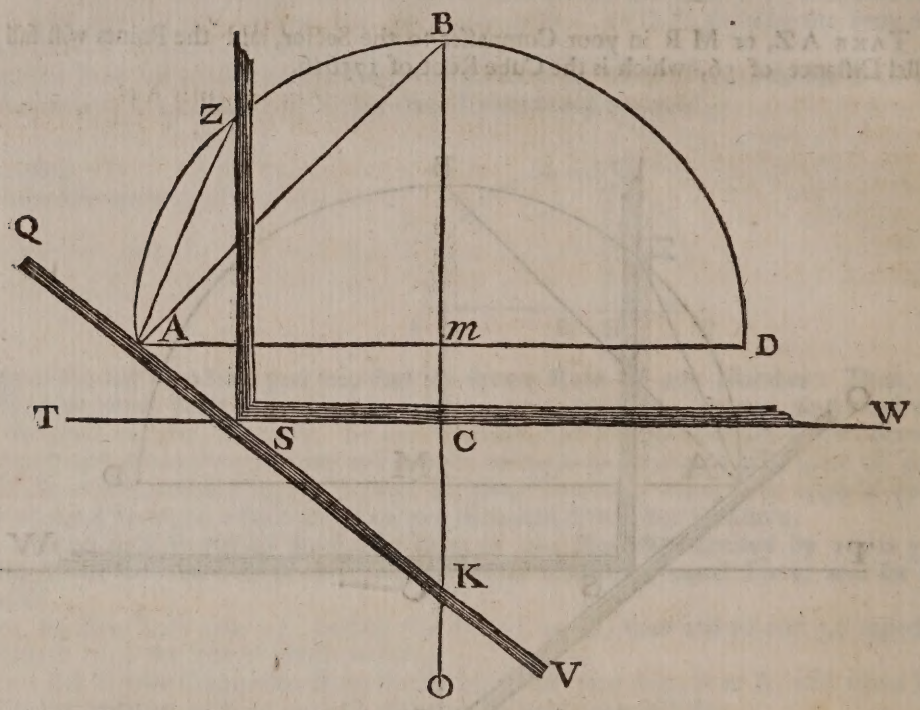


AGAIN, Let it be required to find the Cube Root of 202262003
here a convenient Diviſor is——————————1000000
BY which if you divide the given Number, the Quotient is 202,262003
AND the Square Root of the Diviſor——————————1000

1. HAVING drawn the Semicircle A B D, take A B in your Compaſſes, and ſet the Sector to that Extent, as at the laſt Example; but take notice that what there you call'd 10, at the End of the Line of Lines 100, becauſe that was the Square Root of the Diviſor, for the ſame Reaſon you muſt call it 1000, in this Example.

2. THE Sector being thus ſet, take the parallel Diſtance of 202,26, *&c.* in your Compaſſes, and ſet that from M to C, and draw T W.

3. LAY the Square and Ruler, and moving them as before, until you find M R equal to A Z; then take one of them in your Compaſſes to the Sector, and the Points will fall upon the parallel Diſtance of 587, the Root ſought.



PROB. 20.

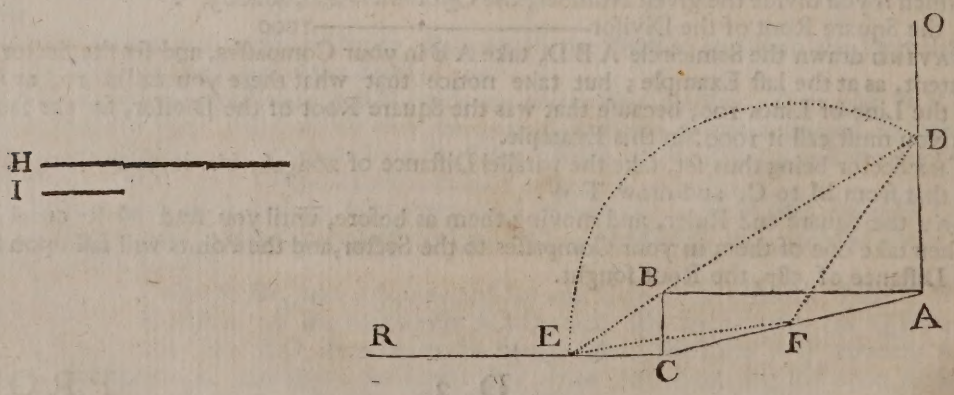
Between two Right Lines given, to find two mean proportional Lines.

LET H and I, be the Lines propos'd, between which it is required to find two Mean Proportionals.

1. DRAW the Line A B, equal to H, and from B let fall the perpendicular B C equal to I, at Right Angles to which draw C R at Pleaſure.

2. Erect the Perpendicular A O, of any convenient Length, and draw the Line A C, which divide into two equal Parts in F.

3. LAY a Ruler upon the Point B, and keeping it upon that Point, move it up and down upon the Line A O, until F E becomes equal to F D; then will A D and C E be the Mean Proportionals between the given Lines H and I.



Another

Another Way to do the ſame.

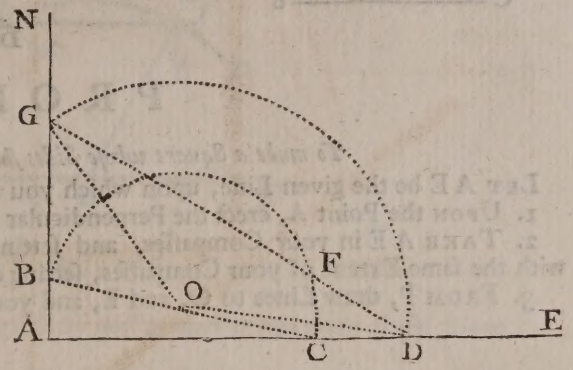
LET H and I be the Lines given as above.

1. DRAW out A E at Pleaſure ; and upon A erect the Perpendicular A N.

2 TAKE the Line H in your Compaſſes, and ſet that from A to C, and the Line I, and ſet that from A to B, and draw the Line B C.

3. DIVIDE B C into two equal Parts in O ; then ſet one Foot of your Compaſſes in O, and extend the other to B, or C, and with that Extent deſcribe the Semicircle B F C.

4. TAKE the Line I in your Compaſſes, and lay that upon the Semicircle from C to F ; then laying a Ruler upon the Point F, move it up and down upon that Point, and the Line A N, until O G is made equal to O D, ſo will B G be one of the Mean Proportionals ſought, and C D the other.



PROB. 21.

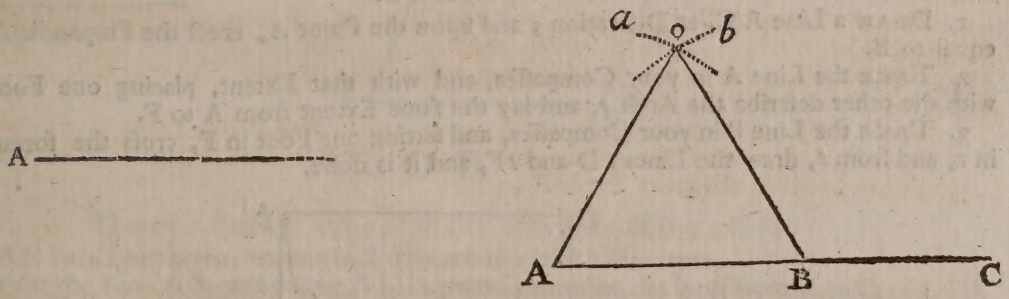
To make a Triangle, each Side whereof ſhall be equal to a given Right Line.

LET A be the given Line, and let it be required to make a Triangle that each Side thereof ſhall be equal to the ſaid Line.

1. DRAW the Line A C at Pleaſure, ſo as to be longer than the given Line A.

2. TAKE this Line A in your Compaſſes, and place that from the Point A to B.

3. With the ſame Extent of your Compaſſes, place one Foot in A, and draw the Arch *a* ; and then with one Foot in B, draw the Arch *b*, to interſect the Arch *a* in the Point *o* ; from whence draw Lines to A and B, ſo will the Triangle be form'd, and have each Side equal to the Line A, as was required. (This is called an equilateral Triangle.)



PROB. 22.

Of three Lines given, ſo that the Sum of the two ſhorteſt ſhall be greater than the Third, to make a Triangle.

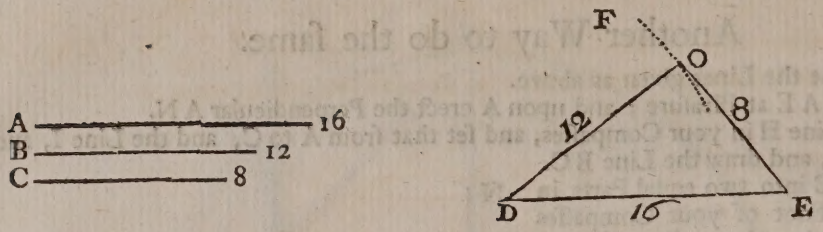
LET A, B and C, be the three Lines given ; whereof it is required to make a Triangle.

1. TAKE the Line A, and lay it down from D to E.

2. TAKE the Line B in your Compaſſes, and ſetting one Foot in D, make the Arch F ; that done take the Line C, and ſetting one Foot at E, with the other croſs that Arch at O.

3. From O, draw the Lines O E, and O D, ſo ſhall the Triangle be form'd, whoſe three Sides are equal to the three Lines A, B, C.

A



PROB. 23.

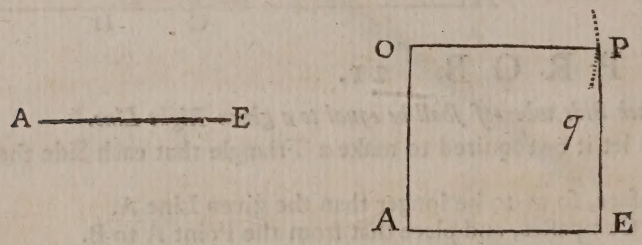
To make a Square whoſe Sides ſhall be equal to a given Line.

LET A E be the given Line, upon which you are to make a Square A E O P.

1. UPON the Point A, erect the Perpendicular A O, equal to A E.

2. TAKE A E in your Compaſſes, and ſetting one Foot in O, deſcribe the Arch *q*; and with the ſame Extent of your Compaſſes, ſetting one Foot in E, croſs that Arch in P.

3. FROM P, draw Lines to O, and E, and you will form the Square required.



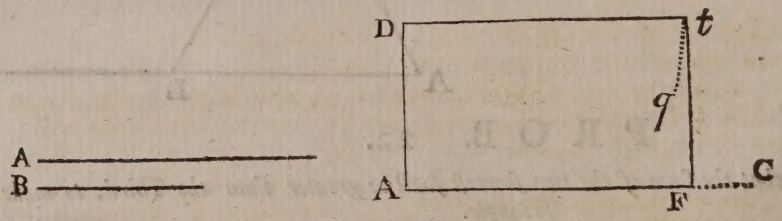
PROB. 24.

To make a Parallelogram, or Long Square, whoſe Length ſhall be equal to a given Line A, *and Breadth to the Line* B.

1. DRAW a Line A C, at Diſcretion; and upon the Point A, erect the Perpendicular A D equal to B.

2. TAKE the Line A in your Compaſſes, and with that Extent, placing one Foot in D, with the other deſcribe the Arch *q*, and lay the ſame Extent from A to F.

3. TAKE the Line B in your Compaſſes, and ſetting one Foot in F, croſs the former Arch in *t*, and from *t*, draw the Lines *t* D and *t* F, and it is done.



PROB. 25.

To make a Rhombus whoſe Sides ſhall be equal to a given Right Line.

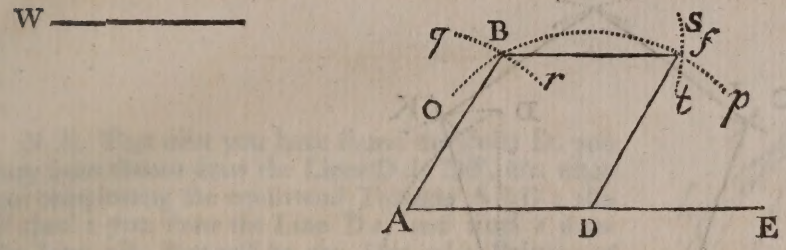
LET the Line given be W.

1. DRAW the Line A E to any Length longer than the given Line W; and then taking the Line W in your Compaſſes, ſet that off from A to D, and with the ſame Extent of your Compaſſes, put one Foot in D, and with the other draw the Arch O P.

2. THE

2. THE Compaſſes continuing as they were, ſet one Foot in A, and with the other deſcribe the Arch *q r*; croſſing the Arch O P, in B, upon which Point ſet one Foot of your Compaſſes, and with the former Extent draw the Arch S *t* which will cut the Arch O P in *f*.

3. DRAW the Lines *f* B, and *f* D, ſo will A B *f* D be the Rhombus required.



PROB. 26.

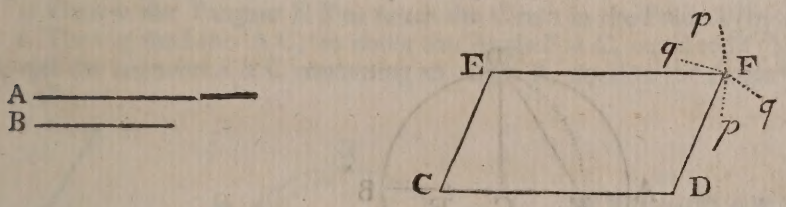
To make a Rhomboides whoſe longeſt and ſhorteſt Sides ſhall be equal to two given Right Lines.

LET the given Lines be A and B.

1. TAKE the Line A in your Compaſſes, and ſet that from C to D; alſo take B in your Compaſſes, and (if you are not limited by a given Angle) ſet that from C to any Point as E, and draw C E.

2. TAKE the Line A in your Compaſſes, and ſetting one Foot in E, with the other draw the Arch *p p*.

3. TAKE B in your Compaſſes, and ſet one Foot in D, and draw the Arch *q q*, which will cut the former Arch in F; from which draw the Lines F E and F D, ſo is C E F D the Rhomboides required.



PROB. 27.

To make a Pentagon upon a given Line without the Help of a Circle.

LET A C be a Line given, upon which you are to make a Pentagon.

1. DRAW the Line A B, and taking A C in your Compaſſes, ſet it off from A to C.

2. RAISE the Perpendicular C D equal to A C, and divide A C into two equal Parts in E.

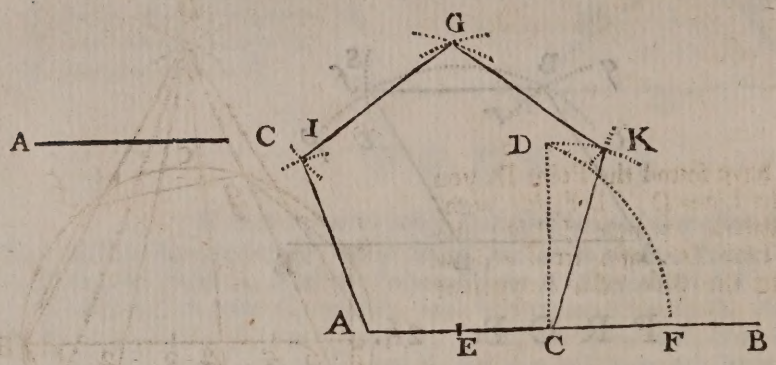
3. SET one Foot of your Compaſſes in E, and extend the other to D, and with that Extent draw the Arch D F.

4. TAKE A F in your Compaſſes, and ſetting one Foot in A, and then in C, upon theſe two Points, as Centers deſcribe two Arches to cut one another in G.

5. TAKE A C in your Compaſſes, and upon the Points A and G, draw two Arches to croſs one another in I, then (with the ſame Extent) upon the Points C, and G, draw two other Arches to cut one another in K.

6. JOIN

6. JOIN A I, I G, G K, and K C together, by Right Lines, and it is done.



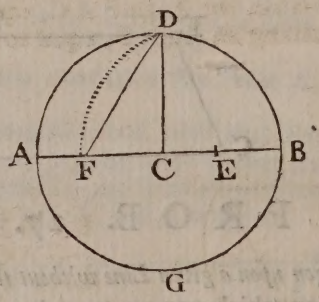
PROB. 28.

To inſcribe a Pentagon within a Circle. (Euclid. Schol. *Prop.* 10. *Lib.* XIII.

LET A D B G, be a Circle in which you are to draw a Pentagon.

1. DRAW the Diameter A B, and upon the Center C erect the Perpendicular C D.

2. DIVIDE C B into two equal Parts in E; then ſet one Foot of your Compaſſes in E (as a Center) and extending the other to D, deſcribe the Arch D F, and draw the Line F D, and this is one Side of the Pentagon requir'd, by which you may draw the reſt.



Renaldinus, *has given us an univerſal Method of inſcribing in a Circle a Polygon of any Number of Sides whatever:* Thus,

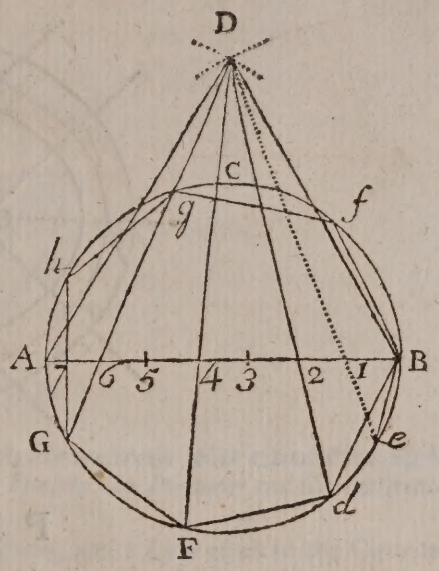
THE Circle A C B F being drawn, and alſo its Diameter A B, let it be required to inſcribe a Heptagon, or Figure of Seven equal Sides.

1. MAKE the Equilateral Triangle A B D upon the Diameter A B. (by *Prob.* 18.) and divide the Diameter into as many equal Parts (by *Prob.* 8.) as the Figure to be inſcribed has Sides (in this Caſe Seven)

2. FROM the Point D, draw a Line thro' the ſecond Diviſion (2) to the oppoſite Concave Part of the Circumference at *d*, and from *d*, draw another Line to the End of the Diameter at B, ſo B *d* is the Side of the Polygon required (here of a Heptagon) and ſo of any other.

3. IF thro' 4 and 6 you draw the Lines D F, and D G, and from F draw the Lines F *d*, and F G, you will have two Sides more, and if you would complete the Heptagon take B *d* in your Com-

Compaſſes, and lay that from B to *f*, and from *f* to *g*, and the ſame from G to *b*, and ſo draw the Sides B*f*, *fg*, &c.

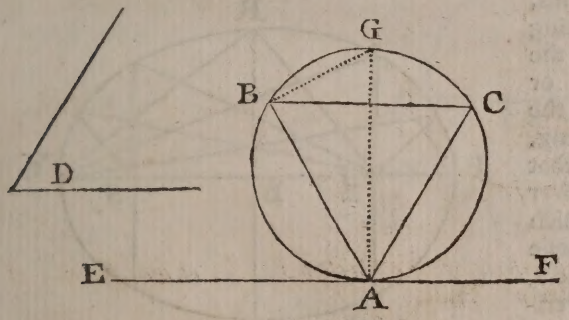


N. B. That after you have found the Point D, you may from thence draw the Lines D *d*, D F, &c. without compleating the equilateral Triangle A B D; alſo if thro' 1 you draw the Line D *e*, and from *e* draw the Line *e* B, that will be the Side of a Polygon of 14 equal Sides.

PROB. 29.

FROM a Circle given, A B C to cut off a Segment A B C to contain an Angle B, equal to a right lin'd Angle given D.

1. DRAW the Tangent E F to touch the Circle in the Point A (by *Prob.* 15.)
2. DRAW the Line A C, to make the Angle F A C, equal to D (by *Prob.* 8.) this Line will cut off the Segment A B C containing an Angle B, equal to the Angle C A F.



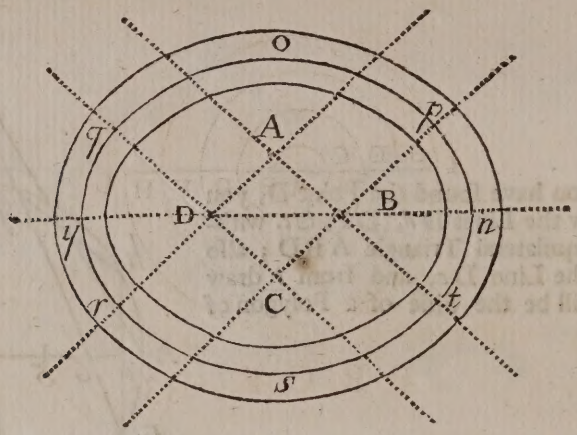
G B A = G A F both right Angles, and G B C = G A C by the 17th of Chap. 3d But G B A — G B C = C B A, and G A F —G A C = C A F, therefore C B A = C A F by Axiom 3d.

PROB. 30.

To deſcribe one or more Mechanic Ovals.

1. MAKE a four ſided Figure A B C D, and produce its Sides both Ways at Diſcretion.
2. PLACE one Foot of your Compaſſes at C, and with the other deſcribe the Arch *q o p*.
3. WITH the ſame Extent ſet one Foot in A, and with the other deſcribe the Arch *r s t*.
4. CONTRACT your Compaſſes ſo as that one Foot being ſet in B or D, the other may reach to *q* or *p*, and at that Extent deſcribe *q y r*, (upon the Center D,) and *p n t*, (upon the Center B) meeting the former Arches in *q*, *r*, and *p*, *t*, ſo will the Oval be completed; and

thus upon the ſame Centers you may deſcribe other Ovals either greater or leſs than this now drawn.



PROB. 31.

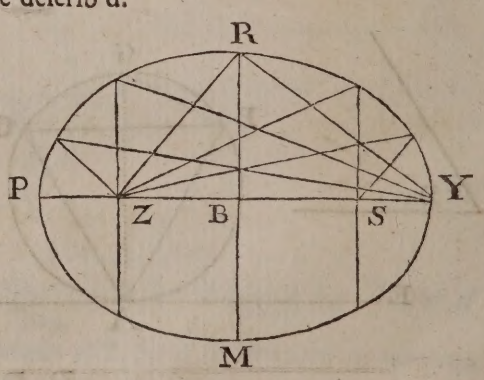
To deſcribe an Ellipſis more exact.

ABOUT any two Right Lines given for the two Diameters of an Ellipſis; to deſcribe ſuch an Ellipſis.

1. LET the Line P Y, be the longer, and R M the ſhorteſt Diameter given, and let them croſs one another at Right Angles in B.

2. TAKE half the longeſt Diameter P B, or B Y in your Compaſſes, and ſetting one Foot in R or M, the other will reach upon the longeſt Diameter to the Points Z and S, which are the Centers upon which the Ellipſis P R Y M muſt be deſcrib'd.

3. IN the two Points Z and S, fix two Pins, Nails, or the like, and about them put a String ſo long that being doubled it may reach from the Pin at S, to the End of the Diameter at P, or from the Pin at Z to the End Y, ſo then the whole Length of the String will be twice as long, as the Diſtance S P, or S Y, which String at that Length, join at both Ends, then putting it over the two Pins at Z and S, with a third Pin (which may be a Black Lead Pencil, or ſuch like) move the String about upon the two fixed Pins Z and S, and it will by its Motion deſcribe the true Periphery of an Ellipſis, or mathematical Oval.



PROB. 32.

To draw a Spiral Line about a given Line.

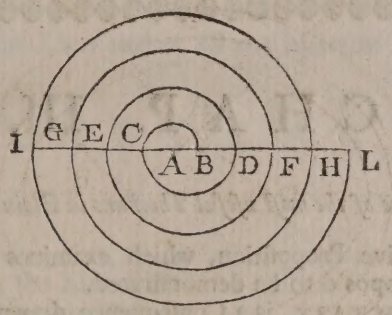
LET I L be the Line about which the Spiral Line is to be deſcrib'd.

1. DIVIDE half the Line I B, or L B into as many equal Parts as there are to be Revolutions in C, E, G, I, (*viz* four.)

2. DIVIDE B C into two equal Parts in A, and upon the Point A deſcribe the Semicircles B C, D E, F G, H I.

3. UPON

3. Upon the Point B describe the Semicircles CD, EF, GH, IL, and you will have the Spiral requir'd.



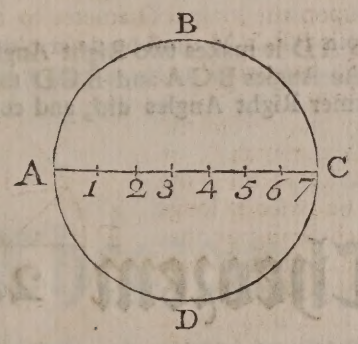
PROB. 33.

The Diameter of a Circle being given, to find the Length of the Circumference when extended to a Right Line; admitting, with Archimedes, *that the Proportion between the Diameter and Circumference of a Circle is near, as 7 to 22.*

Let the Circle be ABCD, whose Diameter AC is given, and a Line equal to the Circumference ABCD, requir'd.

1. Divide the Diameter AC into 7 equal Parts, and having drawn the Line EG to any convenient Length take the Diameter in your Compasses, and lay that Extent from E to 1, from 1 to 2, and from 2 to 3; so is E 3, thrice the Diameter.

2. Take one of the 7 Parts into which the Diameter was divided, as A 1; and set that off from 3 to N, then EN is a Line nearly equal to the Circumference ABCD, as was requir'd.



CHAP. III.

A Collection *of ſome of the moſt uſeful Theorems in Plain Geometry demonſtrated.*

A THEOREM is a ſpeculative Propoſition, which examines the Properties of Things, and wherein ſomething is propos'd to be demonſtrated.

A COROLLARY, or CONSECTARY, is a Conſequence drawn from ſomething that has been already demonſtrated.

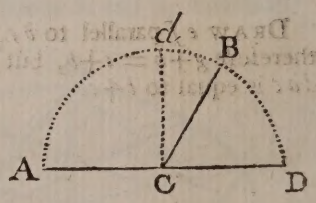
A LEMMA is a Sort of a Preparatory Propoſition which is brought in on purpoſe to demonſtrate (or facilitate) the Demonſtration of ſome enſuing Theorem, or to conſtruct ſome Problem.

Theorem 1.

IF a Right Line (B C) ſtands upon (or meets with) another Right Line (A D) it either makes two Right Angles, or Angles equal to two Right Angles.

DEMONSTRATION.

IF C *d* ſtands perpendicular to A D it makes two Right Angles A C *d*, D C *d*, if it lean as B C, the Angles B C A and B C D take up the ſame Place as the two former Right Angles did, and conſequently are equal to them.



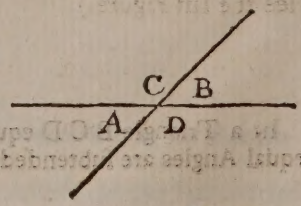
Theorem 2.

IF two Lines interſect each other, the two oppoſite Angles will be equal.

DEMONSTRATION.

D + A = 2 ∟ and D + B = 2 ∟ therefore (D being common) A = B

AGAIN A + C = 2 ∟ and A + D = 2 ∟, therefore (A being common) C = D



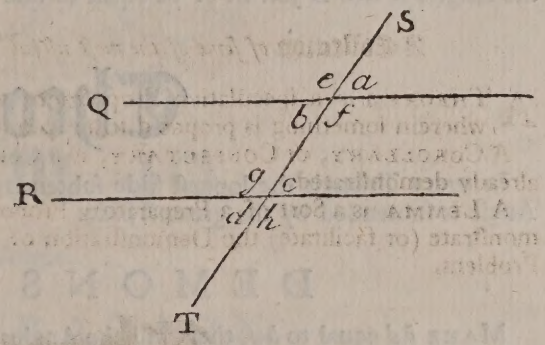
Theorem

Theorem 3.

A RIGHT Line cutting Parallel Lines makes all the oppoſite Angles equal $a=b=c=d$ and $e=f=g=h$.

DEMONSTRATION.

FOR Q and R being Parallel, the Line S T muſt have the ſame Inclination to one as it has to the other, and conſequently make equal Angles with both.

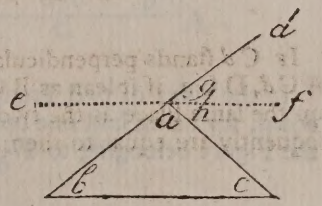


Theorem 4.

THE external Angle ($d\,a\,c$) of any Triangle $a\,b\,c$, is equal to the two oppoſite internal Angles b and c

DEMONSTRATION.

DRAW $e\,f$ parallel to $b\,c$, then $g=b$ and $h=c$ (*per* the laſt) therefore $g+h=c+b$, but $g+h$ is alſo equal to $d\,a\,c$, therefore $d\,a\,c$ is equal to $b+c$.



Theorem 5.

THE three Angles of every Triangle, are equal to two Right Angles.

DEMONSTRATION.

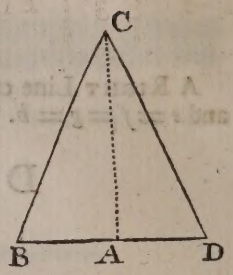
$a+b+g=2\,L$ (by the firſt) but $g+h=b+c$ (by the fourth) therefore $b+a+c=2\,L$ (ſee the laſt Figure.)

Theorem 6.

IN a Triangle B C D equal Sides (B C and D C) ſubtend equal Angles (D. B.) conſequently equal Angles are ſubtended by equal Sides.

DEMON-

DEMONSTRATION.

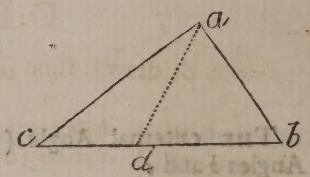


Divide B D in the Middle, at A, and draw the Line C A, which will divide the Triangle B C D into two other Triangles, *viz.* B C A and D C A, which have all their Sides equal, ſo that if the Triangle D C A was turn'd round upon the Line C A, until the Angle D comes to lie upon B ; then the Line D C would exactly cover the Line B C (as being equal in Length) and D A would juſt cover B A, ſo would the Angles B and D juſt fit or be equal to one another.

Theorem 7.

In every Triangle the longeſt Side ſubtends the greateſt Angles, conſequently the greateſt Angle is ſubtended by the greateſt Side.

DEMONSTRATION.



Make *b d* equal to *b a*, then will the Angles *d*, and *a*, be equal (by the 5th) but the Angle *c a b*, is greater than the Angle *d a b*, (by *Axiom* 9) now *b d a* is an external Angle to the Triangle *a d c*, and therefore greater than the Angle at *c* (which is ſubtended by the leaſt Side *a b*) tho not ſo big as the Angle *c a b* (which is ſubtended by the greateſt Side *c b*.

Theorem 8.

If the Sides of two Trangles are equal, the Angles oppoſite to thoſe equal Sides will be equal, for being laid one upon the other they will fit in every Part (ſee the two Triangles of the Figure belonging to *Theorem* 6th.)

Theorem 9.

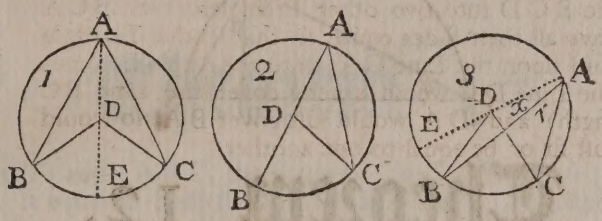
In a Circle A B C the Angle B D C at the Center, is double the Angle B A C ar the Circumference, when the ſame Arch of the Circle (B C) is the Baſe of the Angles.

DEMONSTRATION.

In Figure 1, from A thro' D, draw A E, then the external Angle B D E = D B A + B A D = 2 B A D (by the 4, and 6) and the external Angle C D E = D A C + D C A = 2 D A C conſequently the whole Angle B D C is equal to twice the Angle B A C.

In Fig 2, B D C is an external Angle to the Triangle D A C therefore equal to D A C + D C A = 2 D A C. Hence in Fig. 3 you have.

	1	EDC = 2r + 2x
	2	EDB = 2x
1—2	3	EDC—EDB = BDC = 2r



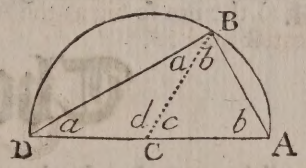
Theorem 10.

An Angle (D B A) in a Semicircle is a Right Angle.

DEMONSTRATION.

From B, draw a Line to the Center C, as B C ; then

	1	$d + c = 2\angle$ (by the 1)
	2	$d = 2b$ (by the 4)
	3	$c = 2a$
2 × 3	4	$d + c = 2b + 2a = 2\angle$
4 ÷ 2	5	$b + a = \angle$



Theorem 11.

In a Right angled Triangle (Q R S) a Perpendicular let fall from the Right Angle (R) to the Hypothenuse will divide that Triangle into two right Angled Triangles, which will be both similar or like to the first Triangle, and to each other.

DEMONSTRATION.

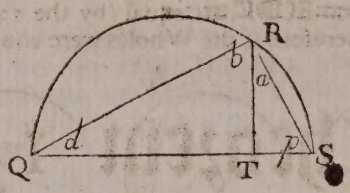
Let R T be the Perpendicular, then the Angles Q T R and S T R are Right Angles, and also $d + b$ and $p + a$ are two more. (by the 5) now

	1	$b + a = \angle$ (by the 10)	} b is common therefore $a = d$
	2	$b + = \angle$	}
Again	3	$a + b = \angle$	} a is common therefore $b = p$.
	4	$a + p = \angle$	}

Hence it appears that the Angles of the three Triangles are equal, and consequently the Triangles themselves are similar, so it will be as QT : TR :: TR : TS. ∴ QT × TS = TR □.

Hence

Hence it alſo appears that the ſaid Perpendicular is a Mean Proportional between the Segments of the Hypothenuſe Q T and T S.



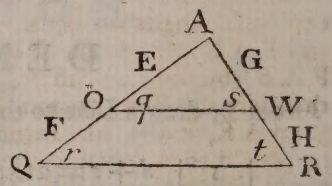
Theorem 12.

If a Right Line (O W) be drawn in any plain Triangle parallel to one of the Sides, it will cut off a Triangle (O A W) ſimilar or like to the whole Triangle Q A R.

N. B. That Triangles whoſe Angles are equal are ſimilar; and the Sides about the equal Angles are proportional.

DEMONSTRATION.

The Line O W being drawn parallel to the Side Q R, then will the Angle *q* be equal to *r*; and Angle *s*, equal to *t*, (by the 3) and the Angle at A being common, it follows that all the Angles of both the Triangles are equal, alſo that F and H are like Parts of A Q and A R, and by Conſequence, as E + F: F :: G + H : (H) and by diſſolving the Compoſition, it is E : F :: G : H.

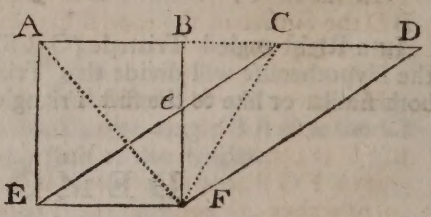


Theorem 13.

All Parallelograms that ſtand upon the ſame or equal Baſes and of the ſame Height (or between the ſame Parallels) are equal, to one another.

DEMONSTRATION.

The Triangles A E C and B F D are equal, for the Side A E = B F and E C = F D, and B C being common the Side A C muſt be equal to B D; now if from both you take the Triangle B C *e*, there will remain the Trapezium A B *e* E, equal to the Trapezium C D *e* F, and if to theſe you add the Triangle E *e* F the Sums will be A B E F = E C D F.



Theorem 14.

Triangles which ſtand upon the ſame Baſe E F, and between the ſame Parallels E F and A D (or of the ſame Height) are equal.

DEMON.

DEMONSTRATION.

THE Parallelograms ABEF, and ECDF, are equal (by the 13) but the Triangles EAF, ECF, are half thoſe Parallelograms, therefore if the Wholes were equal, their Halves muſt be ſo; (See the laſt Figure.)

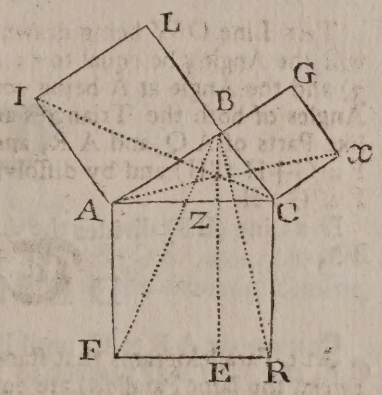
Theorem 15.

IN a Right angled Triangle A B C, the Square of the Hypothenuſe (or Side ſubtending the Right Angle) A C, is equal to both the Squares L A and G C, which are made upon the Sides A B and B C that contain the Right Angle.

DEMONSTRATION.

THE Angles I A B and F A C being Right Angles, and C A B Common, Then $\left.\begin{matrix} CAB + IAB = IAC \\ CAB + FAC = FAB \end{matrix}\right\}$ therefore I A C = F A B.

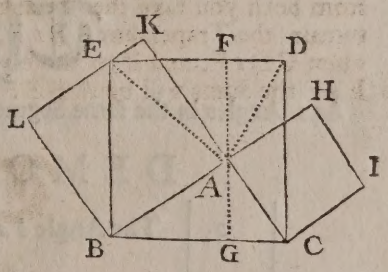
BUT the Sides A I and A C, alſo A B and A F about theſe two equal Angles are equal, therefore the Triangle I A C and F A B are equal. Now the Parallelogram A E = 2 F A B (by the 14) and the Parallelogram A L = 2 I A C, therefore the Parallelogram A L is equal to the Parallelogram A E, and in like manner the Parallelogram E C may be prov'd to be equal to the Parallelogram G C; therefore the Whole F C = A L + G C.



Another *Way to demonſtrate the Truth of the ſame* Theorem *from* Sturmius's Matheſis Enucleata.

IN the next Figure A I is the Square of the Side A C, A L, ſquare of the Side A B, and B D the Square of the Side B C, which he proves to be equal to the other two Squares A I and A L; thus,

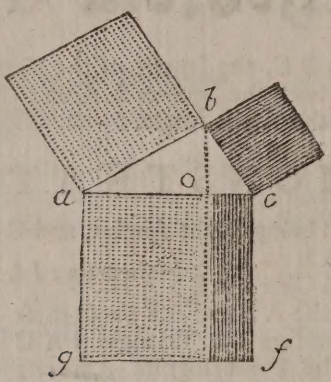
HAVING made the Squares, and drawn the pointed Lines, as in the Figure, you may ſee that the Triangle B A E is common either to the Square A L, or Parallelogram E F G B, and therefore it is the Half of either, conſequently they are equal, ſo the Triangle C A D being common to the Square A I, and Parallelogram F D G C, is half either of them; therefore they are equal, *&c.*



A Third Demonſtration of Theorem 15.

HAVING let fall the Perpendicular *b o*, it will divide the Triangle *a b c* into two ſimilar Triangles *a b o*, and *c b o*, which are alſo ſimilar to the given Triangle *a b c* (by the 11); therefore it will be

	1	$ac : ab :: ab : ao$
∴	2	$ac (= ag) \times ao = ab \square$
Again	3	$ac : bc :: bc : co$
∴	4	$ac (= cf) \times co = bc \square$
2 + 4	5	$ag \times ao + cf \times co = ba \square + bc \square$
But	6	$ag \times ao + cf \times co = ac \square$
5 = 6	7	$ac \square = ba \square + bc \square$ as by the two former.

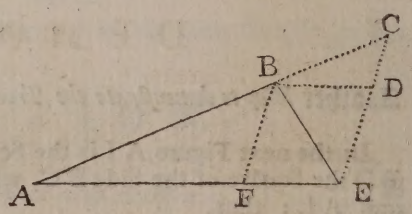


Theorem 16.

If a Line (B F) divides the Angle B into two equal Parts, it will be as F E : B E :: F A : B A.

DEMONSTRATION.

Continue A B to C, until B C = B E join the Points C E, and draw B D parallel to A E, then will the Triangles B C D and A B F be similar, consequently B D (= F E) : B C (= B E) :: F A : B A as above.

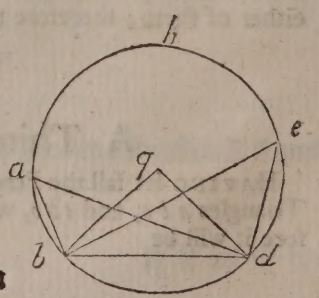


Theorem 17.

All Angles in the same Segment ($b\,h\,d$) are equal.

DEMONSTRATION.

	1	The Angle $b\,a\,d = \frac{b\,q\,d}{2}$
	2	$b\,e\,d = \frac{b\,q\,d}{2}$ (by the 9th.)
1 = 2	3	$b\,a\,d = b\,e\,d$ (by Axiom 1.)



Theorem

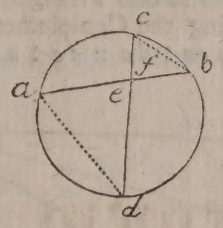
Theorem 18.

1. LINES in a Circle (*ab. dc*) cut themſelves proportionally.
2. AND their Parts make equal Rectangles, *viz.* $ae + be = ce + de$.

DEMONSTRATION.

JOIN the Points *c. b.* and *a. d.*

then	1	$e = f$ (by the 2.)
	2	$dab = dcb$ } (by the 17.) { Therefore the Triangles cbf and adf are ſimilar.
	3	$adc = abc$ }
	4	$ae : ce :: de : be$
4 ∴	5	$ae \times be = ce \times de$.



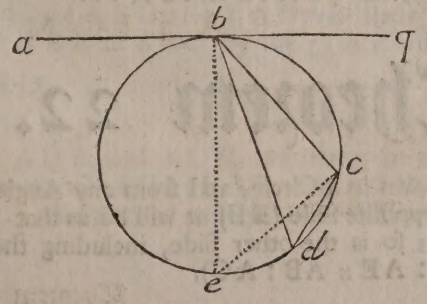
Theorem 19.

IF a Right Line (*aq*) touch a Circle, and from the Point of Contact (*b*) 2 Lines (*bdbc*) be drawn, they will make Angles with the Tangent (*aq*) equal to thoſe in the alternate Segment, that is, $abd = bcd$ and $cbq = cdb$.

DEMONSTRATION.

DRAW the Diameter *be*, and the Line *ce*, then

	1	$abe = bce$ (ſee the 10.) both being Right Angles.
	2	$ebd = ecd$, both ſtanding on the Arch *ed* (p. 17.)
1 + 2	3	$abe + ebd = bce + ecd$.
but	4	$abe + ebd = abd$.
and	5	$bce + ecd = = bcd$.
4 = 5	6	$abd = bcd$.
AGAIN,	7	$cbe + cbq = \text{L}$. } Hence $ceb = cbq$.
	8	$cbe + ceb = \text{L}$. }
BUT	9	$ceb = cdb$ (by the 17.) Therefore $cdb = cbq$.



Theorem

Theorem 20.

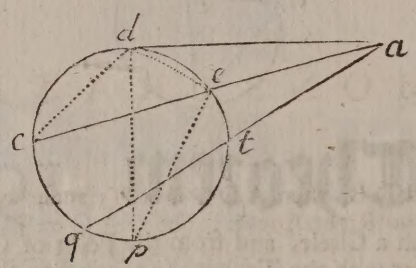
THE Rectangle of the whole Secant ca into its external Part ea, is equal to the Square of the Tangent da, drawn from the End of the Secant, *i.e.* $ca \times ae = ad \times ad = ad\,\square$.

DEMONSTRATION.

DRAW the Diameter dp, alſo the Lines dc, de, and pe, then will pda and ped be Right Angles, and cda and dea ſimilar Triangles; for the Angle dac is common to both, and the Angle $ecd = eda$. For

$pde + eda = \angle$ and $pde + epd = \angle$ } therefore $eda = dpe$. { But $dce = dpe$ (by the 17.) then $dce = eda$.

Now having prov'd that two Angles in one Triangle, are equal to two Angles in the other, the third in each muſt be equal, as being the Complement of the other two to 180 Degrees (or a Semicircle); ſo then it will be as $ca : da :: da : ae$. $\therefore ca \times ae = da \times da = da\,\square$.



Theorem 21.

IF two Right Lines ca and qa, be drawn from a Point (a) out of the Circle (ſee the laſt Figure) the Rectangle of one whole Line (ca) into its Part out of the Circle (ea) will be equal to the Rectangle of the other whole Line (qa) into its Part (ta) out of the Circle, that is $ca \times ea = qa \times ta$. Then it may be as $ca : ta :: qa : ae$.

DEMONSTRATION.

IT was demonſtrated by the laſt *Theorem*, that $ca \times ea = da\,\square$, it might be demonſtrated in like manner that $qa \times ta = da\,\square$; But Things that are equal to a third, are equal to one another. Hence it follows, that $qa \times ta = ca \times ea$.

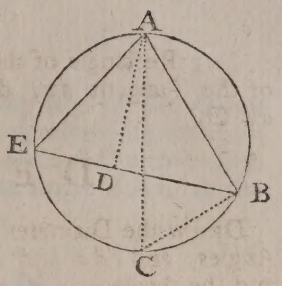
Theorem 22.

IF a Triangle (ABE) be drawn in a Circle, and from any Angle thereof you draw a Line (AD) at Right Angles to the oppoſite Side (EB) it will be as that Perpendicular is to one of the Sides, including the Angle; ſo is the other Side, including the ſame Angle, to the Diameter of the Circle (*i.e.* AD : AE :: AB : AC).

DEMON-

DEMONSTRATION.

DRAW the Diameter A C, and join BC, ſo you have two Right angled Triangles A D E, and A B C, which are alſo ſimilar, becauſe the Angles A E D, and A C B are equal (by the 17) conſequently E A D and C A B, muſt be ſo; hence it will be A D : A E :: A B: A C.

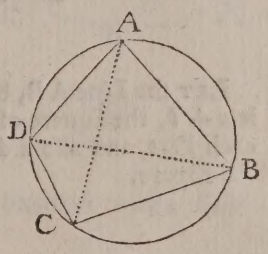


Theorem 23.

THE oppoſite Angles (A C, D B) of any four ſided Figure inſcrib'd in a Circle, are equal to two Right Angles.

DEMONSTRATION.

DRAW the Lines A C and D B, then A D C is a Triangle whoſe three Angles are equal to two Right Angles; but D A C = D B C, and D C A = D B A; therefore A D C + D B A + D B C = 2 L.



Theorem 24.

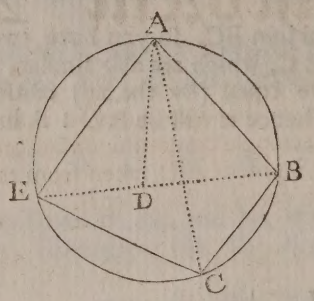
IF any four ſided Figure (A B C E) be inſcrib'd in a Circle, the Rectangle of the Diagonals (E B × A C) is equal to the two Rectangles of the oppoſite Sides (*i. e.* of B C × A E + C E × A B.)

DEMONSTRATION.

MAKE the Angle E A D equal to the Angle B A C (by *Prob.* 8) and it will make the Triangle A D E like to the Triangle A B C; for E A D was made equal to B A C, and A E D = A C B (by 17); therefore A D E = A B C (by the 5) ſo it will be A C : B C :: A E : E D, therefore $\frac{BC \times AE}{AC} = ED$.

AGAIN, the Triangles A D B, and A C E, are like; for the Angle E A D = B A C, and C A D being common, therefore E A C = D A B, alſo A C E = A B E (by the 17) and conſequently A D B = A E C; therefore it will be A C : C E :: A B : B D. *Ergo* $\frac{CE \times AB}{AC} = BD$.

BD. But ED + DB = EB, hence BC × AE + CE × AB = EB × AC; which was to be demonſtrated.



Theorem 25.

If a Right Line AB be divided into two Parts, the Square of the whole Line is equal to the Square of both Parts, and alſo a double Rectangle of the Parts (*i.e.* AB □ = $aa + bb + 2ab$)

DEMONSTRATION.

Let the Line AB, be divided into two Parts, *viz.* *a* and *b*, whoſe Sum is $a + b$, the Square of which is $aa + bb + 2ab$ that is the Square of each Part, and alſo a double Rectangle of the Parts, as you may ſee by the Figure.

A *a*	*b* *B*
ab	*bb*
aa	*ab*

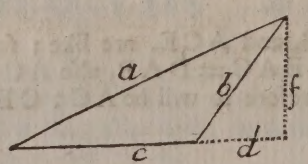
Theorem 26.

In an obtuſe angled Triangle (*abc*) the Square of the Subtendent of the obtuſe Angle is more than the Squares of the other two Sides by $2cd$.

DEMONSTRATION.

$aa = bb + cc + 2cd$ by the *Theorem*.

	1	$aa = ff + cc + dd + 2cd$ (by the 25)
	2	$bb = ff + dd$
1 − 2	3	$aa - bb = cc + 2cd$
3 + bb	4	$aa = cc + bb + 2cd$; which was to be demonſtrated.



Theor.

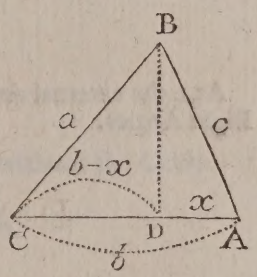
Theorem 27.

In an acute angled Triangle the Square of any Side (B C) ſubtending any of the Angles (as A) is equal to the Squares of the other two Sides (A B and A C) comprehending the acute Angle B A C. Leſs by a double Rectangle contain'd under one of the Sides A C upon which the Perpendicular B D falls, and a Segment A D taken from the ſaid acute Angle B A C to the Perpendicular B D let fall from the vertical Angle at B.

DEMONSTRATION.

$aa = cc + bb - 2bx$ by the *Theorem*.

	1	$cc - xx =$ BD □
	2	$aa - bb + 2bx - xx =$ BD □
$1 = 2$	3	$aa - bb + 2bx - xx = cc - xx$
$3 + x^2$	4	$aa - bb + 2bx = cc$
4 ±	5	$aa = cc + bb - 2bx$

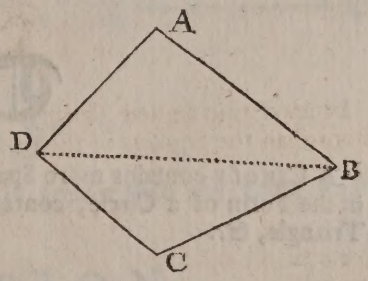


Theorem 28.

In every Quadrangular (or four ſided Figure) the four Angles taken together make four Right Angles.

DEMONSTRATION.

Let the four ſided Figure be A B C D, draw D B, which ſhall divide the Figure into two Triangles, each being two Right Angles (by the 5) then both has four.

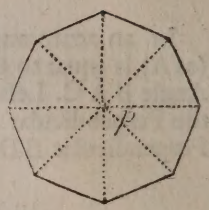


Theorem 29.

All the Angles of every Right lin'd Figure taken together make twice ſo many Right Angles abating four, as the Figure has Sides.

D E-

DEMONSTRATION.

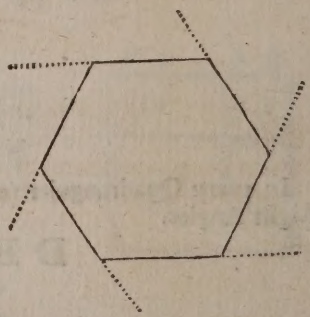


LET the Figure be that next following, consisting of eight Sides: Then having drawn Lines from any Point (as *p*) to each Corner, by them you will divide it into eight Triangles, each of which has two Right Angles (by *Theorem* 5.) that is 16 in all: But the Angles about *p* are four, which take from 16, and there will remain 12, equal to the Right Angles made at the Perifery of the Figure, agreeing with twice the Number of Sides, wanting four.

Theorem 30.

ALL the external Angles of any Right lin'd Figure whatsoever taken together, do make four Right Angles.

DEMONSTRATION.



IN the following Figure each internal and its external Angle make two Right Angles (by *Theorem* 1). that is (in this Case) 12 in all: But by *Theorem* 29, the internal Angles are only Eight, therefore all the external Angles are but four.

Theorem 31.

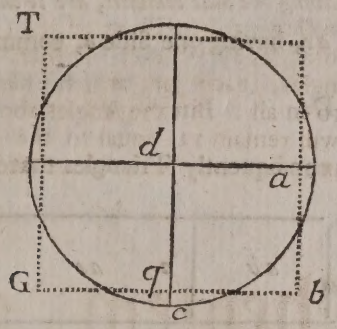
A CIRCLE contains more Space than any Figure of equal Compass (*i. e.*) a Line, when put in the Form of a Circle, contains more Space than when put in the Form of a Square, Triangle, *&c.*

DEMONSTRATION.

Let	1	$ab = c = dq$
1×8	2	$8c =$ the Circumference of the Square, or Circle.
$2 \div 4$	3	$2c =$ F G the Side of the Square.
$3 \odot 2$	4	$4cc =$ the Area of the Square.
Let	5	$qc = e$.
then	6	$dc = c + e$ is half the Diameter of the Circle.
$2 \div 2$	7	$4c =$ half the Circumference.

6+7

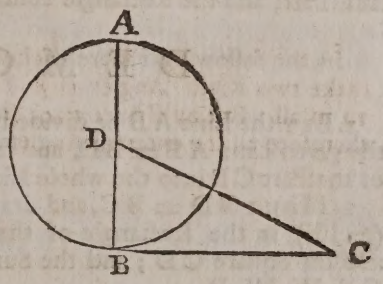
6 + 7 | 8 | $4cc + 4ce$ = the Area of the Circle.
8 − 4 | 9 | $4cc + 4ce - 4cc = 4ce$. Hence it appears, that the Area of the Circle exceeds the Area of the Square by $4ce$. W. W. D.



Theorem 32.

A Perpendicular B C to the End of the Diameter B A falls entirely without the Circle.

DEMONSTRATION.

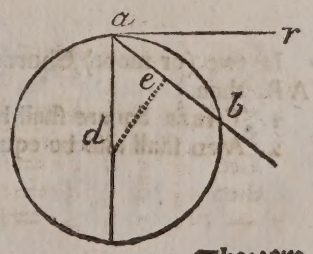


Let B C be the Perpendicular; from the Center D, draw the Secant D C, the Angle D B C is a Right Angle, and therefore greater than the Angle at C, therefore the Subtendent D C is greater than D B (by the 7th); and consequently the Point C is without the Circle. The same Reason will hold good for all Points between C and B. W. W. D.

Theorem 33.

The Angle (*d a b*) of the Radius (*d a*) and Circumference, is greater than any acute Angle (*d a c*); therefore it is a Right one.

DEMONSTRATION.



Draw *d e* perpendicular to *a b*, then the Angle *d e a* is a Right Angle, and so greater than *d a e*; therefore its Subtendent *d a*, is greater than *d e*, so that *d e* must fall within the Circle; and by Consequence the Angle *d a e* is but a Part of *d a b*, and so less than it. And the same way of Reasoning will hold quite up to the Tangent *a r*. W. W. D.

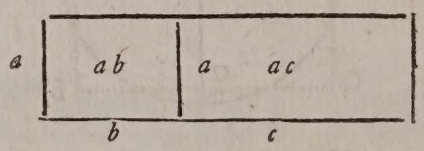
Theorem 34.

Rectangles or Products, having one Side common, are to one another as the other Sides.

LET the Products be ab, and ac, having one Side a, common.

Then | 1 | $b : c :: ab : ac$
∴ | 2 | $bac = bac$, W. W. D.

HENCE Parallelograms, and consequently Triangles that have the same Height, are to one another as their Bases.



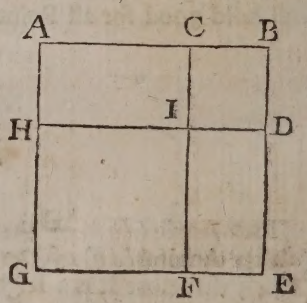
Theorem 35.

IF a Line A B be divided any how into two Parts, as A C and C B, the Rectangle contain'd under the whole Line A B, and one of its Parts (suppose C B) is equal to the Square of the same Part, and the Rectangle contain'd under both Parts.

DEMONSTRATION.

1. LET the Line A B be divided into two Parts in C, make a Square A E, whose Side shall be the given Line A B = B E, and from C let fall the Perpendicular C F ; so B F is the Rectangle of the Part C B into the whole Line B E.

2. MAKE B D = B C, and draw D H parallel to A B, then C H (= I E) is the Rectangle of the Parts A C, and C I, to which add the Square C D ; and the Sum will be the Rectangle A D = C E. W. W. D.



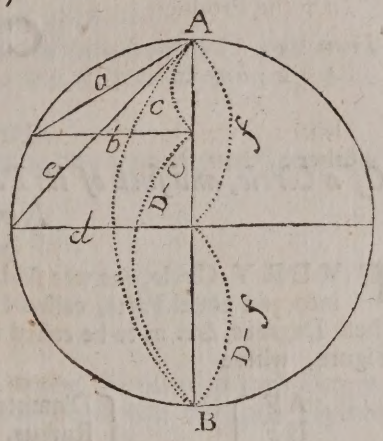
Theorem 36.

IF two (or more) Chords, as a, and e, issue from the same End of any Diameter of a Circle A B, then

1. THEIR Square shall be directly as the versed Sines (c and f.)
2. AND shall also be equal to the Rectangles under the Diameter, and such versed Sines.

	1	$aa = bb + cc$ (by 15.)
$1 - cc$	2	$bb = aa - cc$
	3	$ee = ff + dd$
$3 - ff$	4	$dd = ee - ff$
	5	$bb = D - c \times c = Dc - cc$ (by 11.)
and	6	$dd = D - f \times f = Df - ff$.
2 and 5 by Subst.	7	$Dc - cc = aa - cc$
$7 + cc$	8	$Dc = aa$
4 and 6 by Subst.	9	$ee - ff = Df - ff$.
$9 + ff$	10	$Df = ee$
8 and 10	11	$c : f :: aa : ee$. (by 34.)

The second Part is prov'd by the 8th and 10th Steps.



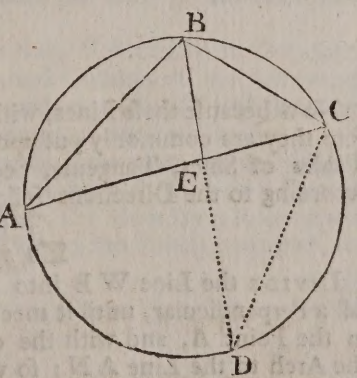
Theorem 37.

If any Right Line B E biſſect any Angle, (as A B C) of the Triangle A B C, I ſay the Square of the ſaid Line B E ſhall be equal to A B × B C − A E × E C.) See *Newton's Univerſal Arithmetick*, Page 90th.)

DEMONSTRATION.

Having deſcrib'd a Circle about the Triangle, and continued out the Line B E till it cuts the Circle in D, and drawn the Line D C, the Triangle A B E and B C D will be ſimilar, becauſe equiangled; it will therefore be,

	1	A B : B E :: D B : B C
1 ∴	2	A B × B C = B E × D B.
Again,	3	A E × C E = B E × D E (by *Theor.* 18.)
and	4	B D × B E = BE^2 + B E × D E (by 35.)
2 and 4 by Subst.	5	A B × B C = BE^2 + B E × D E.
3 and 5 by Subst.	6	A B × B C = BE^2 + A E × C E
6 − A E × C E	7	BE^2 = A B × B C − AE×CE. W.W.D.



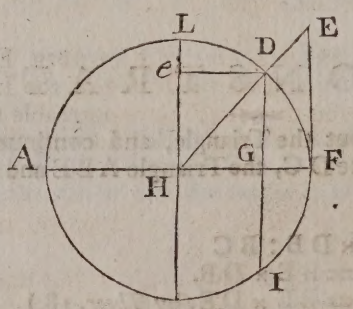
CHAP.

CHAP. IV.

Of a Circle, and ſuch of its Parts, as are of moſt Uſe in Trigonometry, Aſtronomy, Navigation, Dialling, &c.

EVERY Circle, (as was ſaid before) of what Radius ſoever, is ſuppos'd to be divided into 360 equal Parts, called Degrees, *&c.* And there are ſeveral Right Lines ſo related to theſe Degrees, *&c.* as to be called their Sines, Tangents, Secants, or verſed Sines, as in the next Figure, where

A F	is the	Diameter.
H F		Radius, or Semidiameter.
D I		Chord of the Arch D F I.
D G		Sine of the Arch D F.
D *e*		Sine Complement of the Arch D F, or Sine of the Arch L D.
H E		Secant of the Arch D F.
F E		Tangent of the Arch D F.
G F		Verſed Sine of the ſame.



Now becauſe theſe Lines, with ſeveral others hereafter mention'd, are of frequent Uſe, therefore they are commonly put upon Scales, and divided either by a Line of equal Parts, and the Tables of Sines, Tangents, Secants, *&c.* or from a Circle actually divided into 360 Degrees, according to the Directions following.

To *make the Line of* Longitudes A N.

DIVIDE the Line W B into ſix equal Parts, 1, 2, 3, 4, 5, B, and from each Point let fall a Perpendicular, until it meets the Quadrant A B; then place one foot of your Compaſſes in the Point A, and with the other transfer the Points where thoſe Perpendiculars interſect the Arch to the Line A N; ſo will you divide it into its primary Parts; but for the Minutes you muſt divide each ſixth Part of the Line W B into ten other equal Parts, and then transfer them to A N, as you did the primary Diviſions.

To *make the Line of* Rhombs A B.

DIVIDE the Quadrant A B into eight equal Parts, and each of theſe into as many equal Parts as you pleaſe, (four is enough) then ſet one foot of your Compaſſes in the Point A, and with the other transfer thoſe Diviſions from the Quadrant to the Line A B, and ſet Figures to it, as in the Diagram.

To make *the Line of* half Tangents C H.

LAY a Ruler upon the Point A, and over every Degree of the Quadrant B E, and where it interſects the Line C H, make Marks agreeable to thoſe in the Arch or Quadrant and 'tis done.

To make *the Line of* Chords B E.

DIVIDE the Quadrant B E into 90 equal Parts, then ſetting one foot of your Compaſſes in the Point E, transfer thoſe Diviſions to the Line B E.

To make *the Line of* Tangents E F.

LAY a Ruler upon the Center C, and upon every Degree of the Arch B E, and where it interſects the Line E F make Marks, and ſet Figures to them, as in the Scheme.

To make *the Line of* Secants C F.

SET one foot of your Compaſſes in the Center at C, and extend the other to each Degree of the Tangent Line, which transfer to the Line C F, and figure it with the ſame Figures that the Tangent Line is mark'd with.

To make *the Line of* Sines C D.

DIVIDE the Quadrant D E into 90 equal Parts or Degrees, beginning at E, and ending with 90 at D, and alſo the Quadrant A D into 90 equal Parts, beginning at A, and ending at D as before. Then, if you lay a Ruler at 10 in the Quadrant E D, and alſo on 10 in the Quadrant A D, it will cut the Line C D in a Point, which call the Sine of 10 Degrees; and ſo you may divide it into all its other Parts.

To make *the Line of* Latitudes E D.

LAY a Ruler upon the Point A, and every Degree of the Line of Sines C D, and where it interſects the Quadrant D E, you may ſet Figures agreeable to thoſe in the Line of Sines; then ſet one foot of your Compaſſes in the Point E, and with the other transfer thoſe Diviſions from the Arch to the Line D E, and figure it as in the Scheme.

To *make an Hour Scale, or Line qr, to be us'd in Dialling, with the Line of* Latitudes *above deſcrib'd.*

THE Line of Tangents F E being already made to the Radius, C E = D q = D r, take thence in your Compaſſes the Degrees and Minutes, as the ſecond Column of the following Table directs, and lay them ſeverally from D, both Ways, towards q, and r; ſo will the Line be divided into the Times expreſs'd in the firſt and third Columns of the ſaid Table.

N. B. THAT if you have not a Tangent Line already made, you may ſet the ſeveral Degrees and Minutes by Help of your Line of Chords, as you have them in the ſecond Column, and lay them upon the Quadrants D A, and D E from D; then lay a Ruler upon the Center at C, and thoſe Points, and it will cut the Hour-Line in the Points required, as you ſee it mark'd or figur'd.

A TABLE

A TABLE *to ſhew what Number of Degrees and Minutes, muſt be ſet off from* D, *both Ways, towards* A *and* E, *to make the Hour Line qr.*

Time.			*Deg. Min.*			*Time.*		
h.		*gr.*				*h.*		*gr.*
3	:	0	00	:	00	3	:	00
2	:	3	03	:	45	3	:	01
2	:	2	07	:	30	3	:	02
2	:	1	11	:	15	3	:	3
2	:	0	15	:	00	4	:	0
1	:	3	18	:	45	4	:	1
1	:	2	22	:	30	4	:	2
1	:	1	26	:	15	4	:	3
1	:	0	30	:	00	5	:	0
0	:	3	33	:	45	5	:	1
0	:	2	37	:	30	5	:	2
0	:	1	41	:	15	5	:	3
0	:	0	45	:	00	6	:	0

To make the Hour-line A D, *to divide a Circle into* 24 *equal Parts, us'd when a Dial is to be made by the Line of Contingence*, &c.

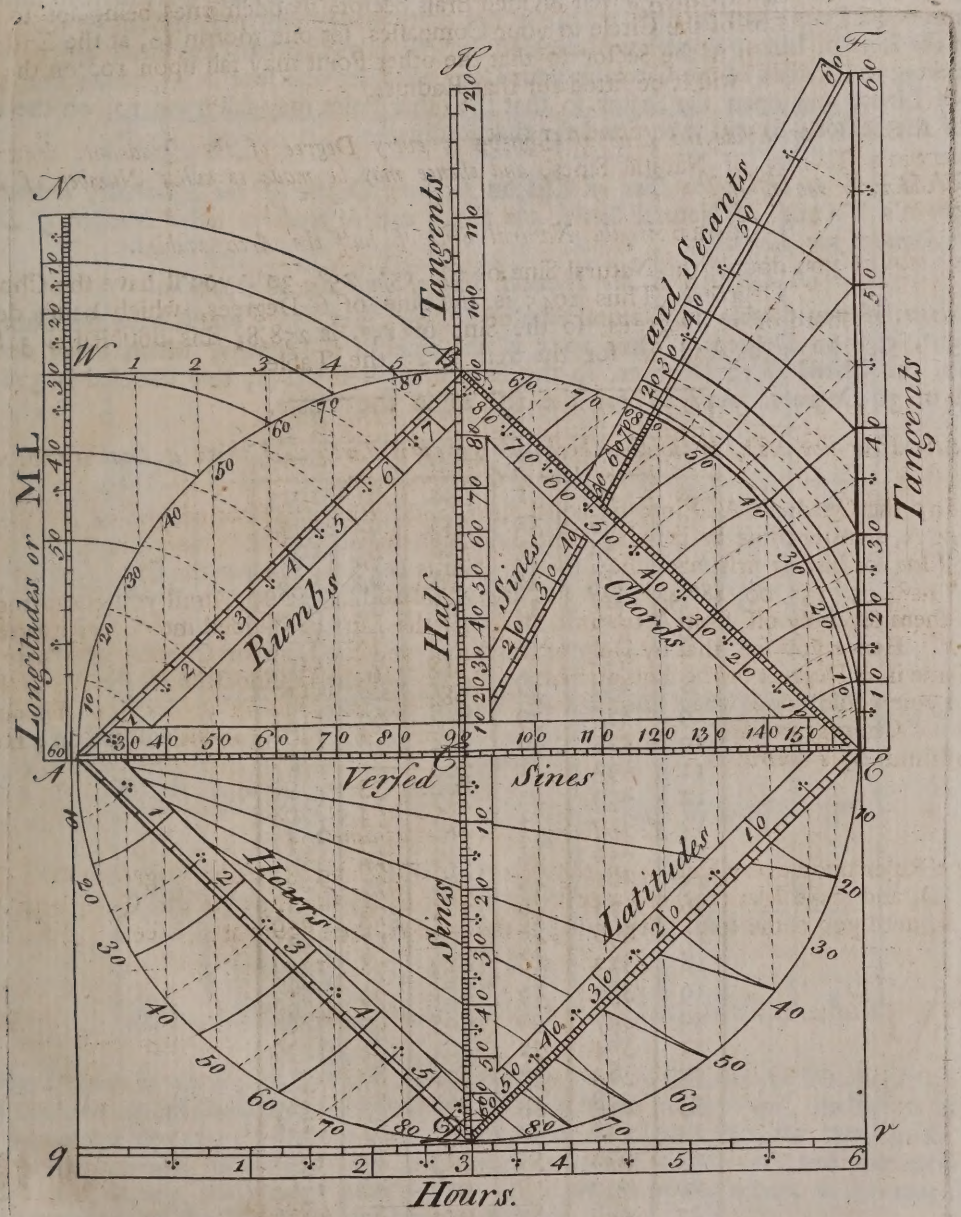
DIVIDE the Quadrant A D into ſix equal Parts, and ſetting one Foot of your Compaſſes in the Point A, transfer thoſe Parts to the Hour-line A D, ſo will you have the primary Diviſions of that Line, or having firſt made the Line of Chords B E, take thence the Chord of 3° 45′ 7° 30′ 11° 15′ 15° 00′ 18° 45′ 22° 30′, *&c.* Still adding 3° 45′, until you come to 90°, and ſet them ſeverally off from A towards D, ſo will the Line be divided into every quarter of an Hour. Hence you ſee, that by Help of this Line, and the Line of Chords, you may reduce Time into Degrees of the Equinoctial, or Degrees of the Equinoctal into Time; as thus: Suppoſe you would know what Time anſwers to 37° 30′, take this in your Compaſſes out of the Line of Chords, and apply that Extent to the Hour-line from A, and it will fall on 2 Hours and 30 Minutes, *e contra.*

To make the Line of verſed Sines.

LAY a Ruler upon every Degree of the Quadrant E B, and the ſame Degrees of the Quadrant E D, and make Marks where it cuts the Line A E, do the ſame by the Quadrants A B, and A D, until you come to the Beginning of the Line A, then figure as you ſee.

The

The Diagram.



THE

THE foregoing Method of making the Line of Longitudes, Rhombs, *&c.* (which we did begin with, becauſe they are the firſt in Order in the Diagram) depend originally upon the Circles being truly divided into 360 equal Parts, or Degrees; but this being ſo hard a Task, that there are few who have attempted it that have finiſh'd, with Satisfaction. And if that was exactly done, yet the Geometrical Way there propos'd is not ſufficiently exact, tho' it does make what is intended plainer to a Learner than by Tables: But that he may have his Choice, I will inſert ſuch Tables as may be moſt convenient for his Purpoſe, and the Uſe of them in the eaſieſt Manner that we can. Now before we can begin this, we muſt have a Line of equal Parts in Length to the Radius of the Circle (ſuppoſe C E) and truly divided into 100, or 1000 equal Parts; but as ſuch a Line is not eaſily made, you may inſtead of that, make uſe of the Line of Lines, upon a well-divided Braſs Sector (wooden ones being apt to warp); then taking the Radius of the Circle in your Compaſſes, ſet one foot in 10, at the End of the Line of Lines, and open the Sector ſo that the other Point may fall upon 10, on the other Leg of the Sector; ſo will it be fitted for that Radius.

A TABLE *to divide the Line of* Chords *to every Degree of the Quadrant, drawn from* Sherwin's TABLE of Natural Sines, *and thence may be made to what Number of Degrees and Minutes you pleaſe.*

A CHORD *is the Natural Sine of half the Arch doubled.*

EXAMP. if you double the Natural Sine of 6°. 15°. 25°. 30°. you'll have the Chords of 12°. 30°. 50°. 60 Degrees. Thus 1045 is the Sine of 6 Degrees, which being doubled is 2090, the Chord of 12 Degrees, ſo the Sine of 15°. is 258,8, this doubled is 518, the Chord of 30 Degrees. And ſo for the reſt as in the Table.

Deg.	*Chords*	*Deg.*	*Chords*	*Deg.*	*Chords*
1	17	31	534	61	1015
2	35	32	551	62	1030
3	52	33	568	63	1045
4	70	34	585	64	1060
5	87	35	601	65	1074
6	105	36	618	66	1089
7	122	37	635	67	1104
8	139	38	651	68	1118
9	157	39	668	69	1133
10	175	40	684	70	1147
11	192	41	700	71	1161
12	209	42	717	72	1176
13	226	43	733	73	1190
14	244	44	749	74	1204
15	261	45	765	75	1217
16	278	46	781	76	1231
17	296	47	797	77	1245
18	313	48	813	78	1259
19	330	49	831	79	1273
20	347	50	845	80	1286
21	364	51	861	81	1299
22	382	52	876	82	1312
23	398	53	892	83	1325
24	416	54	908	84	1338
25	432	55	923	85	1351
26	450	56	939	86	1364
27	466	57	954	87	1377
28	484	58	970	88	1389
29	501	59	984	89	1402
30	518	60	1000	90	1414

THE

THE Table being finiſhed, draw the Line B E in the Diagram for the Line of Chords, and having divided a Line equal in Length to the Radius C E into 1000 equal Parts, or inſtead thereof take the Radius C E in your Compaſſes, and open the Sector to that Extent as before directed; then ſuppoſe you would lay on 30 Degrees upon your Line of Chords, the Number in the Table againſt 30 Degrees is 518; take this in your Compaſſes from the Sector, and lay it down upon the Line of Chords from E to 30. Again, Suppoſe you would put on 45 Degrees, the Number in the Table againſt 45 is 765; take this in your Compaſſes from the Sector, and lay off that Extent upon the Line B E, from E to 45. And ſo for the reſt.

A TABLE *for the Angles which every Rhomb makes with the Meridian, and the Chords of every Quarter of the Compaſs.*

North.	*South.*	*Deg.*	*Min.*	*Sec.*	*Chords.*	*South.*	*North.*	*Parts.*
		2	48	45	49			
		5	37	30	98			
		8	26	15	147			
N *b* E	S *b* E	11	15	00	195	S *b* W	N *b* W	1
		14	03	45	244			
		16	52	30	293			
		19	41	15	333			
NNE	SSE	22	30	00	390	SSW	NNW	2
		25	18	45	427			
		28	07	30	485			
		30	56	15	533			
NE *b* N	SE *b* S	33	45	00	580			
		36	33	45	627	SW *b* S	NW *b* N	3
		39	22	30	673			
		42	11	15	720			
NE	SE	45	00	00	765			
		47	48	45	811	SW	NW	4
		50	37	30	855			
		53	26	15	899			
NE *b* E	SE *b* E	56	15	00	942			
		59	03	45	985	SW *b* W	NW *b* W	5
		61	52	30	1028			
		64	41	15	1069			
ENE	E. S. E.	67	30	00	1111			
		70	18	45	1151	WSW	WNW	6
		73	07	30	1190			
		75	56	15	1230			
E *b* N	E *b* S	78	45	00	1268			
		81	33	45	1305	W *b* S	W *b* N	7
		84	22	30	1343			
		87	11	15	1378			
East	East	90	00	00	1414	West	West	8

THE Chords in the fourth Column are double the Sines of half the Arches that ſtand in the third Column againſt them thus: Suppoſe I want the Chord of 22° : 30′, its half is 11° : 15′, and the natural Sine of 11° : 15′ : is 195, which being doubled is 390, for the Chord of 22° 30′. So if you would have the Chord of 90 Degrees its half is 45° : 00′, and the natural Sine of 45° : 00′, is 707, the Double of which is 1414 : And ſo of any other, as you may ſee was done in compoſing the Table for the Line of Chords.

Now to make the Line of Rhombs A B, (in the Diagram.)

1. TAKE the Radius A C in your Compaſſes, and ſetting one Foot in 10, at the End of the Line of Lines on one Leg of the Sector, open the other ſo wide that the other Foot of the Compaſſes may fall in 10 at the End of that.

2. THEN, ſuppoſe you would ſet the firſt Point upon the Line of Rumbs, take the parallel Extent of 195 in your Compaſſes from the Line of Lines, and ſet that A to 1: Again, if you would put on the third Point (the Sector remaining as before) take the parallel Extent of 580 out of the Line of Lines, and ſet that from A to 3, upon the Line of Rumbs: And ſo on with the other Points and Quarters.

A TABLE *to divide the Line of Longitudes* A N.

Min.	Chords.		Min.	Chords.		Min.	Chords.	
	°	′		°	′		°	′
1	89	02	21	69	30	41	46	54
2	88	06	22	68	29	42	45	35
3	87	08	23	67	28	43	44	12
4	86	11	24	66	25	44	42	49
5	85	14	25	65	23	45	41	26
6	84	16	26	64	20	46	40	00
7	83	18	27	63	15	47	38	28
8	82	20	28	62	11	48	36	54
9	81	22	29	61	06	49	35	16
10	80	24	30	60	00	50	33	34
11	79	26	31	58	53	51	31	45
12	78	27	32	57	46	52	29	58
13	77	29	33	56	38	53	27	59
14	76	30	34	55	29	54	25	53
15	75	31	35	54	18	55	23	33
16	74	31	36	53	09	56	21	06
17	73	32	37	51	56	57	18	13
18	72	32	38	50	02	58	14	50
19	71	32	39	49	26	59	10	26
20	70	31	40	48	11	60	00	00

THIS Line is divided by help of the Line of Chords: Thus, ſuppoſe I would put 10 Minutes upon the Line of Longitudes, I look in the Table for 10′ in the Column of Minutes, and againſt that I find a Chord of 80° : 24′ which I take in my Compaſſes out of the Line of Chords B (in the Diagram) and then ſetting one Foot in the Point A, I lay the other towards N, and it will reach to a Point in that Line which I muſt mark with 10′.

AGAIN, to put on 30′, I look in the Column of Minutes 30, and againſt that I find 60° 00′, which I take in my Compaſſes out of the Line of Chords, and lay that Extent from A towards N, and it will reach to a Point, which muſt be number'd with 30′, *&c.*

THIS Line of Longitude is to find how many Minutes of the Equator go to make one Degree of Longitude in any Latitude. Thus, ſuppoſe I would know how many Minutes of the Equator will go to make one Degree of Longitude in the Latitude of 50 Degrees; take in your Compaſſes 50 Degrees out of the Line of Chords, and that Extent being laid off from 60 in the Line of Longitudes, will reach to 38 Minutes, and a little above an half for the Anſwer.

A TABLE

A TABLE *to divide the Line of Latitudes* DE.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.	Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
1	247	16	3758	31	6475	46	8259	61	9311	80	9924
2	493	17	3969	32	6622	47	8348	62	9360	85	9982
3	739	18	4176	33	6764	48	8436	63	9408	90	10000
4	984	19	4378	34	6900	49	8519	64	9454		
5	1228	20	4577	35	7036	50	8600	65	9496		
6	1470	21	4772	36	7186	51	8678	66	9539		
7	1711	22	4961	37	7292	52	8753	67	9578		
8	1949	23	5146	38	7414	53	8825	68	9615		
9	2186	24	5328	39	7532	54	8895	69	9651		
10	2419	25	5505	40	7647	55	8962	70	9685		
11	2650	26	5678	41	7758	56	9016	72	9745		
12	2879	27	5846	42	7865	57	9881	74	9801		
13	3109	28	6010	43	7968	58	9147	75	9825		
14	3325	29	6169	44	8068	59	9203	76	9846		
15	3543	30	6325	45	8165	60	9258	78	9888		

TAKE the Line D E in your Compaſſes, and ſetting one Foot in 10, on the Line of Lines upon one Leg of the Sector, open the other ſo that the other Point of the Compaſſes may fall in 10 at the End of that, ſo is the Sector fitted for dividing the Line: Thus, ſuppoſe you would ſet on 30 Degrees; look 30 in the Table, and againſt that you will find 6325, which take in your Compaſſes from the Sector, and lay that from E to 30, and ſo for the reſt.

THIS Table was calculated thus,

As Radius is to the Chord of 90 Degrees (equal to D E in the Diameter); ſo are the Tangents of the reſpective Degrees in the Table to the Tangents of other Arches; then the natural Sines of theſe Arches are the Numbers, or Parts to be ſet againſt the Degrees, as in the Table.

EXAMPLE, Let it be required to calculate for the Diviſion of 15 Degrees.

To the Sine 45 Degrees ——— 9,8494850
ADD the Logarithm of 2 ——— 0,3010300

THE Sum is the Chord of 90 Degrees ——— 10,1505150

THEN, as Radius is to the Chord of 90 Degrees ——— 10,1505150
So is the Tangent of 15 Degrees ——— 9,4280525

To the Tangent of 20° : 45′ ——— 9,5755675

AND the natural Sine of 20° : 45′ is 3543, as in the Table againſt 15 Degrees; and ſo for all the reſt.

A TABLE *to divide the Line of natural Sines* CD.

Deg.	Sines	Deg.	Sines	Deg.	Sines	Deg.	Sines	Deg.	Sines	D.g.	Sines
1	17	16	276	31	515	46	719	61	875	76	970
2	35	17	292	32	530	47	731	62	883	77	974
3	52	18	309	33	545	48	743	63	891	78	978
4	70	19	326	34	559	49	755	64	899	79	982
5	87	20	342	35	574	50	766	65	906	80	985
6	105	21	358	36	588	51	777	66	914	81	988
7	122	22	375	37	602	52	788	67	921	82	990
8	139	23	391	38	616	53	799	68	927	83	993
9	156	24	407	39	629	54	809	69	934	84	995
10	174	25	423	40	643	55	819	70	940	85	996
11	191	26	438	41	656	56	829	71	946	86	997
12	208	27	454	42	669	57	839	72	951	87	999
13	225	28	469	43	682	58	848	73	956	88	999
14	242	29	485	44	695	59	857	74	961	89	1000
15	259	30	500	45	707	60	866	75	966	90	1000

TAKE the Line CD in your Compaſſes, and open the Sector ſo wide that 10 and 10, at the End of the Line of Lines upon each Leg may anſwer to this Extent, then ſuppoſe you would ſet 10 Degrees upon the Line of Sines, look for 10 Degrees in the firſt Column of the Table, and againſt that you will find 174, take this in your Compaſſes out of the Line of Lines, and ſetting one Foot in C, turn the other towards D, and it will fall upon a Point at which you muſt ſet 10, for the Sine of 10 Degrees.

AGAIN, for the Sine of 45 Degrees, find this in the fifth Column, and againſt that in the ſixth you have 707, which take in your Compaſſes out of the Line of Lines, and ſet that Extent from C towards D, ſo will it fall upon a Point, at which ſet 45, for the Sine of 45 Degrees, *&c.*

A TABLE *to divide the Line of verſed Sines* A E.

Deg.	Ver. Sines	Deg.	Ver. Sines	Deg.	Ver Sines	Deg.	Ver. Sines	Deg.	Ver. Sines	Deg.	Ver. Sines
1	1	16	387	31	1428	46	3053	61	5152	76	7581
2	6	17	437	32	1519	47	3280	62	5305	77	7750
3	14	18	489	33	1613	48	3308	63	5461	78	7921
4	24	19	545	34	1709	49	3439	64	5616	79	8092
5	38	20	603	35	1808	50	3572	65	5774	80	8263
6	55	21	664	36	1910	51	3707	66	5932	81	8436
7	74	22	728	37	2013	52	3843	67	6092	82	8608
8	97	23	795	38	2120	53	3982	68	6254	83	8781
9	123	24	864	39	2228	54	4122	69	6416	84	8955
10	152	25	937	40	2339	55	4264	70	6580	85	9128
11	184	26	1012	41	2453	56	4408	71	6744	86	9302
12	218	27	1090	42	2568	57	4553	72	6910	87	9476
13	256	28	1170	43	2686	58	4701	73	7076	88	9651
14	297	29	1254	44	2806	59	4849	74	7244	89	9825
15	341	30	1340	45	2920	60	5000	75	7412	90	10000

TAKE the Radius A C, or E C in your Compaſſes, and open the Sector ſo that 10 and 10 at the End of the Lines upon each Leg may anſwer to this Extent, then ſuppoſe you would ſet 30 Degrees upon this Line againſt 30 Degrees in the third Column, you will find 1340 in the fourth; take this in your Compaſſes out of the Line of Lines, and ſetting one Foot in A, turn the other towards C, and it will fall on a Point in the Line A C, at which ſet 30 for the verſed Sine of 30 Degrees, and with the ſame Extent ſet one Foot in E, and turn the other towards C, ſo will it reach to a Point at which ſet 150 for the verſed Sine of 150 Degrees.

AGAIN, if you would ſet on 70 Degrees, look for 70 Degrees in the Table, and againſt that you have 6580, which take in your Compaſſes out of the Line of Lines (the Sector remaining as before) and ſet that Extent from A towards C, and it will fall upon a Point, where ſet 70 Degrees; with the ſame Extent ſet one Foot in E, then turn the other towards C, and it will fall on a Point in the Line E C, at which ſet 110; where note, that the Degrees on E C are the Complements of thoſe on the Line A C to 180 Degrees.

A TABLE

A TABLE *to divide the Line of Secants* CF.

Deg.	Secants.	Deg.	Secants.	Deg.	Secants.	Deg.	Secants.	Deg.	Secants.
1	1000	19	1058	37	1252	55	1743	73	3420
2	1001	20	1064	38	1269	56	1788	74	3628
3	1001	21	1071	39	1287	57	1836	75	3864
4	1002	22	1078	40	1305	58	1887	76	4133
5	1004	23	1086	41	1325	59	1942	77	4445
6	1005	24	1095	42	1346	60	2000	78	4810
7	1007	25	1103	43	1367	61	2063	79	5241
8	1010	26	1113	44	1390	62	2130	80	5759
9	1012	27	1123	45	1414	63	2203	81	6392
10	1015	28	1132	46	1439	64	2281	82	7185
11	1019	29	1143	47	1466	65	2366	83	8205
12	1022	30	1155	48	1494	66	2458	84	9567
13	1026	31	1167	49	1524	67	2559	85	11474
14	1031	32	1179	50	1556	68	2669	86	14335
15	1035	33	1192	51	1589	69	2790	87	19107
16	1040	34	1206	52	1624	70	2924	88	28654
17	1046	35	1221	53	1662	71	3071	89	57299
18	1051	36	1236	54	1701	72	3236	90	Infinite

HERE it will be convenient to let the Learner know how to call the ſeveral Diviſions upon the Line of Lines, that ſo what has been ſaid about it relating to ſome of the foregoing Tables, and thoſe that follow, may be the more intelligible.

NUMERATION upon this Line is the ſame with one of the Radiuſes of *Gunter*'s Line, only the Diviſions there are unequal, and theſe equal, either is numbered with 1, 2, 2, 4, 5, 6, 7, 8, 9, 10, which you may call Diviſions of the firſt Order; each of theſe is divided into 10 other equal Parts, theſe are the ſecond Order; and if you imagine each of theſe laſt to be divided into 10 equal Parts, call theſe of the third Order, *&c.* and all theſe have different Values according to what you have Occaſion to call 10, at the End of the Line: Thus if that 1000, then 1 next the Beginning of the Line is 100, the 2 next that is 200, the 3 is 300, *&c.* And each of the Diviſions of the ſecond Order is 10, and of the third Order Units; ſo if 10 be called 1000, and I have Occaſion to take off 435, I call 4 of the firſt 400, the next 3 Lines of the ſecond Order 30, and 5 Lines of the third Order 5 Units.

Now in our preſent Caſe, you muſt ſuppoſe the whole Line of Sines C 60 = CE to be divided into 1000 equal Parts; take this Line in your Compaſſes, and open the Sector ſo that 10 (which you may now call 1000) at the End of the Lines may agree to this Extent, then ſuppoſe you are to ſet 30 Degrees upon the Line of Secants CF, look in the Table, and againſt 30 you will find 1155, that is once the Radius of the Circle C 60 = 1000, and 155 more; ſo take 155 in your Compaſſes out of the Line of Lines, and ſet that from the upper End of the Line of Sines, towards F, and it will reach to a Point at which ſet 30; the reſt are done in like manner.

A TABLE

A TABLE *to divide the Line of natural Tangents* F E.

Deg.	Tangents.	Deg.	Tangents.	Deg.	Tangents.	Deg.	Tangents.	Deg.	Tangents.
1	17	19	344	37	754	55	1428	73	3271
2	35	20	364	38	781	56	1483	74	3487
3	52	21	384	39	810	57	1540	75	3732
4	70	22	404	40	839	58	1600	76	4011
5	87	23	424	41	869	59	1664	77	4331
6	105	24	445	42	900	60	1732	78	4705
7	123	25	466	43	933	61	1804	79	5145
8	141	26	488	44	966	62	1881	80	5671
9	158	27	510	45	1000	63	1963	81	6314
10	176	28	532	46	1036	64	2050	82	7115
11	194	29	554	47	1072	65	2145	83	8144
12	213	30	577	48	1110	66	2246	84	9514
13	231	31	601	49	1150	67	2306	85	11430
14	249	32	625	50	1192	68	2475	86	14301
15	268	33	649	51	1235	69	2605	87	19081
16	287	34	675	52	1280	70	2747	88	28636
17	306	35	700	53	1327	71	2904	89	57290
18	325	36	727	54	1371	72	3078	90	Infinite

To divide this Line, you muſt take the Radius C E in your Compaſſes, and open the Sector to that Extent, then if you would put on 30 Degrees, look in the Table, and againſt 30 you will find 577, which take and ſet it up from E towards F, *&c.*

A TABLE *to divide the Line of Half-Tangents* H C.

Deg.	H.Tan.	Deg.	H.Tan.	Deg.	H.Tan.	Deg.	H.Tan.	Deg.	H.Tan.	Deg.	H.Tan.
1	9	25	227	49	456	73	740	97	1130	121	1767
2	17	26	231	40	466	74	753	98	1150	122	1804
3	26	27	240	51	477	75	767	99	1171	123	1842
4	35	28	249	52	488	76	781	100	1192	124	1881
5	44	29	259	53	498	77	785	101	1213	125	1921
6	52	30	268	54	509	78	810	102	1235	126	1963
7	61	31	277	55	520	79	824	103	1257	127	2006
8	70	32	287	56	532	80	839	104	1280	128	2050
9	79	33	296	57	543	81	854	105	1303	129	2096
10	87	34	306	58	554	82	869	106	1327	130	2144
11	96	35	315	59	566	83	885	107	1351	131	2194
12	105	36	325	60	577	84	900	108	1376	132	2246
13	114	37	334	61	589	85	916	109	1402	133	2300
14	123	38	344	62	601	86	932	110	1428	134	2356
15	132	39	354	63	613	87	949	111	1455	135	2414
16	140	40	364	64	625	88	966	112	1482	136	2471
17	149	41	374	65	637	89	983	113	1511	137	2539
18	158	42	384	66	649	90	1000	114	1540	138	2605
19	167	43	394	67	662	91	1018	115	1570	139	2675
20	176	44	404	68	674	92	1035	116	1600	140	2744
21	185	45	414	69	687	93	1054	117	1632	141	2824
22	194	46	424	70	700	94	1072	118	1664	142	2904
23	203	47	435	71	713	95	1091	119	1698	143	2989
24	212	48	445	72	728	96	1111	120	1732	144	3236

To

To divide this Line, take C B in your Compaſſes, and call that 1000, to which open the Sector, and then if you would ſet on 50 Degrees you will find in the Table againſt that, 466, which take in your Compaſſes, and ſet that off from C towards H, and it will reach to the Half-Tangent of 50 Degrees.

AGAIN, ſuppoſe you would ſet on 120 Degrees, againſt that in the Table you will find 1732, of which C B is 1000, then take 732 in your Compaſſes, and ſet that from B towards H, and it gives a Point, which will be for the Half-Tangent of 120 Degrees.

HAVING thus ſhewn the making and Origin of theſe Lines of natural Sines, Tangents, Secants, *&c.* as they are moſt commonly put upon one Side of the plane Scale, *Gunter*'s Scale, *&c.* that they may be the more ready for the ſeveral Purpoſes for which they are made,

NOW, as one Side of the Scale is thus furniſh'd with the ſeveral natural Lines before deſcrib'd, the other Side is uſually ſet off with the Line of Numbers (commonly call'd *Gunter*'s Line) together with the Lines of artificial Sines and Tangents for working Proportions in plane and ſpherical Trigonometry, and many other uſeful Parts of the Mathematicks. And as theſe Lines are divided by the Table of Logarithms, and Logarithmick Sines and Tangents, we will tranſcribe ſo much of theſe Tables as may be ſufficient for our preſent Purpoſe, having taken Sines and Tangents no nearer than every 10th Minute, and leave thoſe who would go nearer to conſult the Tables themſelves.

A TABLE *to divide the Line of Numbers.*

Num.	Log. Pts.	Num.	Log. Pts.	Num.	Log. Pts.	Num.	Log. Pts.	Num.	Log. Pts.
1	00	21	322	41	613	61	785	81	908
2	30	22	342	42	623	62	792	82	914
3	48	23	362	43	633	63	799	83	919
4	60	24	380	44	643	64	806	84	924
5	70	25	398	45	653	65	813	85	929
6	78	26	415	46	663	66	819	86	834
7	84	27	431	47	672	67	826	87	838
8	90	28	447	48	681	68	832	88	944
9	95	29	462	49	690	69	839	89	948
10	100	30	477	50	760	70	845	90	954
11	41	31	491	51	707	71	851	91	958
12	79	32	505	52	716	72	857	92	964
13	114	33	518	53	724	73	863	93	968
14	146	34	531	54	732	74	869	94	973
15	176	35	544	55	740	75	875	95	978
16	204	36	556	56	748	78	881	96	982
17	230	37	568	57	756	76	886	97	987
18	255	38	580	58	763	77	892	98	991
19	279	39	591	59	771	79	898	99	996
20	301	40	602	60	778	80	903	100	1000

A TABLE

A TABLE *to divide the Line of artificial Sines.*

Min.	Sines Deg. 0	Sines Deg. 1	Sines Deg. 2	Sines Deg. 3	Sines Deg. 4	Sines Deg. 5	Sines Deg. 6	Sines Deg. 7	Sines Deg. 8	Sines Deg. 9	Sines Deg. 10
0	00	242	543	719	843	94[illegible]	1019	1086	1143	1194	1240
10	4[illegible]	309	577	742	861	954	1031	1096	1152	1202	1247
20	7[illegible]	367	610	764	878	968	1043	1106	1161	1210	1254
30	94	418	640	786	895	981	1054	1116	1170	1218	1261
40	106	464	668	806	910	994	1065	1125	1178	1225	1267
50	163	505	694	825	926	1007	1075	1134	1186	1232	1274
Min.	11	12	13	14	15	16	17	18	19	20	21
0	1280	1318	1352	1384	1413	1440	1466	1490	1513	1534	1554
10	1287	1323	1357	1389	1418	1445	1470	1494	1516	1537	1558
20	1293	1329	1363	1394	1422	1449	1474	1498	1520	1540	1561
30	1300	1335	1368	1398	1427	1453	1478	1501	1523	1544	1564
40	1306	1341	1373	1403	1431	1457	1482	1505	1527	1548	1567
50	1312	1346	1378	1408	1436	1462	1486	1509	1530	1551	1570
Min.	22	23	24	25	26	27	28	29	30	31	32
0	1573	1592	1609	1626	1642	1657	1672	1685	1699	1712	1724
10	1577	1595	1612	1629	1644	1659	1674	1688			
20	1580	1598	1614	1631	1647	1662	1676	1690	1703	1716	1728
30	1583	1601	1618	1634	1649	1664	1679	1692			
40	1586	1603	1620	1637	1652	1667	1681	1694	1708	1720	1732
50	1589	1606	1623	1639	1654	1669	1683	1697			
Min.	33	34	35	36	37	38	39	40	41	42	43
0	1736	1747	1758	1769	1779	1789	1799	1808	1817	1825	1834
20	1740	1751	1762	1773	1783	1792	1802				
40	1744	1755	1766	1776	1786	1786	1805				
Min.	44	45	46	47	48	48	50	51	52	53	54
0	1842	1849	1857	1864	1871	1871	1884	1890	1896	1902	1908
Min.	55	56	57	58	59	59	61	62	63	64	65
0	1913	1918	1923	1928	1933	1933	1942	1946	1950	1954	1957
Min.	66	67	68	69	70	71	72	73	74	75	76
0	1961	1964	1967	1970	1973	1976	1978	1980	1983	1985	1987
Min.	77	78	79	80	81	82	83	84	85		90
0	1988	1990	1992	1993	1995	6661	1997	1998	1998		2000

A TABLE

A TABLE *to divide the Line of artificial Tangents.*

Min.	Tan. Deg. 0	Tan. Deg. 1	Tan. Deg. 2	Tan. Deg. 3	Tan. Deg. 4	Tan. Deg. 5	Tan. Deg. 6	Tan. Deg. 7	Tan. Deg. 8	Tan. Deg. 9	Tan. Deg. 10	Tan. Deg. 11	Tan. Min.
0	00	242	543	719	845	942	1022	1089	1148	1200	1246	1289	60
10	46	308	578	743	862	956	1034	1099	1157	1208	1254	1295	50
20	76	367	610	765	879	970	1045	1110	1166	1216	1261	1302	40
30	94	418	640	786	896	983	1057	1119	1174	1224	1268	1308	30
40	106	464	668	807	912	997	1068	1129	1183	1231	1275	1316	20
50	166	505	694	826	927	1009	1078	1138	1191	1239	1282	1321	10
Min.	89	88	87	86	85	84	83	82	81	80	79	78	Min.
	12	13	14	15	16	17	18	19	20	21	22	23	
0	1327	1363	1390	1428	1457	1485	1512	1537	1561	1586	1606	1628	60
10	1334	1369	1402	1433	1462	1490	1516	1541	1565	1588	1610	1631	50
20	1340	1375	1407	1438	1467	1494	1520	1545	1569	1592	1614	1635	40
30	1346	1380	1413	1443	1472	1499	1524	1549	1573	1595	1617	1638	30
40	1352	1386	1418	1448	1476	1503	1529	1553	1576	1599	1621	1642	20
50	1358	1391	1423	1453	1480	1507	1533	1557	1580	1603	1624	1645	10
Min.	77	76	75	74	73	72	71	70	69	68	67	66	Min.
	24	25	26	27	28	29	30	31	32	33	34	35	
0	1648	1669	1688	1707	1726	1744	1761	1779	1796	1812	1829	1845	60
10	1652	1672	1691	1710	1729	1747							50
20	1655	1675	1694	1713	1732	1750	1767	1784	1801	1818	1834	1850	40
30	1659	1678	1698	1716	1735	1753							30
40	1662	1682	1701	1719	1738	1755	1873	1790	1807	1823	1840	1856	20
50	1665	1685	1704	1723	1741	1758							10
Min.	65	64	63	62	61	60	59	58	57	56	55	54	Min.
	36	37	38	39	40	41	42	43	44	45			
0	1861	1877	1893	1908	1924	1939	1954	1970	1985	2000			
20	1866	1882	1898	1913	1929	1944	1959	1975	1990				
40	1872	1887	1903	1919	1934	1949	1964	1980	1995				
Min	53	52	51	50	49	48	47	46	45				Min.

CHAP. V.

Trigonometry.

THERE are two Sorts or Kinds of Triangles, *viz.* plain or right-lin'd, and ſpherical or circular, either of which does conſiſt of ſix Parts, *namely* of three Sides and three Angles.

General MAXIMS *belonging to right lin'd Triangles.*

1. A TRIANGLE is a Figure having three Sides, as A C, C B, B A, and three Angles A B C, and in reſpect to its Angles is either right or oblique.

2. A RIGHT angled Triangle hath one right Angle, and two acute Angles; and in its makeing hath a Perpendicular erected, or let fall, as B C.

3. THOSE Sides containing the right Angle are called Legs, as the Leg A B, and Leg B C; of theſe one is ſometimes called the Baſe, as A B, and the other the Perpendicular, as B C; the Side oppoſite to the right Angle is called the Hypothenuſe, as A C.

4. THE three Angles of every Triangle is equal to two right Angles (ſee *Theorem* 5th) or 180 Degrees; therefore having in a Right angled Triangle either of the acute Angles, the other is the Compliment thereof to 90 Degrees (ſee Figure firſt next following.)

5. In a Right-angled Triangle there are always two things given beſides the Right-angle, and one of thoſe a Side to find a fourth.

6. IN oblique angled Triangles there are three things always given to find a fourth; but one or more of thoſe three muſt be a Side.

7. The three Angles of every Triangle being equal to 180 Degrees, if in an oblique angled Triangle, you have any two Angles, the third is the Supplement of their Sum to 180 Degrees; or if you ſubſtract any one of them out of 180 Degrees, the Remainder will be the Sum of the other two. So in Fig. 2, the Angle at A is 28°, and that at B is 46°, their Sum is 74°, which being taken from 180°, there will remain the Angle A C B = 106°, *&c.*

8. The external Angle of every Right-lin'd Triangle is equal to the two oppoſite internal Angles (by *Theorem* 4th).

9. The two ſhorteſt Sides of every Triangle is longer than the longeſt Side thereof.

10. THE longeſt Side of any Triangle ſubtends the greateſt Angle.

11. AN Angle may be beſt repreſented by three Letters, the middlemoſt of which reſpects the Angle, and that and the other two expreſſes the Sides which make that Angle (tho' that is ſet at the angular Point may be ſufficient); ſo if one of the Angles of the Triangle next following was ſet down thus B A C, it would ſignify the Angle at A made by the Sides B A and C A. (ſee Fig. 2.)

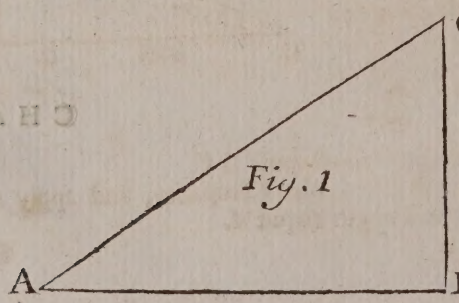


Fig. 1

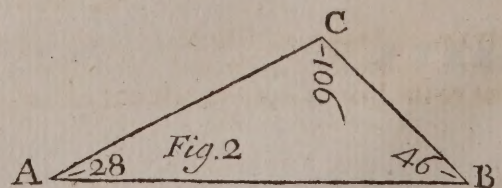


Fig. 2

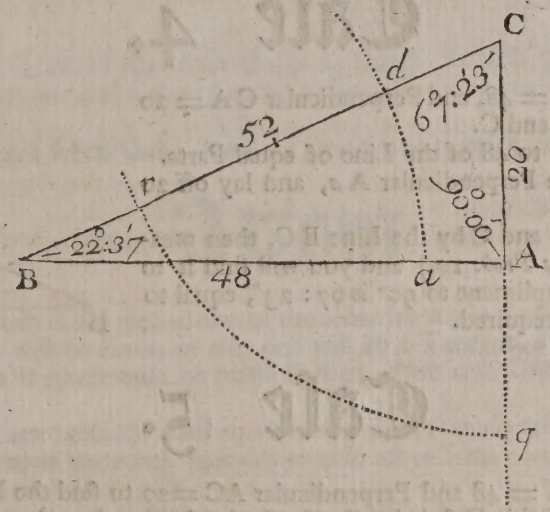
The Solution of the ſeveral Caſes of a Right angled plain Triangle by the Line of *Chords and equal Parts.*

THE Triangle which I ſhall make uſe of in the ſeveral Caſes ſhall be the next following, of which

A B, A C, B C	is the	Baſe, Perpendicular, Hypothenuſe	equal to	48, 20, 52 Yards Miles, Leagues, *&c.*

AND

A, B, C	is the	Right Angle, Angle at the Baſe, Angle at the Perpendicular	equal to	90°: 00′, 22 : 37, 67 : 23

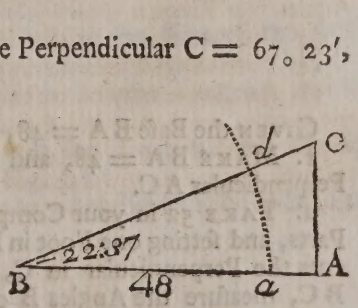


Caſe 1.

GIVEN the Angle at the Baſe B = 22° 37′, the Angle at the Perpendicular C = 67° 23′, and the Baſe B A = 48, to find the Perpendicular C A.

1. MAKE A B equal to 48 by the Line of equal Parts.

2. Erect the Perpendicular A C, and by Help of your Line of Chords make the Angle at B, equal to 22° : 37′ producing the Line B *d* until it cuts the Perpendicular in C, ſo is the Triangle compleated; then the Line A C taken in your Compaſſes and apply'd to the Line of equal Parts will fall upon 20, the Length required.



Caſe 2.

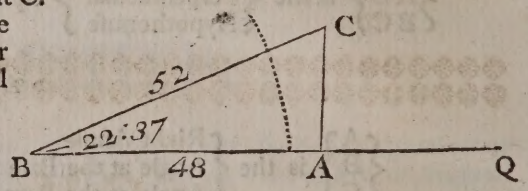
GIVEN the Angles and Baſe (as at *Caſe* 1.) to find the Hypothenuſe A C.

THIS is conſtructed as the firſt; then if you take B C in your Compaſſes, and apply that Extent to the Line of equal Parts it will fall on 52 the Length requir'd.

Caſe 3.

GIVEN the Angles with the Hypothenuſe to find the Baſe.

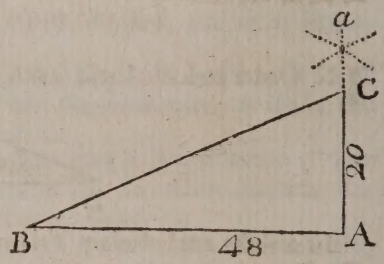
1. DRAW the Line B Q at pleaſure.
2. UPON the Point B make an Angle of 22° : 37′, and continue B C until it be equal to 52 of the Line of equal Parts, which will terminate at C.
3. LET fall the Perpendicular C A, and the Triangle is compleat; then A B, taken in your Compaſſes, will reach upon the Line of equal Parts to 48, the Anſwer.



Caſe 4.

GIVEN the Baſe B A = 48, and Perpendicular C A = 20 to find the Angles at B and C.

1. MAKE B A equal to 48 of the Line of equal Parts.
2. UPON A erect the Perpendicular A *a*, and lay off 20 from A to C.
3. JOIN the Points B and C by the Line B C, then meaſure the Angle at B (by *Prob.* 10.) and you will find it to be 22° : 37′ whoſe Compliment to 90° is 67 : 23′, equal to the Angle at C, as was required.



Caſe 5.

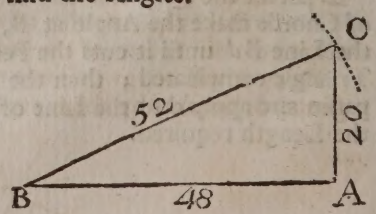
GIVEN the Baſe B A = 48 and Perpendicular AC = 20 to find the Hypothenuſe B C.

THE Conſtruction of this Caſe is in all reſpects the ſame as the 4th, and if you take B C in your Compaſſes, the Line of equal Parts, it will reach to 52, the Anſwer.

Caſe 6.

GIVEN the Baſe B A = 48, and Hypothenuſe B C = 52 to find the Angles.

1. MAKE B A = 48, and upon the Point A erect the Perpendicular A C.
2. TAKE 52 in your Compaſſes out of the Line of equal Parts, and ſetting one Foot in B, turn the other Foot until it cuts the Perpendicular in C, and having drawn the Line B C, meaſure the Angles B or C by help of your Line of Chords.



Case 7.

GIVEN the Base B A = 48, and Hypothenuse B C = 52, to find the Perpendicular A C.

THE Construction of this Case is the same with the sixth, so the Perpendicular A C, taken in your Compasses to the Line of equal Parts, will reach to 20, the Answer.

CHAP. VI.

The Arithmetical Solution of the seven Cases of Right Angled plain Triangles by the *Logarithms and Canon of artificial Sines, Tangents and Secants.*

IN Order to solve these seven Cases, it will be requisite to premise the following

Axiom.

IN a right angled plain Triangle, if one of the Sides be made Radius (or Semidiameter of a Circle), the other two will be Sines, or else one will be a Tangent, and the other a Secant.

THAT is, 1 If the Hypothenuse be made Radius, then each Leg represents the Sine of its opposite Angle.

2. IF one of the Legs be made Radius, then the Hypothenuse respects the Secant, and the other Leg, the Tangent of the Angle opposite to that other Leg.

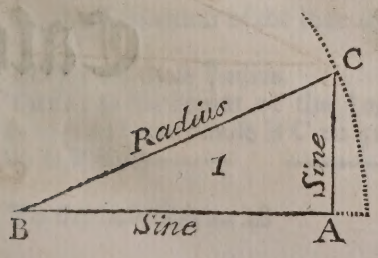
Examples.

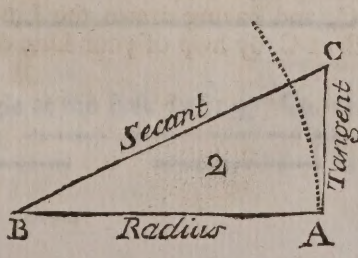
1. IF the Hypothenuse B C be made Radius, then the Base B A, is the Sine of the Angle at the Perpenicular C, and the Perpendicular is the Sine of the Angle at the Base B (see Fig. 1st.)

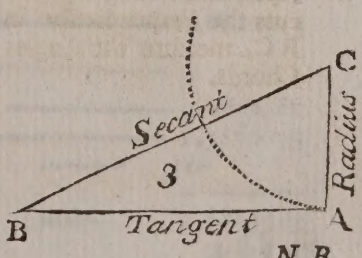
2. IF the Base B A be made Radius, then the Perpendicular A C is the Tangent, and the Hypothenuse B C, the Secant of the Angle at the Base B. (Fig. 2.)

3. IF the Perpendicular be made Radius, then the Base B A is the Tangent, and the Hypothenuse the Secant of the Angle at the Perpendicular C (see Fig. 3.)

THEN it will be, as the Side made Radius is to Radius; so is the other Sides to the Sines, Tangents, or Secants by them represented, and the contrary.







N. B.

N. B. In the following Proportions, it must be observed, that when a Side is wanting, then Radius, or an Angle is made the first Term; but if an Angle be wanting, then the first Term must be a Side.

1. To find a Side, any Side may be made Radius, saying,
THUS, As the Word on the Side given:
Is to the Side given:
:: So is the Word, on the Side required:
To the Side required.

2. To find an Angle, one of the given Sides must be made Radius.
THEN, As one given Side
Is to the Word on it
So is the other given Side
To the Word on it.

Case 1.

GIVEN the Base BA=48, and Angle at the Base B=22° : 37′, to find the Perpendicular AC.

1. MAKE the Hypothenuse Radius (as at *Examp.* 1st.)
THEN, As the Co-sine of the Angle at the Base 22° : 37′ Co. ar. 0347520
Is to the Logarithm of the Base 48——1,6812412
So is the Sine of the Angle at the Base 22° : 37——9.5849685

To the Perpendicular 20——1,3009617

2. MAKE the Base B A Radius (as at *Examp.* 2.)
THEN, As Radius——10.
Is to the Logarithm of the Base 48——1,6812412
So is the Tangent of the Angle at the Base 22° : 37′ 9,6197205
To the Logarithm of the Perpendicular 20——1,3009617

3. MAKE the Perpendicular Rodius (as at *Examp.* 3.)

THEN as Tangent Comp. of the Angle at the Base 22° : 37′ Co. ar. 9,6197205
Is to the Base 48——1,6812412
So is Radius——10,

To the Logarithm of the Perpendicular 20——1,3009617

Case

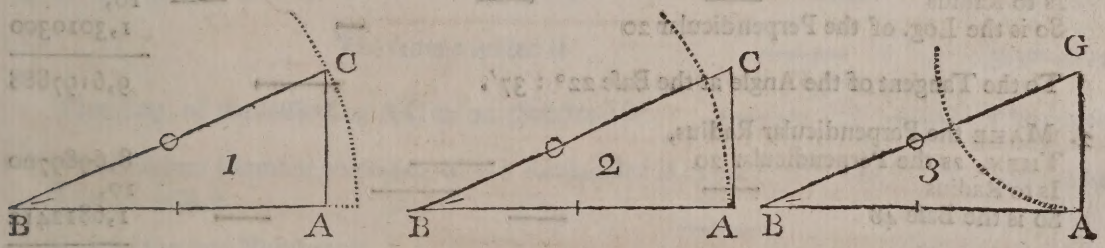
Case 2.

GIVEN the Base A B = 48, and Angle at the Base B = 22° 37, to find the Hypothenuse BC.

1. MAKE the Hypothenuse B C Radius.
THEN, As the S *c* of the Angle at the Base 22° : 37′ Co. ar. — ,0347520
Is to the Log. of the Base 48 — 1,6812412
So is Radius — 10
To the Log. of the Hypothenuse BC = 52 — 1,7159932

2. MAKE the Base Radius.
THEN, as Radius — 10,
Is to the Logarithm of the Base = 48 — 1,6812412
So is the Secant of the Angle at the Base 22° : 37′ — 10,0347520
To the Logar. of the Hypothenuse B C = 52 — 1,7159932

3. MAKE the Perpendicular Radius.
THEN it is, as T. *c.* of the Angle at the Base 22° : 37′ Co. ar. — 96197205
Is to the Log. of the Base 48 — 1,6812412
So is the Co-secant of the Angle at the Base 22° : 37′ — 10,4150315
To the Log. of the Hypothenuse B C = 52 — 1,7159932



Case 3.

GIVEN the Hypothenuse B C = 52, and Angle at the Base B = 22° : 37′, to find the Base.

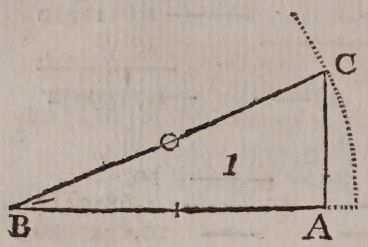
1. MAKE the Hypothenuse Radius.
THEN, as Radius — 10,
Is to the Logarithm of the Hypothenuse 52 — 1,7160033
So is the Co-sine of the Angle at the Base 22° : 37′ — 9,9652480
To the Logarithm of the Base 48 — 1,6812513

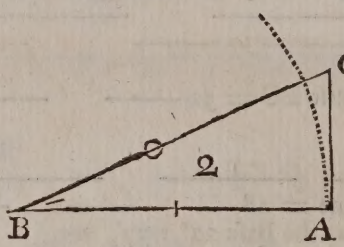
2. MAKE the Base Radius.
THEN, as the Secant of the Angle at the Base 22° : 37′ Co. ar. — 9,9652480
Is to the Hypothenuse B C = 52 — 1,7160033
So is Radius — 10
To the Base B A = 48 — 1,6812513

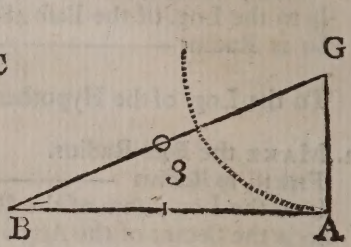
3. MAKE

3. MAKE the Perpendicular Radius.
THEN, as the Secant of the Angle at the Perpendicular 67° 23′ Co. ar. —— 9,5849685
Is to the Hypothenuſe B C = 52 —— 1,7160033
So is the Tangent of the ſame Angle 67° : 23′ —— 10,3802795

To the Baſe 48 —— 1,6812513







Caſe 4.

GIVEN the Baſe B A = 48, and Perpendicular A C = 20, to find the Angles.

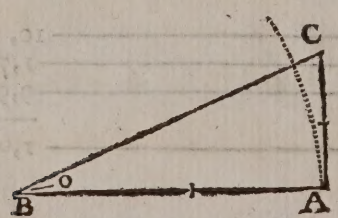
1. MAKE the Baſe Radius.
THEN, as the Log. of the Baſe 48, Co. ar. —— 8,3187588
Is to Radius —— 10,
So is the Log. of the Perpendicular 20 —— 1,3010300

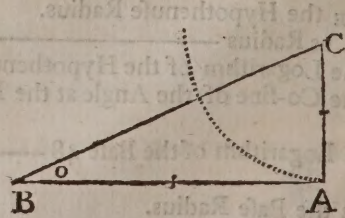
To the Tangent of the Angle at the Baſe 22° : 37′. —— 9,6197888

2. MAKE the Perpendicular Radius.
THEN, as the Perpendicular 20 —— 8,6989700
Is to Radius —— 10,
So is the Baſe 48 —— 1,6812412

To the Tangent of the Angle at the Perpendicular 67° : 23′, whoſe Compliment to 90° is the Angle at the Baſe required 22 : 37. } 10,3802112

Note, That the Hypothenuſe could not be made Radius in this Caſe, becauſe as it was not given, it would have taken away the only Aſſiſtant that was to accompany either of the given Sides, to find the Angle required.





Caſe

Case 5.

GIVEN the Base A B = 48, and the Perpendicular A C = 20, to find the Hypothenuse B C.

1. MAKE the Base Radius.

THEN, as the Base 48, Co. ar.	8,3187588
Is to Radius	10,
So is the Perpendicular 20	1,3010300
To the Tangent of the Angle at the Base B = 22 : 37	9,6197888

AGAIN, making the Hypothenuse Radius.

As the Sine of the Angle at the Base 22° : 37′, Co. ar.	0,4150315
Is to the Perpendicular A C = 20	1,3013000
So is Radius	10,
To the Hypothenuse B C = 52	1,7160615

Note, that this Case may be answer'd by the 47 of *Eu.* 1, and by the Logarithms thus: From twice the Log. of the greater Leg substract the Log. of the lesser Leg, and add the absolute Number belonging to the Remainder unto the lesser Leg. then half the Log. of that Sum added to the Logarithm of the lesser Leg is the Log. of the Hypothenuse. Thus, Log. of the greatest Leg, A B = 48 — 1,6812412

The same doubled is — 3,3624824

THE Log. of the lesser Leg A C = 20 (Substra.) is — 1,3010300

THE absolute Number belonging to the Remainder is 115,2 — 2,0614524

LESSER Leg is 20

SUM of the two Numbers is 135,2 Log. 2,1309767

LOG. of the lesser Leg 20 is — 1,3010300

SUM of the Log. is — 3,4320067

HALF the Sum is the Log. of the Hypothenuse 52 — 1,7160033

Case 6.

GIVEN the Base B A = 48, and the Hypothenuse B C = 52, to find the Angles.

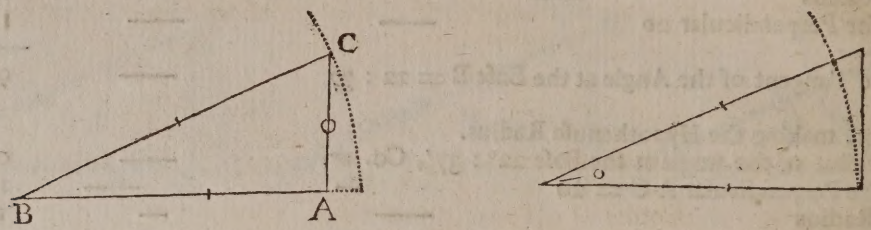
1. MAKE the Hypothenuse Radius.

Then, as the Hypothenuse 52, Co. ar.	8,2839966
Is to Radius	10,
So is the Base 48	1,6812412
To the Sine of the Angle at the Perpendicular 67° 23′	9,9652378

2. MAKE the Baſe Radius.

Then as the Baſe 48, Co. ar.	8,3187588
Is to Radius	10,
So is the Hypothenuſe 52	1,7160033
To the Secant of the Angle at the Baſe 22° 37′	10,0347621

Note here, as at the fourth, that the Perpendicular cannot be made Radius, for the Reaſon given there.



Caſe 7.

GIVEN the Hypothenuſe B C = 52, and the Baſe A B = 48, to find the Perpendicular A C.

1. MAKE the Hypothenuſe Radius

THEN, as the Hypothenuſe 52, Co. ar.	8,2839966
Is to Radius	10,
So is the Baſe 48	1,6812412
To the Sine of the Angle at the Perpendicular 67° 23′ } Whoſe Compliment to 90° is 22 37′ }	9,9652378

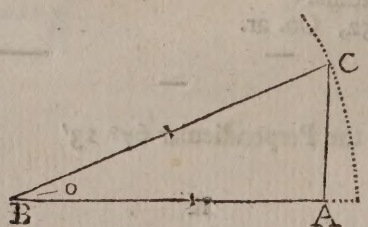
Again,

2. LETTING the Hypothenuſe continue to be Radius,

Say, as Radius	10,
Is to the Hypothenuſe 52	1,7160033
So is the Sine of the Angle at the Baſe 22° 37′	9,5849685
To the Perpendicular 20	1,3009718

To do this by the Logarithms, obſerve the following Rule: Take the Sum and Difference of the Hypothenuſe and given Log. then half the Sum of their Logarithms is the Log. of the Leg requir'd.

The Hypothenuſe is	52 } their	Sum is 100, Log. is	2,
The Baſe	48 }	diff. 4 Log. is —	0,6020600
Sum of the Log. is			2,6020600
Half the Sum is the Log. of the Perpendicular 20			1,3010300



CHAP. VII.

The Uſe of Right-angled, plain Triangles, in ſeveral Problems about taking the Height *of Objects when acceſſible.*

PROBLEM 1.

TO take an acceſſible Altitude at one Station,

LET A B repreſent the Corner of a Tower whoſe Height A B is required.

1. MAKE choice of any Station, as at D, and directing a Quadrant (or any other Inſtrument) ſo as that you may ſee the Point A thro' both the Sights, then obſerve where the Thread falls, if you uſe a Quadrant, ſuppoſe upon 40 Degrees of the Limb, this is the Angle A p q = A E B, whoſe Compliment to 90° is 50° = E A B = p A q.
2. MEASURE B D, the Diſtance between the Station and Bottom of the Tower, which let be 74 Yards = p q, then, to find the Height A q,

Say by Caſe the firſt, as Radius	10
Is to the Diſtance B D = 74 Yards	1,8692317
So is the Tangent of the Angle A p q = 40°	9,9238135
To A q = 62 Yards	1,7930452

HAVING thus found A q, which is the Height of the Tower above the Level of the Eye, to be 62 Yards, you muſt add thereto to the Line B q = D p, the Height of your Eye, which let be 5 Feet, then the Sum will be 63 Yards, and two Feet, and ſo high is the Tower (ſee the next Figure following.)

PROB. 2.

Yds. f. f.

BY knowing the Height of the Tower A B to be 63 : 2 : = 191, and being upon the Top thereof at A, to find how far any Object, ſuppoſe at E, is from the Baſe B of the ſame.

1. FIRST from A, direct the Sights of your Quadrant to E, and find the Angle E A B = 50° (ſuppoſe.)
2. MAKE the Height of the Tower A B = 191 Feet Radius, then ſay,

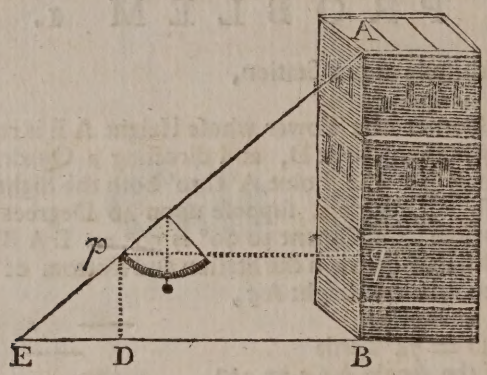
As Radius	10,
Is to the Height of the Tower 191	2,2810334
So is the Tangent of the Angle at A = 50°	10,0761865
To the Diſtance of the Object at E = 227, 6 Feet	2,3572199

PROB. 3.

By knowing the Height of the Tower A B to be 191 Feet, to find how far off it you are when at the Station D.

1. Subſtract the Height of your Eye, ſuppoſe 5 Feet out of the Height of the Tower, and there will remain 186 *f.* = 62 *Yd.* = A *q.*
2. FIND the Angle A *p q.* 40° (as above) then
3. SAY, as the Sine of the Angle at *p.* = 40°. Co. ar. —— 0,1919325
   Is to A *q.* = 62 Yards —— —— 1,7923917
   So is the Co-ſine of 40 Degrees —— 9,8842540

   To the Diſtance *p q.* = B D = 73,89 —— —— 1,8685782



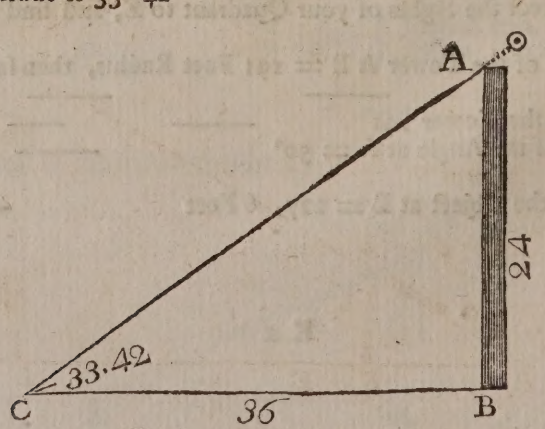
PROB. 4.

By Help of a two Foot Rule, or any thing elſe of a known Length, or Height, to find the Sun's Altitude.

LET A B be a two Foot Rule, erected perpendicular to the horizontal Line B C, caſting its Shadow to C, meaſure the Length of the Shadow B C, and ſuppoſe it to be 36 Inches, then you will have the Baſe B C, and Perpendicular A B, to find the Angle at the Baſe A C B (by *Caſe* the 4th) thus making the Baſe B C, Radius, And

Say, as 36 —— —— 1,5563025
Is to Radius —— —— 10,
So is the Perpendicular A B = 24 —— 1,3802112

To the Tangent of the Angle A C B = 33° 42′ —— 9,8239087
So then the Sun's Altitude is 33° 42′

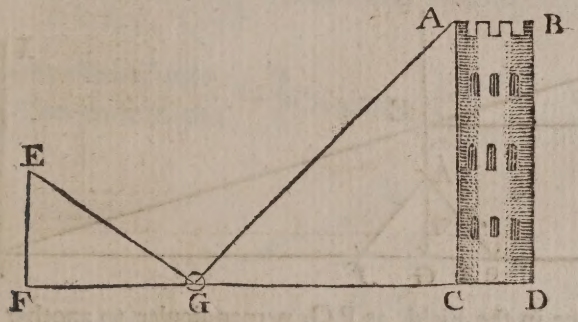


THERE are ſeveral Ways uſed to find the Height of Objects, or their Diſtance from the Place of Obſervation, without the Help of Trigonometry; ſo a few of them are as follows:

PROB. 5.

LET it be required to find the Height of the Tower A B C D, by help of a Looking-glaſs, or a Bowl of Water.

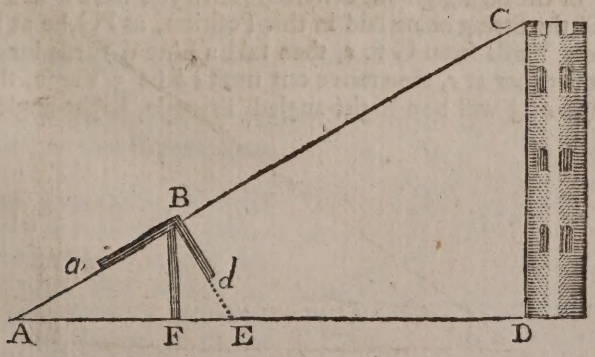
1. Being upon a Level with the Bottom of the Tower, lay down a Looking-glaſs, or Bowl of Water, or rather a flat Piece of poliſh'd Steel, or ſome other Metal, at ſome convenient Diſtance from the Foot of the Tower, ſuppoſe at G.
2. WALK backward, leaving the Glaſs in a Right line, between yourſelf and the Corner of the Tower A C, until you can ſee the Picture of the Top at A in the middle of the Glaſs, your Eye being then at E, and Feet at F.
3. MEASURE G C and G F, and alſo the Height of your Eye E F; then ſay by *The Rule of Three*, as F G is to F E, ſo is G C to C A, the Height requir'd.



PROB. 6.

To find the Height of a Houſe, Tree, Steeple, *&c.* by a Carpenter's Square, or any two ſtraight Rulers ſet at Right-angles to one another, the angular Point being ſet upon a Staff of a known Length, in ſuch a manner that you may ſee the Points A and C along the Side B *a*, and the Point E, along the Side B *d*.

1. THE Square being ſet, and Lines imagin'd to be drawn from B to A, C, and E, there will be form'd ſeveral Triangles, as A B E, A B F, and E B F, all being ſimilar Triangles, and like unto the Triangle A C D.
2. WHEN you have meaſured F E, and A D in ſuch Parts as B F is divided into, as Feet, Inches, *&c.* then it will be as B F is to F E, ſo is A D to C D, the Height of the Houſe, Tree, Steeple, *&c.*

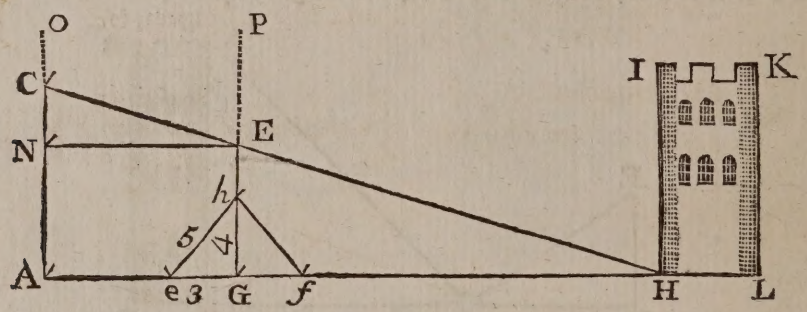


PROB.

PROB. 7.

LET it be required to find the Diſtance of the Tower I H K L from a Station at A or G.

1. SET up a Staff at G, and go any convenient Diſtance, ſuppoſe 120 Feet from thence to E, ſo that the Line E G may be at right Angles with the Line A H, and there ſet up another Staff.
2. WALK from G, leaving it in a right Line with H, any Diſtance, ſuppoſe to A = 176 Feet, and there ſet up a third Stick.
3. Go from A towards O, at right Angles with A H, until you ſee the Stick that you ſet at E, in the ſame Line with H, and that will be when you come to C; then meaſure A C, which let be 171 Feet.
4. From A C = 171 Feet, Subſtract G E = A N = 120 Feet, and there will remain N C = 51 Feet, then ſay, as C N is to C A, ſo is N E = A G, to A H = 590 $\frac{2}{17}$ Feet the Diſtance required.



To draw a draw Line in the Field, as P G, perpendicular to another Line, ſuppoſe A H, you may proceed thus.

1. Go from G, any convenient Diſtance, ſuppoſe to *f*, 10 Feet towards H, leaving G in a right Line with A, and at *f* put up a Stick.
2. Go from G 10 Feet to *e*, leaving G in a right Line with H, and at *e* ſet up another Stick, which will be 20 Feet off that at *f*.
3. TAKE a Piece of Packthread of about 30 or 40 Feet long, with a Loop at each End, and double it, ſticking a Pin in the Midle thereof, and then put the Stick at *f*, thro' one Loop, and that at *e*, thro' the other; then move from the Line A H, as far as the String will let you, going nearer or farther from the Tower, until you are at the middle of the String where you ſtuck the Pin, each half between the Sticks at *e*, and *f*, being held pretty tight, and kept ſo, lay it gently upon the Ground, and ſuppoſe it fell upon the Point *n*, then in that Point ſet up another Stick.
4. HAVING done thus, take the Loops off *e* and *f*, and put the Stick at G, thro' one of them, and with the reſt of the String go out towards P, until you have *h* in a right Line between your ſelf and G, ſo will the String being laid in this Poſition, as PG, be at Right-angles with A H. Or thus, Meaſure 3 Yards from G to *e*, than take a Line 9 Yards long, faſtning one End by a Loop at G, and the other at *e*, then move out until *e h* be 5 Yards, then will G *h* be 4 Yards, and ſo the Triangle *e h g* will be a Right-angled Triangle, Right-angled at G.

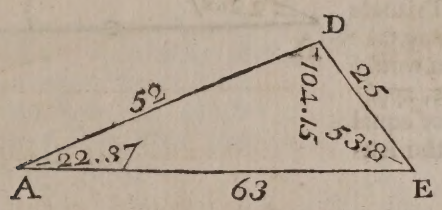
CHAP. VIII.

The Solution of the six Cases of oblique angled Triangles by help of a Line of Chords and equal Parts.

LET the Examples be taken from the Triangle next following, of which

The Side { A E, A D, D E } is { 63, 52, 25 } Yards, Miles, Leagues, *&c.*

And { D, is the obtuse Angle; A, E, Two acute Angles } Containing { 104° : 15; 22 : 37; 53 : 8 }

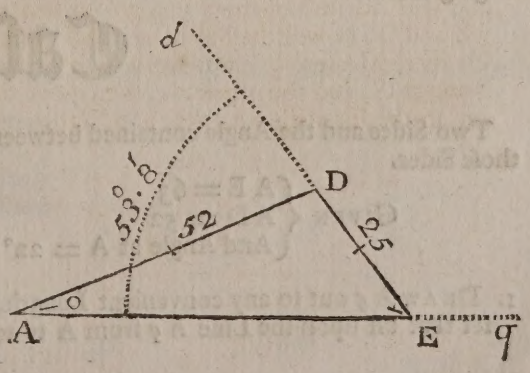


Case I.

Two Sides, and an Angle opposite to one of them, being given to find the Angle opposite to the other Side.

GIVEN { A D = 52; D E = 25; And Angle at E = 53° : 8′ } To find the Angle at A or D.

1. DRAW the Line A *q* out at pleasure, and from the Point E, taken in any convenient Part of A *q*, draw the Line E *d*, so as to make an Angle of 53° : 8′ with the Line A E, as at *Prob.* 9. *Chap.* 2.
2. TAKE 25 in your Compasses out of the Line of equal Parts, and set that Extent upon the Line E *d*, from E to D.
3. TAKE 52 from the same Line of equal Parts, and setting one Foot of your Compasses in D, turn the other until it cuts the Line A E in A, then draw D E and D A, so will the Triangle be truly form'd, and then you may measure the Angle at A or D by *Prob.* 10. of *Chap.* 2.



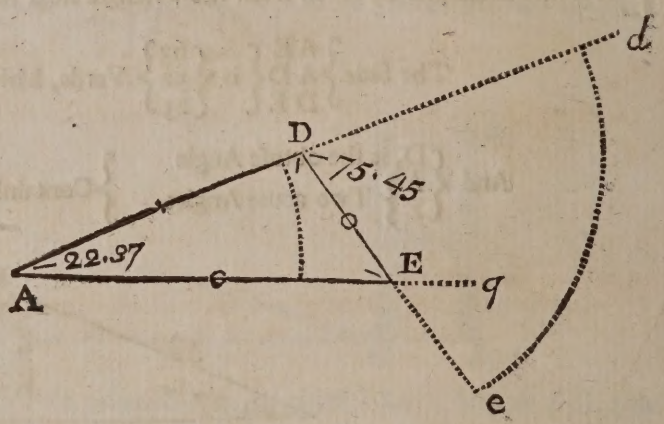
Case 2.

THE Angles and one Side being given to find either of the other Sides.

GIVEN { D = 104° : 15′ / A = 22° : 37 / E = 53 : 08 } To find A E or D E.

And Side A D = 52

1. DRAW A *q* at pleaſure, and from the Point A, draw A D, ſo as to make an Angle of 22° 37′ with the Line A *q*.
2. TAKE 52 in your Compaſſes out of the Line of equal Parts, and ſet that Extent off on the Line A *d* from A to D.
3. BECAUSE the external Angle E D *d* is equal to the Sum of the two oppoſite internal Angles A, and E = 75° 45′ make the Angle *e* D *d* equal to 75° 45′, by drawing the Line D *e*, which will cut the Line A *q* in E, the Triangle being thus form'd, you may take the Side whoſe Length you would know in your Compaſſes, apply that Extent to the Line of equal Parts, and you will have the Anſwer.



Case 3.

Two Sides and an Angle, oppoſite to one of them, being given to find the other Side.

GIVEN { A D = 52 / D E = 25 / And Angle at E = 53 : 8′ } To find A E.

THIS is conſtructed in the ſame manner in all reſpects as the firſt Caſe. See the Figure belonging thereto.

Case 4.

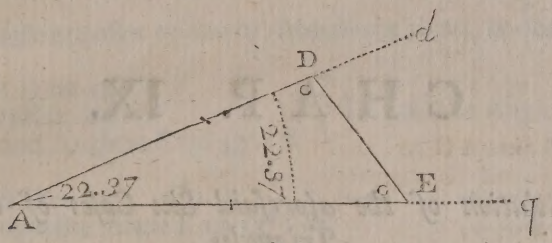
Two Sides and the Angle contained between them being given to find the Angles oppoſite to thoſe Sides.

GIVEN { A E = 63 / A D = 52 / And Angle at A = 22° : 37 } To find the Angles E and D.

1. DRAW A *q* out to any convenient Length, then take 63 out of the Line of equal Parts, and ſet that off upon the Line A *q* from A to to E.

2. DRAW

2. DRAW the Line A *d* out at pleaſure ſo as to make an Angle of 22° : 37′ with the Line A *q*, upon which from A to D, ſet 52 by help of the Line of equal Parts, then join D and E, and you will have the Triangle complete, which being done, you may meaſure the Angles D or E by the Line of Chords.

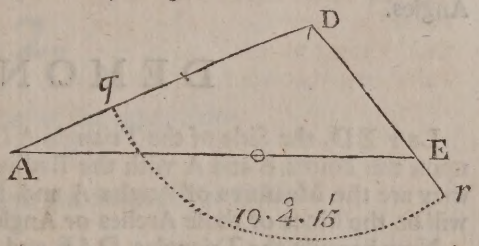


Caſe 5.

ANY two Sides, and the contain'd Angle, being given to find the third Side.

GIVEN { A D = 52 : / D = 104° : 15′ / D E = 25 } To find the Side A E.

1. DRAW the Line A D = 52, and upon D, with 60 Degrees of a Line of Chords in your Compaſſes, deſcribe the Arch *q r*, upon which from *q* ſet 104° : 15′ to *r*.
2. DRAW the Line D *r*, upon which ſet 25 from D to E, and from A draw the Line A E, which will complete the Triangle A D E, then you may meaſure A E, and find it to be 63.

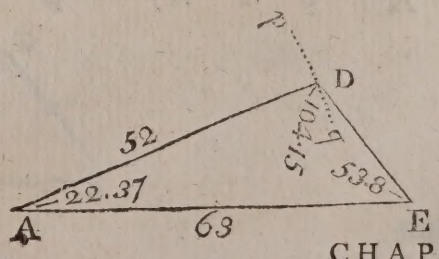


Caſe 6.

The three Sides being given to find the Angles.

GIVEN { A D = 52 / D E = 25 / A E = 63 } To find the Angles { A / D / E } or either of them.

1. MAKE A E = 63 by the Line of equal Parts.
2. FROM the ſame Line take 52, the Length of AD, in your Compaſſes, and ſetting one Foot in A, with the other deſcribe the Arch *p q*.
3. TAKE 25 in your Compaſſes, and ſetting one Foot in E, turn the other ſo as to interſect the Arch *p q* in D, then draw A D and D E ; thus the Triangle will be form'd, and the Angles may be meaſured by the Line of Chords, as before.



CHAP. IX.

The Arithmetical Solution of the aforeſaid Six Caſes of oblique angled plain Triangles.

THAT the Learner may more clearly apprehend this Part, it will be neceſſary to demonſtrate ſome few Properties of a plain Right-lin'd Triangle, (which here may be called *Axioms*), as we ſhall have Occaſion for them.

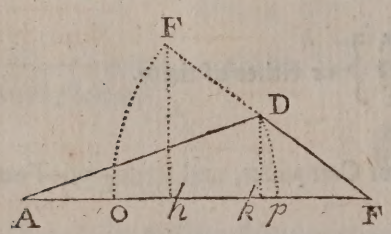
Axiom I.

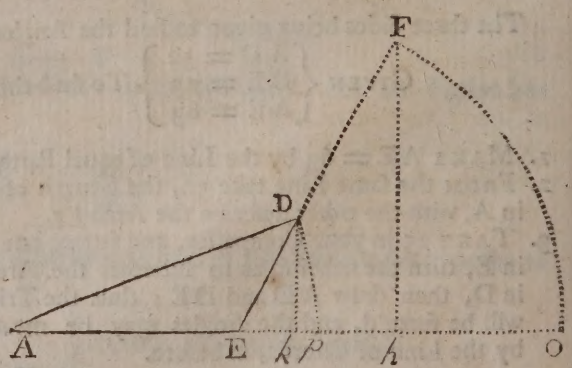
In all Right-lin'd plain Triangles, the Sides are proportional to the Sines of their oppoſite Angles.

DEMONSTRATION.

LET E D, the Side of the Triangle A D E, be produc'd until E F becomes equal to A D, and upon the Points E and A with the Radius E F = A D, deſcribe the Arches F *o*, and D *p*, and they are the Meaſures of Angles A and E; then let fall the Perpendicular F *h*, and D *k*, theſe will be the Sines of thoſe Arches or Angles to the Radius A D or E F.

Now becauſe the Triangles D *k* E and F *h* E, are ſimilar, it will be (by *Theorem* 12) E D: E F (=A D):: D *k* : F *h*, that is, as the Side E D is to the Side A D, ſo is the Sine of the Angle at A, to the Sine of the Angle at E. Q. E. D.





Caſe

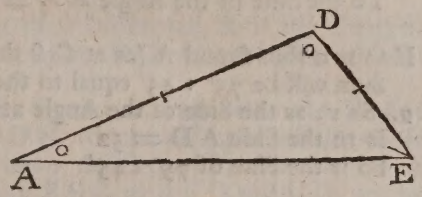
Case 1.

Two Sides and an Angle oppoſite to one of them being given, to find the Angle oppoſite to the other Side.

GIVEN { A D = 52, D E = 25, And Angle at E = 53° : 8′ } To find the Angle at A or D.

Say (by *Axiom* 1) as A D = 52	Co. ar.	8,2839967
Is to the Sine of its oppoſite Angle E = 53° : 8′		9,9031084
So is the Side D E = 25		1,3979400
To the Sine of the oppoſite Angle at A = 22 : 37		9,5850451

THE Angle at A being known, and alſo the Angle at E, you may thence find the Angle at D by *Theorem* 5, to be 104° : 15′.



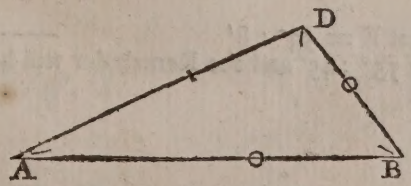
Case 2.

THE Angles, and one Side being given, to find either of the other Sides.

GIVEN { D = 104° : 15′, A = 22 : 37, E = 53 : 08 } To find the Side A E or D E.

And Side A D = 52.

Say, as the Sine of the Angle at E = 53° : 8′	Co. ar.	0,0968916
Is to A D = 52		1,7160033
So is the Sine of the Angle at A = 22° : 37′		9,5849685
To the Side D E = 25		1,3978634
If the Side A E had been required		
Then it would be, as the Sine of E = 53° : 8′	Co. ar.	0,0968916
Is to the Side A D = 52		1,7160033
So is the Sine of 75° : 45′ (the Supplement of 104° : 15′)		9,9864273
To the Side A E = 63		1,7993222



Caſe 3.

Two Sides, and an Angle oppoſite to one of them, being given to find the other Side.

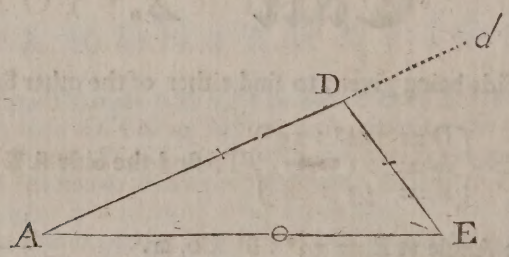
GIVEN { A D = 52, D E = 25, And Angle at E = 53° : 8′ } To find A E.

This Caſe requires two Operations.

1. SAY, As A D = 52 Co. ar.	8,2839967
Is to the Sine of the Angle at E = 53° : 8′	9,9031084
So is the Side D E = 25	1,3979400
To the Sine of the Angle at A = 22° : 37′	9,5850451

HAVING thus found A (as at *Caſe* the firſt), add the Angle at E = 53° : 8′ thereto, and the Sum will be 75° : 45′ equal to the external Angle *d* D E, then,

2. SAY, as the Sine of the Angle at E = 53° : 8′ Co. ar.	0,0968916
Is to the Side A D = 52	1,7160033
So is the Sine of 75° : 45′	9,9864273
To the Side A E = 63	1,7993222



EXAMPLE 2.

GIVEN { A D = 52, D E = 63, And A D E = 104 : 15′ } To find D E.

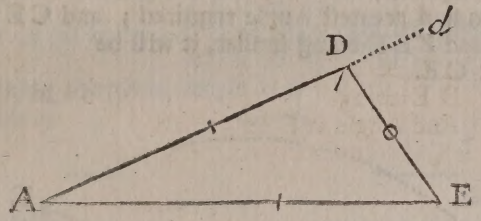
1. As A E = 63 Co. ar.	8,2006595
Is to the Sine of *d* D E = 75° : 45′ (the Supplement of 104° : 15′)	9,9864273
So is A D = 52	1,7160033
To the Sine of the Angle at E = 53° : 8′	9,9030901

Take this 53° : 8′ out of 75° : 45′ and the Remainder will be 22° : 37, for the Angle at A.

Now ſay,

2. As

2. As the Sine of 75° : 45′ = *d* D E. Co. ar.	0,0135727
Is to A E = 63	1,7993405
So is the Sine of the Angle at A = 22° : 37′	9,5849685
To D E = 25	1,3978817



Axiom 2.

In all plain Triangles, as the Sum of any two Sides is to their Difference, ſo is the Tangent of half the Sum of the two oppoſite Angles to the Tangent of the Difference of either of them above or under the half Sum.

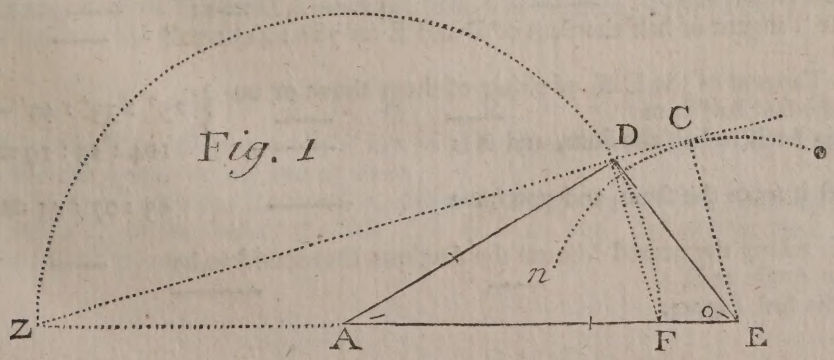
Produce E A, and make A Z = A D, then upon the Center A, and with the Radius A D, deſcribe the Semicircle Z D F, and draw the Line D F, which will make a Right-angle with Z D, and parallel thereto draw the Line E C, then upon the Center E, with the Radius E C, deſcribe the Arch *n* C *o*, ſo will Z C be the Tangent of Z E C = A F D and C D, will be the Tangent of D E C = F D E.

DEMONSTRATION.

1. The Angles A D E + A E D = Z A D, here Z A D, (by *Theorem* 4.) and A D F + A F D = Z A D, here Z A D being common, therefore A D E + A E D = A D F + A F D. (by *Axiom* 1. *Chap.* 1) But A D F = A F D (by *Theorem* 6.) therefore A D F, or A F D is equal to half A D E + A E D, and ſo is A E C, or Z E C.
2. D E C is the Difference between the ſaid half Sum Z E C, and leaſt of the Angles required, *viz.* A E D, and its Equal E D F, is the Difference between the half Sum A D F, and the greater Angle A D E.
3. The Triangle Z D F and Z C E, are ſimilar, and conſequently their Sides proportional (by *Theorem* 12th) then it will be as Z E (= A E + A D) is to F E (= A E − A D) ſo is Z C (= the Tangent of the Angle Z E C) to D C (= the Tangent of the Angle D E C = F D E)

That is, as the Sum of two Sides is to their Diff. ſo is the Tangent of half the Sum of two oppoſite Angles, *&c.* as the *Axiom* expreſſes.

Having by the foregoing Analogy found the Angle D E C, take it from half the Sum of the Angles required, *viz.* Z E C, and the Remainder will be the leſſer of the Angles required A E D, or add its equal F D E to the half Sum A D F, and you will have A D E, the greater of the Angles required, as you will ſee at the fourth *Caſe*.

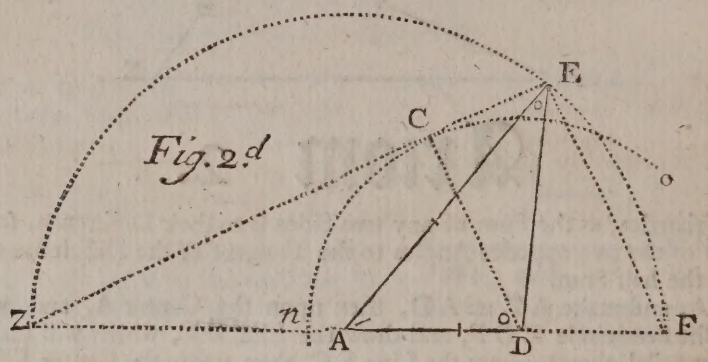


In the Figure next following, where it is ſuppoſed that A D, A E, and contained Angle at *A* are given to find the Angles at E, and D, the Demonſtration of the ſame Axiom may be thus :

Z D is the Sum of the given Sides, and D F their Difference, Z F E = Z D C is half the Sum of the two Angles required, and Z C the Tangent thereof, C D E (= D E F) is the Diff. between the half Sum and greateſt Angle required ; and C E the Tangent thereof.

The Triangles Z C D and Z E F being ſimilar, it will be

As Z D : D F :: Z C : C E.



Caſe 4.

Two Sides, and the Angle contain'd between them, being given to find the Angles oppoſite to thoſe Sides.

GIVEN { A E = 63, A D = 52, And Angle at A = 22° : 37′ } To find the Angles E and D.

THE Side ——— A E = 63
A D = 52

Their Sum is Z D = A E + A D = 115
And Difference D E = A E − A D = 11
The Angle at A is ——— 22° : 37

And its Supplement to 180, is ——— 157 : 23 = D + E
Half thereof is ——— 78 : 41 : 30″

THEN, as the Sum of the containing Sides A E + A D = 115 Co. ar. ——— 7,9393021
Is to their Difference ——— A E − A D = 11 ——— 1,0413927
So is the Tangent of half the Sum of D and E = 78° : 41′ : 30″ ——— 10,6990331

To the Tangent of the Diff. of either of them above or under the ſaid half Sum ——— 25° : 33′ : 49″ — 9,6797279
Add this Arch to half the Sum, and it is ——— 104 : 15 : 19 = the greater Angle D.
Subſtract it from the ſame, and you have ——— 53 : 07 : 41 = the leſſer Angle E.

HENCE, taking the neareſt Minute the Angle at D is found to be ——— 104° : 15′
And the Angle at E ——— 53° : 08′
See the firſt Figure.

Caſe

Case 5.

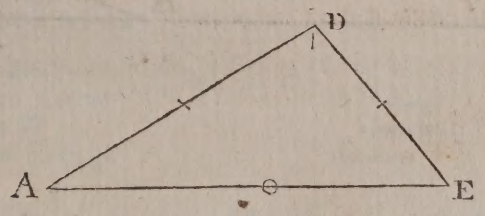
ANY two Sides, and the contain'd Angle, being given to find the third Side.

GIVEN (as before) { A D = 52, D = 104° : 15′, D E = 25 } To find A E.

ONE of the Sides containing the given Angle D is		A D = 52
The other containing Side is		D E = 25
Their Sum is		77
And Difference		27
The contain'd Angle is D =		104° : 15′
The Supplement thereof to 180° is (= A + E)		75 : 45
THEN, as the Sum of the two containing Sides 77 Co. ar.		8,1135093
Is to their Difference 27		1,4313638
So is the Tangent of half A + E =	37° : 52′ : 30″	9,8908557
To the Tangent of the Difference	15 : 15 : 18	9,4357288
Add this to half A + E, and the Sum is	53 : 07 : 48 = the Angle at E.	
Subſtract it from A + E, and the Diff. is	22 : 37 : 12 = the Angle at A.	

HAVING thus found the Angles oppoſite to the given Sides, you may find A E the Side required by the firſt *Axiom*, ſaying,

As the Sine (to the neareſt Min.) of the Angle at A = 22° : 37′ Co. ar.	0,4150315
Is to the Side D E = 25	1,3979400
So is the Sine of 75° : 45′ (the Supplement of D = 104° : 15)	9,9864273
To the Side A E = 63, which was required,	1,7993988



Axiom 3.

IN oblique angled plain Triangles, as the true Baſe is to the Sum of the Sides, ſo is the Difference of the Sides to the alternate Baſe.

CONSTRUCTION.

LET A D E be the Triangle propos'd (in any of the three Varieties) having its three Sides given, to find the Angles or any one of them.

1. Upon the Point D, with the Radius *D E*, deſcribe the Circle F I E Q, and continue *A D* to Q, ſo will *A Q* be the Sum of the Sids *A D* + *D E*, and *A F* will be their Difference, *A E* is the true Baſe, and *A I* the alternate Baſe.

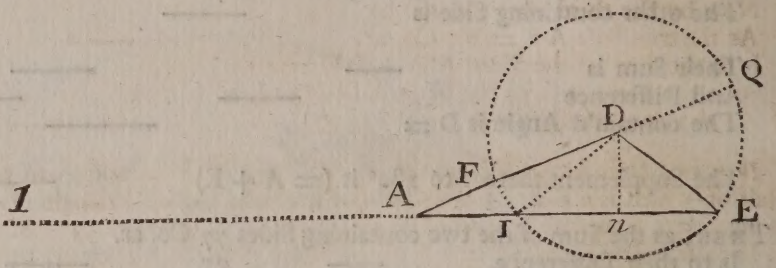
D E-

DEMONSTRATION.

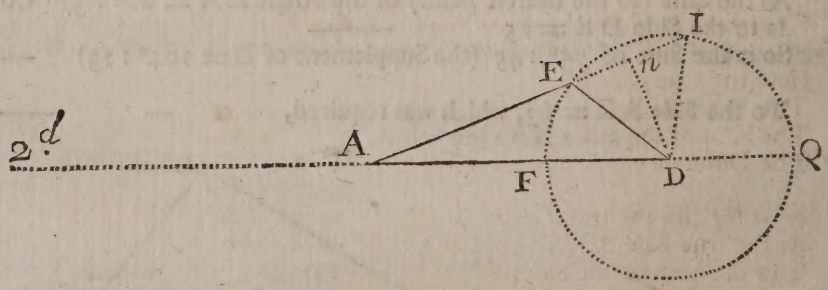
1 in Analog. | 1 | $AQ \times AF = AE \times AI$ (by *Theorem* 20.)
| 2 | $AE : AQ :: AF : AI$.

THAT is, as AE (the true Base) is to $AQ = (AD + ED)$ the Sum of the Sides, so is AF ($= AD - ED$) the Difference of the Sides to AI, the alternate Base.
See the Figures for the three Varieties next following.

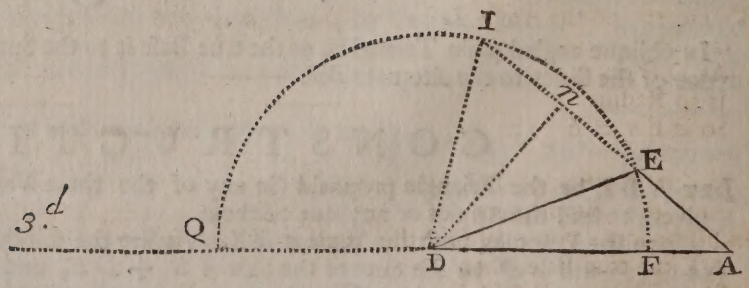
Variety 1



2.d



3.d



Case

Case 6.

THE Sides being given to find the Angles.

A D = 52
D E = 25
A E = 63 } To find the Angles { A, D, E } Variety 1.

Now by the third *Axiom* it will be

As the true Base A E = 63 Co. ar.		8,2006595
Is to the Sum of the Sides A D + D E = 77		1,8864907
So is the Difference of the Side A D — D E =	27	1,4313638
To the alternate Base A I = 33		1,5185140

HAVING thus found A I, you must take it out of A E, and there will remain I E = 30, the Half of which is *n* E = 15, one Side of the right angled Triangle D *n* E, the other Side D E, being given, you may by these find the Angle at E to be = 53° : 8.

IF you add the said 15 = *n* E to the alternate Base A I = 33, the Sum will be 48 = A *n*, one Side of the right-angled-Triangle A D *n*, by which, and the Side A D, you will find the Angle at A = 22° : 37′.

OR when you had found Angle at E, that at A, might have been found by *Axiom* 1. saying, as A D = 52, is to the Sine of the Angle at E = 53 : 8, so is the Side D E = 25 to the Sine of the Angle at A = 22° : 37′, *&c.*

EXAMPLE. 2.

LET there be given the three Sides as before to find the Angle at E, (see the Fig. for Variety the second.)

Here the true Base is	A E = 52
The alternate Base is	A I
The Sum of the Sides is A Q =	A D + D E = 88
The Difference of the Sides is A F =	A D — D E = 38

Now say (by the third *Axiom.*)

As the true Base A E = 52 Co. ar.		8,2839967
Is to the Sum of the Sides 88		1,9444827
So is the Difference of the Sides 38		1,5797836
To the alternate Base A I =	64 : 31	1,8082630
From which take the true Base A E =	52	

And there will remain ——— E I = 12,31, half is En. = 6,15.

THEN to find the Angle D E *n*, of the right-angled-Triangle D *n* E make D E Radius, and say,

As D E = 25		1,3979400
Is to Radius		10,
So is E *n* = 6 : 15		0,7888751
To the Co-sine of D E *n* = 75° : 45′		9,3909351

And the Supplement of 75° : 45′ is 104° : 15 = A E D which was required.

IN like manner you may find the Angle at E (at Variety the third) by making the shortest Side A E the true Base, then

Say as the true Base A E = 25 Co. ar.		8,6020600
Is to the Sum of the Sides A D + E D = 115		2,0606978

So is the Difference of the Sides A D – E D = 11 —— 1,0413927
To the alternate Baſe A I = 50 : 6 —— 1,7041505
From which take the true Baſe A E = 25,
And there will remain E I = 25 : 6 its half is En. = 12,8.

THEN to find the Angle *n* E D of the right-angled-Triangle D *n* E make D E Radius, and ſay,
As D E = 52 —— —— 1,7160033
Is to Radius —— —— 10,
So is E *n* = 12,8 —— —— 1,1072099

To the Co-ſine of the Angle *n* E D = 75° : 45′ —— 9,3912066
The Supplement of which 104° : 15′ equal to the Angle D E A, as was required.

CHAP. X.

The Uſe of oblique angled plain Triangles in taking Heights, Diſtances, &c.

TO take an inacceſſible Altitude from two Stations taken at pleaſure.

SUPPOSE that there is a Tower C D E F G H I, whoſe Altitude C D you would know, and alſo how far it is off the Stations A or B, taken at any convenient Diſtance from one another.

1. STANDING at B, take with your Quadrant (or any other proper Inſtrument) the Angle D B C which let be 25° : 10′.

2. MEASURE 240 Yards (or any other convenient Diſtance) from B to A, leaving B in a right Line between yourſelf and the Corner of the Tower C D.

3. BEING at A, direct your Inſtrument again to C, and find the Quantity of the Angle B A C, which let be 10° : 5′, then take this out of the Angle D B C = 25° : 10′, and there will remain the Angle A C B = 15° : 5.

4. To find how far it is from the Station at A to the Top of the Tower at C, (or Length of the viſual Ray A C) you may
Say, as the Sine of the Angle at C = 15° : 5′ Co. ar. —— 0,5846532
Is to the Diſtance between the Stations A and B = 240 — 2,3802112
So is the Sine of the Angle at B = 25° : 10′ — 9,6286472

To A C = 392,21 —— 2,5935116
To find the Length of the viſual Ray B C, ſay,
As the Sine of the Angle A C B = 15° 5′ — 0,5846532
Is to A B = 240 —— — 2,3802112
So is the Sine of the Angle at A = 10° : 5′ — 9,2432374

To B C = 161,5 —— — 2,2081018

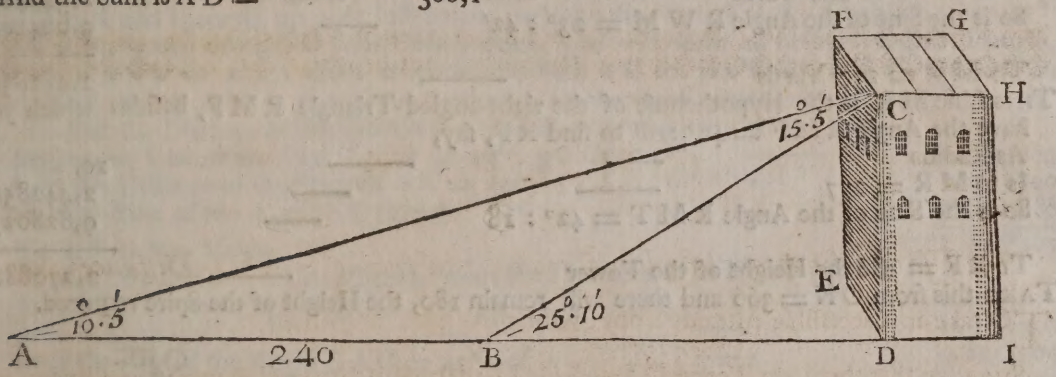
To

To find the Height of the Tower D C, ſay,

As Radius		10,
Is to B C = 161,5		1,2081018
So is the Sine of the Angle at B = 25° : 10′		9,6286472
To the Height of the Tower C D = 68,67		1,8367490

To find how far the Tower at D is from the firſt Station B, ſay as Radius — 10,

Is to B C = 161,5		2,2081018
So is the Co-ſine of the Angle at B = 25° : 10′		9,9566844
To B D =	146,1	2,1647862
To which add A B =	240,	
And the Sum is A D =	386,1	



PROB. 2.

To find the Height of an Object ſtanding upon a Hill, or of a Spire ſtanding upon a Tower, &c.

Suppose O A B C to be a Spire ſtanding upon the Tower A B C D E I, whoſe Height you would know, and alſo that of the Spire.

1. Upon Paper, or the like, draw a right-Line of any Length, as P Q, and therein aſſume any Point at pleaſure, as M for your firſt Station, then directing your Inſtrument to the Top of the Spire at O, you will have the Angle O M N, which let be 60° : 45′, and when you direct it to the Baſe at R, you have the Angle R M T, ſuppoſe of 42° : 18′: Having found theſe two Angles, then protract them from the Point M, by the Lines M *e*, and M *f*, drawn out from M at pleaſure.
2. Go from M any convenient Diſtance, ſuppoſe 220 Yards to W, and then directing your Inſtrument to O, ſuppoſe the Thread to fall upon 40 : 44′, and then directed to R, ſuppoſe it cuts 23° : 42′, then upon the Point W, protract theſe two Angles, and draw the Lines W S and W *r*, until they cut the Lines M *f* and M *e*, in the Points O and R, from which Points let fall the Lines O N and R F perpendicular to P Q, the firſt of which is the Height of the Spire and Tower together, and the other the Height of the Tower; whence the Height of the Spire may be found by the ſame Line of equal Parts by which you laid down M W.
3. By the Interſections of theſe Lines you have two oblique angled plain Triangles W O M, and W R M, and two right-angled-Triangles M R F and M O N.

In the oblique angled Triangle W O M you have the Side W M = 220 Yards, the Angle M W O = 40° : 44, and external Angle O M N = 60° : 45′, from which take 40° : 44′, and there will remain the Angle W O M = 20° : 01′ by theſe find the Line M O,

By ſaying, as the Sine of the Angle W O M = 20° : 1′. Co. ar.		0,4656014
Is to the Side W M = 220 Yards		2,3424226
So is the Sine of the Angle M W O = 40° : 44′		9,8146067
To the Side M O = 419 Yards		2,6226307

This Line M O is the Hypothenuſe of the right-angled-Triangle M O N, by help of which, and the Angle O M N = 60° : 45′ you may find O N, thus,

As Radius		10,
Is to M O = 419 Yards		2,6226307
So is the Sine of O M N = 60° : 45′		9,9407634
To O N = 366, the Height of the Tower and Spire		2,5633941

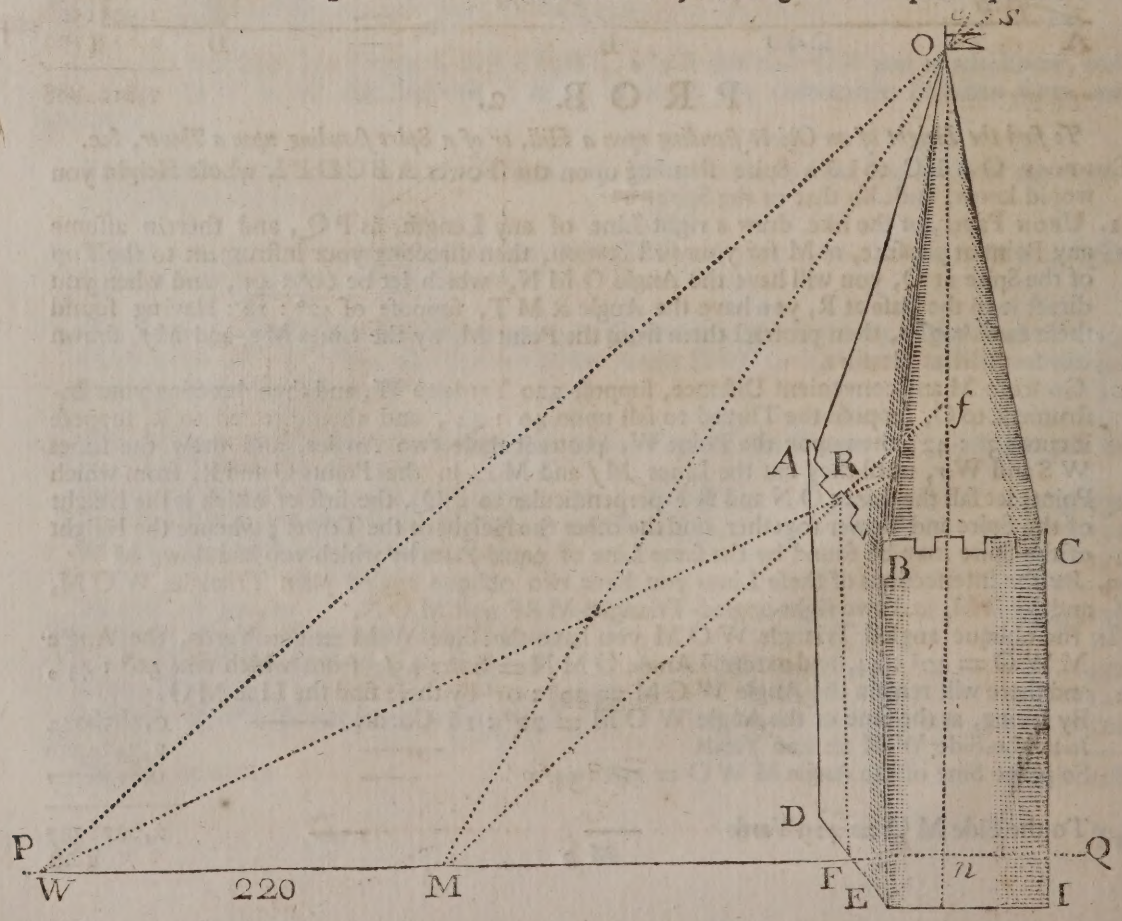
Again, in the oblique angled Triangle W R M, you have the Side W M = 220 the Angle M W R = 23° : 42′, and external Angle R M N = 42° : 18′ from which take M W R = 23° : 42′. and there will remain the Angle W R M = 18 : 36, by theſe find the Side M R, ſaying,

As the Sine of the Angle W R M = 18° : 36′ Co. ar.		0,4962647
Is to W M = 220 Yards		2,3424226
So is the Sine of the Angle R W M = 23° : 42′		9,6041696
To M R = 277 Yards		2,4428569

This Line M R is the Hypothenuſe of the right-angled-Triangle R M F, beſides which you have the Angle R M F = 42° : 18′ to find R F, ſay,

As Radius		10,
Is to M R = 277		2,4428569
So is the Sine of the Angle R M T = 42° : 18		9,8280231
To R F = 186 the Height of the Tower		2,2708800

Take this from O N = 366 and there will remain 180, the Height of the Spire required.



Note, That it may happen ſome Times when you would protract Things of this Nature by a Line of Chords and equal Parts that the Lines (ſuppoſe W O and M O, or W R and M R) will not cut one another on the Paper upon which you are working, tho' very large ; but then in ſuch Caſes you may draw an Eye-draft, and by that find what you want by Calculation, as in the foregoing Page.

PROB. 3.

To take an inacceſſible Diſtance at two Stations.

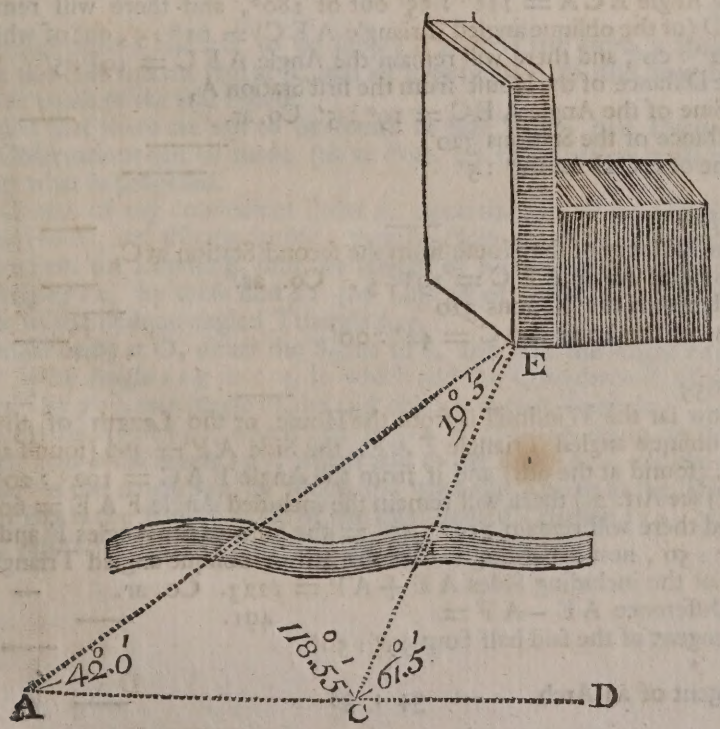
BEING near a River, I ſee a Church on the other Side, whoſe Diſtance from any two Stations A and C, taken at pleaſure I would know.

1. SET up two Marks at A and C, and meaſure their Diſtance, which let be 320 Yards.
2. PLANTING your Theodolite, (or any other convenient Inſtrument) at C, take the Angle E C A, which let be 118° : 55′ whoſe Supplement is E C D = 61° : 5′
3. Go to A and there ſet up your Inſtrument, and take the Angle E A C, ſuppoſe it to be 42° 00′, then having protracted your Obſervations, you will form an oblique angled Tiangle, ſuch as A E C in which there is given the Side A C = 320 Yards, and if you take the Angle E A C = 42° : 0′ out of the Angle E C D, there will remain 19 : 5′ = C E A.
4. To find the Diſtance of the Church from the Station at A, you may

Say, as the Sine of the Angle C E A = 19° : 5′.	Co. ar.	0,4855279
Is to the Diſtance of the Stations A C = 320	---	25051500
So is the Sine of the Angle E C D = 61° : 5′	---	9,9421688
To A E = 857 Yards		2,9328467

5. To find the Diſtance of the Church from the Station at C, ſay,

As the Sine of the Angle C E A = 19° : 5′.	Co. ar.	9,4855279
Is to A C = 320	---	2,5051500
So is the Sine of the Angle E A C = 42° : 0′	---	9,8255109
To E C = 655 Yards	---	2,8161888



PROB.

PROB. 4.

To find the Diſtance of two or more Places from the Stations whence you obſerv'd them, and alſo how far they are off one another.

BEING deſirous to find how far the Windmill at F is from the Houſe at E, and how far either of them is from the Stations A and C taken at pleaſure.

1. SET up two Marks at the Stations A and C, and meaſure their Diſtance, which let be 320 Yards.
2. PLACE your Inſtrument at A, and take the Angle F A C, which let be 102° : 20′, and alſo the Angle C A E = 42° : 00′.
3. REMOVE your Inſtrument to C, and there take the Angle F C A = 41° : 56′ (ſuppoſe) and E C A = 118° : 15′.
4. HAVING protracted, or laid down your Obſervations upon Paper, you will form two oblique angled Triangles, *viz.* F A C and E A C.
5. TAKE the Angle F A C = 102° : 20′ out of 180°, there will remain the external Angle F A B = 77° : 40′ from which take the Angle F C A = 41° 56′, and there will remain the Angle A F C = 35° : 44′, which being found, then,
6. To find how far the Windmill is from your firſt Station, at A,

Say as the Sine of A F C = 35° : 44′, Co. ar.	0,2335771
Is to the Diſtance of the Stations 320	2,5051500
So is the Sine of F C A = 41° : 56′	9,8249490
To A F = 366	2,5636761

7. To find how far the Windmill is from the ſecond Station at C.

Say as the Sine of the Angle A F C = 35° : 44′. Co. ar.	0,2335771
Is to the Diſtance of the Stations 320	2,5051500
So is the Sine of the external Angle F A B = 77° : 40′	9,9898597
To F C = 535	2,7285868

8. TAKE the Angle E C A = 118° : 55′ out of 180°, and there will remain the external Angle E C D (of the oblique angled Triangle A E C) = 61° : 5′, out of which take the Angle E A C = 42° : 00′, and there will remain the Angle A E C = 19° : 5′
9. To find the Diſtance of the Houſe from the firſt Station A,

Say, as the Sine of the Angle A E C = 19° : 5′ Co. ar.	0,4855279
Is to the Diſtance of the Stations 320	2,5051500
So is the Sine of E C D = 61° : 5′	9,9421688
To A E = 857	2,9328467

10. To find the Diſtance of the Houſe from the ſecond Station at C,

Say, as the Sine of Angle A E C = 19° : 5′. Co. ar.	0,4855279
Is to the Diſtance of the Stations 320	2,5051500
So is the Sine of the Angle E A C = 42° : 00′	9,8255109
To E C = 655	2,8161888

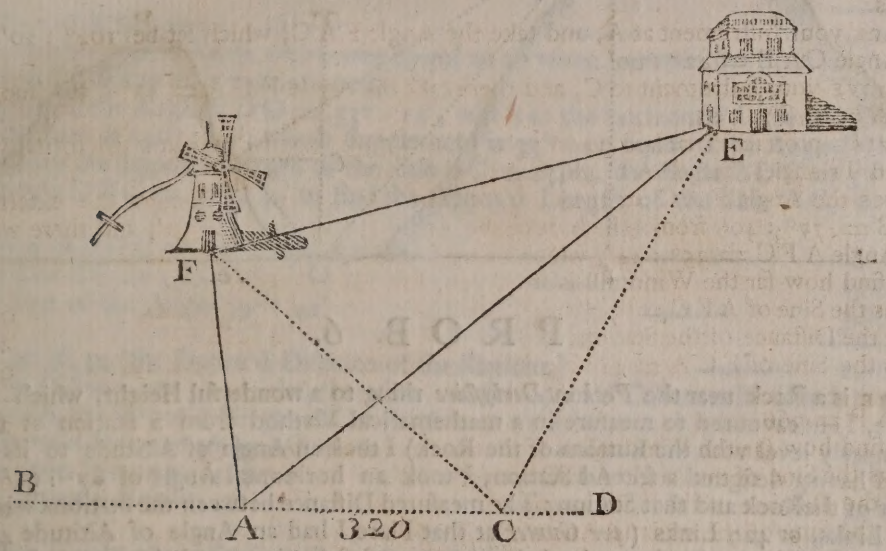
11. To find how far the Windmill is from the Houſe, or the Length of the Line F E, you have in the oblique angled Triangle F A E, the Side A F = 366 (found at Art. 6) the Side A E = 857, (found at the 9th) and if from the Angle F A C = 102° : 20′ you take C A E = 42° : 00′) ſee Art. 2.) there will remain the included Angle F A E = 60° : 20′, this out of 180°, and there will remain 119° : 40′ = the Sum of the Angles F and E, the Half of which is 59° : 50′, now according to *Caſe* the 4th of oblique angled Triangles, ſay,

As the Sum of the including Sides A E + A F = 1223. Co. ar.	6,9125736
Is to their Difference A E − A F = 491.	2,6910815
So is the Tangent of the ſaid half Sum 59° : 50′	10,2356480
To the Tangent of an Arch — 34 : 38	9,8393031

THE

THE Difference between this and the half Sum is 25 : 12 equal to the Angle F E A.
Then ſay, again

As the Sine of the Angle F E A = 25 : 12′	Co. ar.	——	0,3708155
Is to the Side A F = 366	——		2,5634810
So is the Sine of the Angle F A E = 60° : 20′		——	9,9389796
To the Diſtance required, *viz.* F E = 747		——	2,8732761

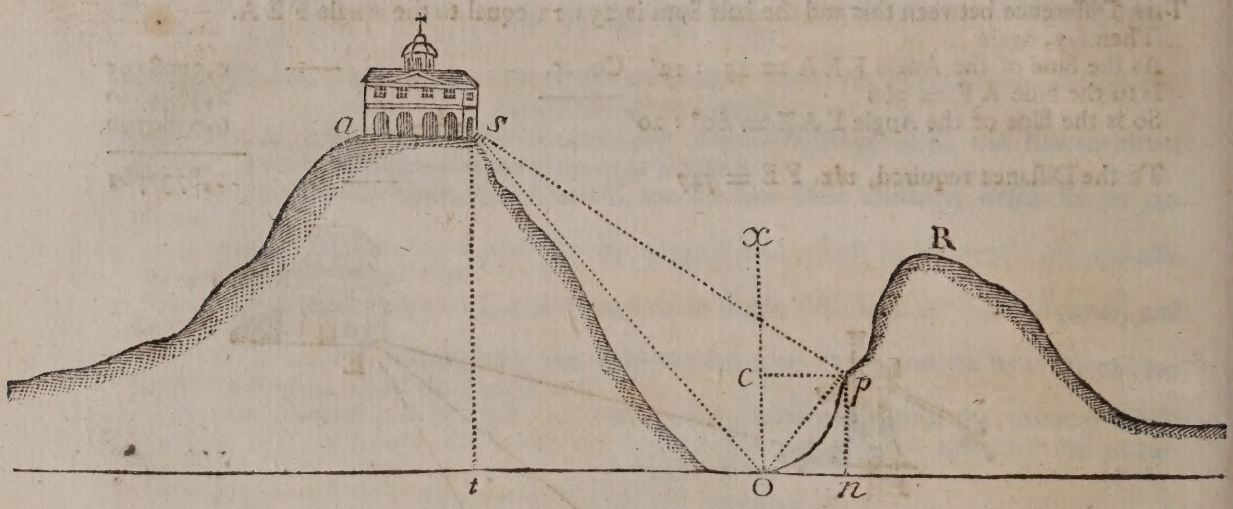


PROB. 5.

LET there be a Houſe upon a Hill *a* S, to be ſupply'd with Water from a Spring at O, in a Valley between this Hill and the Hill at R, and let it be required to find how high the Water muſt be raiſed to come to the ſaid Houſe.

HERE it is ſuppoſed that there are not to be found in this Valley two Places upon a Level from whence Obſervations can be made (as at *Prob.* 1.) therefore we muſt take ſome other Way to anſwer what is propoſed.

1. THUS, make Choice of any convenient Point *p*, upon the Side of the Hill R, from whence you may ſee the Houſe, and alſo the Spring; then by Help of *Siſſon*'s Theodolite, or any other ſuch Inſtrument for Levelling, find the Height of *p n* above the Level of the Spring at O, and the Angle *p o n*, by theſe find *p o* (by *Caſe* 2d of right-angled-Triangles) and this Line is one Side of the oblique angled Triangle *p s o*.
2. YOUR Inſtrument being at O, direct the Sights to S, and take the Angle *t o s*, whoſe Complement to 90° is the Angle *s o x* = *t s o*, to which add the Complement of the Angle *p o n*, and the Sum will be *p o s*, one Angle of the ſaid oblique angled Triangle.
3. WHEN at *p*, take the Angle *c p s*, which add to *p o n* = *c p o* (*c p* being an horizontal Line) the Sum of theſe two is the Angle *s p o*, which add to the Angle *p o s*, and the Supplement of that Sum to 180° is the Angle *p s o*, by which and what elſe is given in the Triangle *s p o*, find the Side *s o*, and this is the Hypothenuſe of the right-angled Triangle *t s o*, with which, and the Angle *t o s*, before found, you may find the Side *ſt* required.



PROB. 6.

THERE is a Rock near the *Peak* in *Derbyſhire* riſing to a wonderful Height, which being inacceſſible, I endeavoured to meaſure in a mathematical Method from a Station at ſome Diſtance (nearly level with the Bottom of the Rock) I took an Angle of Altitude to its Top 47° 30′; and having deſigned a ſecond Station, I took an horizontal Angle of 87° : 05′ between the Foot of the Rock and that Station: The meaſured Diſtance between the Stations was 4 Chains and 29 Links, or 429 Links (*per Gunter*) at that Place I had an Angle of Altitude 41°, 12′; but forgot from hence to take an Angle between my firſt Station, and the Foot of the Rock; yet am in Hopes ſome curious Artiſt, will from this *Data* determine the perpendicular Height of this ſtupendous Work.

THIS was propoſed by Mr. *Henry Travis*, at Page 21 of *The Ladies Diary* for 1740, and anſwered by the ſame ingenious Perſon at Page 6th and 7th of *The Ladies Diary* for 1741. But, ſuppoſing my young *Tyro* (for whoſe Sake this whole Work is compoſed) has not yet arrived to the Knowledge of Algebra, I have given him the Method of anſwering the Queſtion by ſingle Poſition as follows;

HAVING drawn A B to repreſent the Rock, ſuppoſe its Height A B to be 50 Links, C the firſt Station, and B C its Diſtance from the Foot of the Rock at B, and making a right-Angle at B with the Height A B.

LET D be the ſecond Station, and its Diſtance from the Foot of the Rock to be B D, making a right-Angle at B with the Line A B and D C the Diſtance of the two Stations 429 Links, Then,

1. BY Help of the Angle A C B = 47° : 30′, and ſuppos'd Height of the Rock 50 Links, find the ſuppos'd Diſtance of the firſt Station B C.

Thus, as the Sine of the Angle A C B = 47° : 30′. Co. ar.	—	0,1323691
Is to the ſuppos'd Height A B = 50	—	1,6989700
So is the Co-ſine of 47° : 30′	—	9,8296833
To the ſuppos'd Diſtance of the 1ſt Station B C = 4581	—	1,6610224

2. WITH the Angle A D B = 41° : 12, and ſuppoſed Height A B = 50 find the Diſtance of the ſecond Station from the Foot of the Rock, *viz.* B D, ſaying,

As the Sine of A D B = 41° : 12′ Co. ar.	—	0,1813193
Is to the ſuppos'd Height A B = 50	—	1,6989700
So is the Co-ſine of 41° : 12′	—	9,8764574
To B D = 57,11	—	1,7567467

3. Now

3. Now in the oblique angled Triangle D C B, you have the ſuppos'd Length of the Side B C = 45,81, and B D = 57,11 with the horizontal Angle B C D = 87° : 5′ to find the Angle B D C, and this will be the Quantity of the horizontal Angle which was forgotten to be taken between the firſt Station and Foot of the Rock, ſaying,

Thus, as the ſuppos'd Length of the Side B D = 57,11. Co. ar.	8,2432533
Is to the Sine of the Angle B C D = 87° : 5′	9,9994370
So is the ſuppos'd Length of B C = 45,81	1,6609603
To the Sine of B D C = 53 : 14′	9,9036506

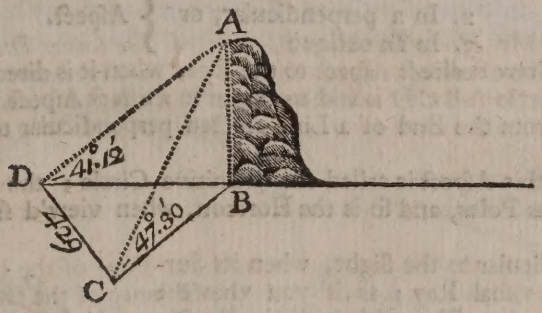
And this is the horizontal Angle which was forgot to be taken between the Foot of the Rock and the firſt Station as before-mentioned,

4. Having found the Angle B D C = 53° : 14′, add it to the horizontal Angle B C D = 87° 5′, and the Sum is 140° : 19′, whoſe Supplement is 39° : 41′ equal to the Angle D B C; ſo now you have the ſuppos'd Length of the Side B C = 45,81, the Angle B D C = 53° : 14′, and the Angle D B C = 39° : 41′ to find the ſuppos'd Length of the Side D C, to which ſay,

As the Sine of B D C = 53° : 14′, Co. ar.	0,0963243
Is to the Side B C = 45,81	1,6609603
So is the Sine of the Angle 39° : 41′	9,8051908
To D C = 36,52 (the ſuppos'd Diſtance of the Stations.)	1,5624754

But the true Diſtance was 429 Links, therefore, as the ſuppos'd Diſtance is found to be too little, ſo muſt the ſuppos'd Height of the Rock be; then ſay,

As the ſuppos'd Diſtance 36,52. Co. ar.	8,4374692
Is to the true Diſtance 429.	2,6324573
So is the ſuppos'd Height 50.	1,6989700
To the true Height of the Rock A B = 587, 4 Links, as was required.	2,7688965



N CHAP.

CHAP. XI.

Stereographic Problems.

In order to project the Sphere, or the ſeveral Caſes in Spherical Triangles.

THO' a Sphere or Globe is the beſt Inſtrument for giving a true Idea of the ſeveral Lines or Circles that are us'd in Aſtronomy, Navigation, Dialling, *&c.* yet it is one of the worſt for reſolving particular Queſtions; therefore, if a Repreſentation of the Sphere, or ſome Part thereof, and the Solution of a Queſtion be both required, it may be done near the Truth by a Delineation of Lines or Circles upon a Plane; and in order to this, it will be requiſite to ſhew,

1. How the Circles of the Sphere both great and ſmall, may in their ſeveral Poſitions be deſcribed upon a Plane.
2. How any of theſe Circles may be meaſured, either in reſpect to itſelf, or to the primitive Circle.
3. How any Angle propounded may be projected, or being projected may be meaſured.

IN the firſt of theſe we are to conſider the ſeveral Varieties of Poſition that the Circles may have in reſpect of that Point from whence the Eye beholds them.

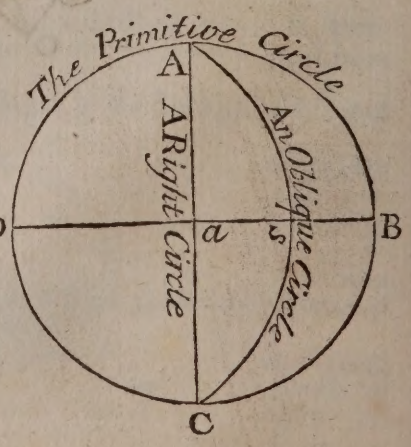
THE Varieties of Poſition are but Three, which are either,

1. In a direct
2. In a perpendicular, or } Aſpect.
3. In an oblique

A CIRCLE is ſaid to have a direct Aſpect to the Eye, when it is directly oppoſite to the Superficies of it; ſo the Circle A B C D is ſaid to appear in a direct Aſpect, or as a perfect Circle to an Eye which views it from the End of a Line erected perpendicular to the Plane of the ſaid Circle in its Center *a*.

Now a Circle having this Aſpect is called the primitive Circle; the Equator is ſuch when viewed from either of its Poles, and ſo is the Horizon, when view'd from the Zenith or Nadir, as A B C D.

A CIRCLE is perpendicular to the Sight, when its Surface is parallel to the viſual Ray; as if you view'd a Circle of Paper, Paſteboard, or Tin, with its Edge directly towards the Eye, it would appear as a right-Line; or if inſtead of an Eye you plac'd a Candle, and then turn'd the Circle in ſuch a Manner that its Shadow becomes a right-Line upon any Plane, ſuch as a Wall, or *&c.* on the other Side, then would that Circle be in a perpendicular Poſition to the Eye, or Candle, and ſo would the Meridian appear at Noon, when projected by the Sun upon the Plane of the Hour of ſix: And this is called a right-Circle, as A C, or D B.



A CIRCLE is ſaid to have an oblique Aſpect to the Eye, when it does not appear as either of the former, ſo the Shadow of the Equator would appear as an obliqueCircle, if caſt upon the Plane of the Circle of the Hour of ſix, by the Sun, he having either north or ſouth Declination, and upon the Meridian as A *s* C.

PROB.

PROB. 1.

THE Pole of a great Circle is a Point 90 Degrees from the Plane of the Circle; to find which there are three Cases.

Case 1.

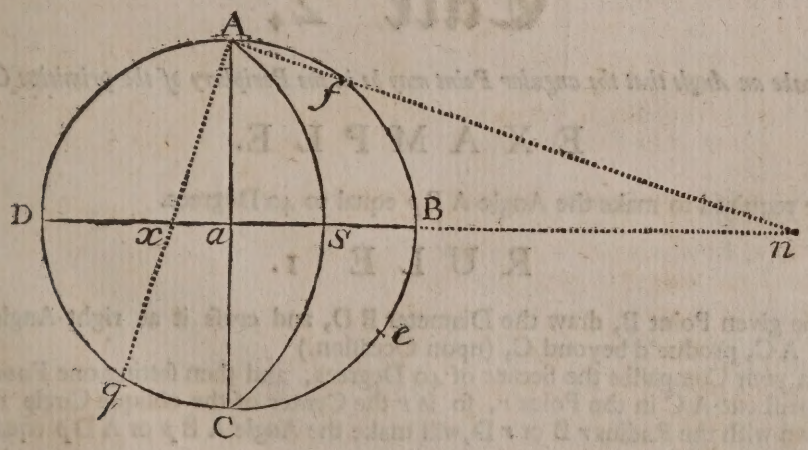
IF the Pole to be found is that of the primitive Circle A B C D, then the Center thereof at *a* is its Pole.

Case 2.

IF it be a right-Circle as A C, then its Poles are D and B, two Points upon the primitive Circle, each 90 Degrees from A or C.

Case 3.

IF you are to find the Pole of an oblique Circle, as A *s* C, which (as all great Circles do,) cut the primitive Circle in two oppoſite Points, as A and C, having one of its Poles between its own Center, and the Center of the primitive Circle, and in a Line perpendicular to its Plane; lay a Ruler upon A and S, and it will cut the primitive Circle in *e*; from *e* ſet off 90 Degrees to *q* and *f*, then lay a Ruler upon A and *q*, and it will cut the right-Circle D B, in the Point *x*, which is one of the Poles required; if the other be wanting, then lay a Ruler upon A and *f*, and it will cut the Line D B (produc'd) in the Point *n* for the other Pole.



PROB. 2.

To lay down on the Projection any Angle required.

A ſpherical Angle is the Inclination of two great Circles (ſuppoſe two Meridians) upon the Sphere, meeting in a Point (as the North or South Pole) and whoſe Meaſure is the Arch of a great

great Circle included between the two Arches which conſtitute the Angle at 90 Degrees diſtant from the angular Point, ſuch is the Equinoctial to the Angles made by the Interſection of the Meridians at either Pole, or the Horizon to the vertical Circles that interſect one another in the Zenith and Nadir.

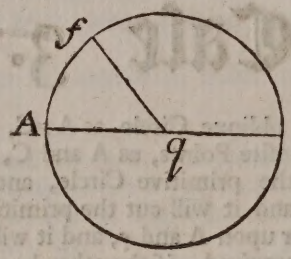
In this Problem there are three Caſes.

1. When the angular Point is at the Center of the primitive Circle.
2. When the angular Point is in the Periphery of the primitive Circle.
3. When the angular Point is any where within the primitive Circle, but not in the Center.

Caſe 1.

To make an Angle when the angular Point is in the Center of the primitive Circle.

This is made in all Reſpects like a plane Angle; ſo if it was required to make an Angle of 50 Degrees with the Line A *q*, you muſt take 50 Degrees in your Compaſſes from the ſame Line of Chords by which you drew the primitive Circle, and lay that Extent from A to *f*, then draw the Line *f q*, and it is done.



Caſe 2.

To make an Angle that the angular Point may be in the Periphery of the primitive Circle.

EXAMPLE.

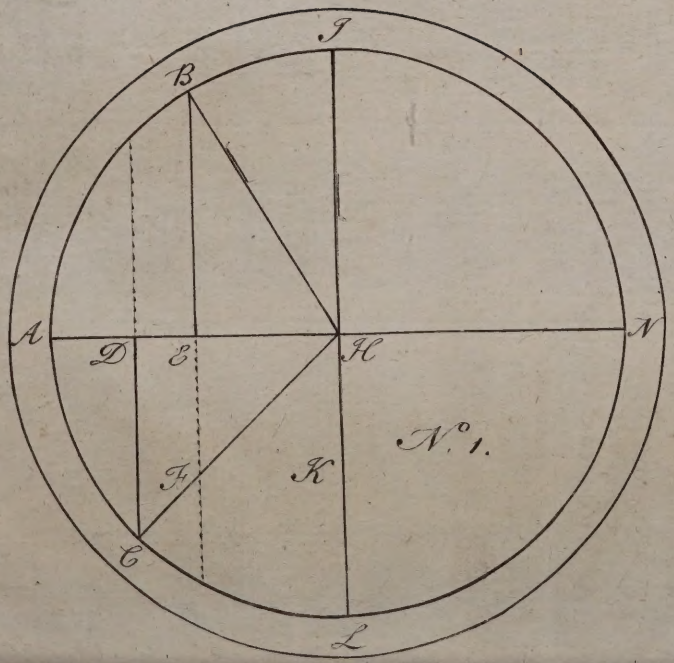
Let it be required to make the Angle A B *p* equal to 40 Degrees.

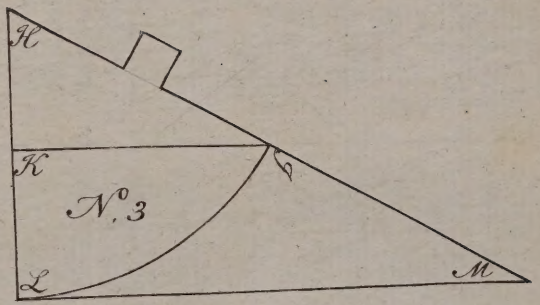
RULE 1.

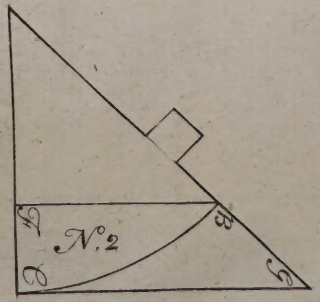
1. From the given Point B, draw the Diameter B D, and croſs it at right-Angles with the Diameter A C, produc'd beyond C, (upon Occaſion.)
2. Take in your Compaſſes the Secant of 40 Degrees, and then ſetting one Foot in B or D, the other will cut A C in the Point *r*, ſo is *r* the Center of the oblique Circle B *p d*, which being drawn with the Radius *r* B or *r* D, will make the Angle A B *p* or A D *p* equal to 40 Degrees, as was required.

RULE 2.

It may happen, that you have a Scale that has not the Line of Secants upon it, but only a Line of Chords, with which you may work thus: Lay 40 Degrees from A to *n*, then lay a Ruler upon

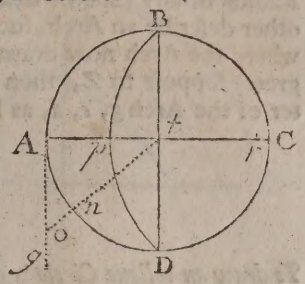

J
B
A
D
E
H
N
F
K
N.° 1.
C
L


R
K
N.° 3
L
M


N.° 2

upon B and *n*, and it will cut A C in *p*, so you have three Points, *viz.* B *p* D, thro' which you may draw the Circle by *Prob.* 12 of *Chap.* 2, and find its Center at *r*, as before.

Or thus: Draw A *q* at Right-Angles to A C, and having found *n* as before, lay a Ruler upon and *n*, and it will cut the Line A *q* in *o*, so is *t o* the Secant of 40 Degrees, and to be set from B, or D to *r*, as before, or take A *o*, the Tangent of the given Arch, and set that from *t* to *r*, so is *r* the Center of the oblique Circle B *p* D, as above.

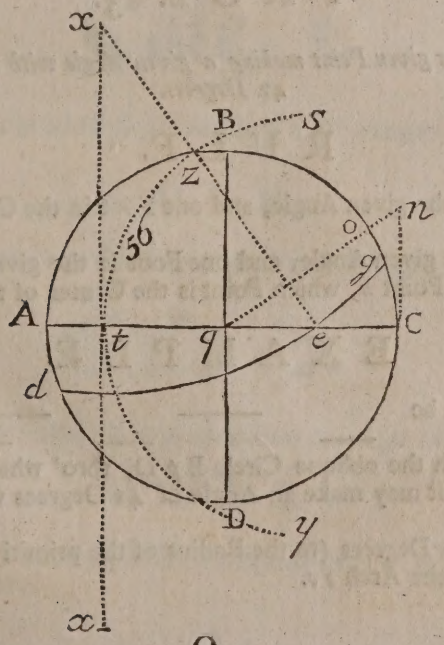


Case 3.

To draw an oblique Circle d e g, *so as to make a given Angle (suppose of* 34 *Degrees) with a Right-Circle* A q C, *at the Point* e.

RULE.

1. Thro' *e*, and *q*, draw the Diameter, or right Circle A *q e* C, and cross it at Right-Angles with B *q* D.
2. Lay a Ruler upon D and *e*, and it will cut the primitive Circle in *o*, produce *q o*, until it cuts the Tangent C *n* in the Point *n*.
3. Take C *n* the Tangent of the Arch C *o* in your Compasses, and lay that on the Line A C from *q* to *t*, then is *t* the Center of a great Circle that would pass thro' the three Points B *e* and D.
4. Upon the Point *t* draw *x x* at right Angles with A C, and lay off the Tangent of the Compliment of the given Angle (to the Radius *e t*) from *t* either way to *x*, as occasion requires.
5. Upon the Point *x*, with the Distance *x e*, describe the Circle *d e g*, the Angle A *e d* is the Angle required to be projected equal to 34 Degrees.



O

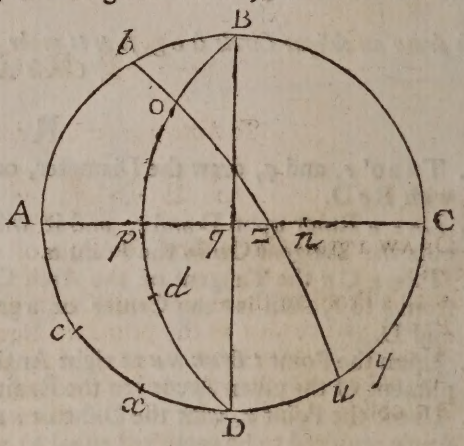
Not

Note, That if you have not a Line of Natural Tangents to the Radius *e t*, you may take the Radius of any Line of Chords in your Compaſſes, and then ſetting one Foot in *e*, with the other deſcribe an Arch, ſuch as *ſ t y*, upon which from *t* (or any other Point in the Line A C where the Arch now drawn ſhall interſect it) ſet down 56 Degrees (the Compliment of 34 Degrees) ſuppoſe to *Z*, then lay a Ruler upon *e* and *z*, and it will cut the Line *x t* in *x*, the Center of the Arch *g*, *e*, *d*, as before.

Caſe 4.

To draw an oblique Circle b o z u, *through a given Point* (*ſuppoſe* o *in the oblique Circle* B o p D), *ſo as to make with the ſaid Circle an Angle, ſuppoſe* D o z, *of* 74 *Degrees.*

1. LAY a Ruler upon the Pole *n* of the Circle *B o p d*, and the given Point *o*, and it will cut the primitive Circle in *b*.
2. FROM *b* ſet off 90 Degrees to *c*, a Ruler laid upon the Pole *n*, and the Point *c*, will cut the oblique Circle B *o p* D in *d*.
3. A RULER laid upon *o* and *d* will cut the primitive Circle in *x*, from whence ſet off the given Angle 74 Degrees to *y*.
4. A RULER laid upon *o* and *y* will cut the right Circle A C in *z*, ſo now you have two Points *o* and *z*, thro' which you may draw a great Circle *b o z u*, as directed at *Prob.* 13 of *Chapter* the 2d, and thereby form the Angle D *o z* = 74 Deg.



PROB. 3.

To draw a great Circle thro' a given Point making a given Angle with the primitive Circle, ſuppoſe of 42 *Degrees.*

RULE.

1. WITH the Tangent of the given Angle, and one Foot in the Center of the primitive Circle *q*, deſcribe an Arch *o o*.
2. WITH the Secant of the given Angle, and one Foot in the given Point *p*, ſtrike an Arch cutting the former in the Point *n*, which Point is the Center of the oblique Circle *x p d*, and its Radius *n p*.

EXAMPLE.

LET the Primitive Circle be ——— ——— ABCD
Its Center ——— ——— ——— *q*

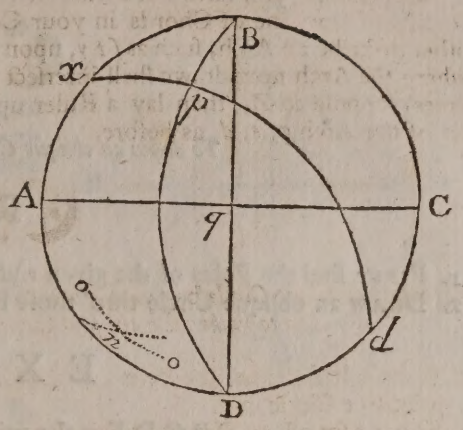
AND the given Point *p* in the oblique Circle B *p* D, thro' which it is required to draw an oblique Circle *x p d*, ſo that it may make an Angle of 42 Degrees with the primitive Circle.

1. With the Tangent of 42 Degrees (to the Radius of the primitive Circle) and one Foot in the Center at *q*, deſcribe the Arch *o o*.

2. WITH

2. WITH the Secant of the ſame Angle, and one Foot in the given Point *p*, make an Arch cutting the Arch *o o* in *n*, then is *n* the Center upon which (at the Diſtance of *n p*) deſcribe the Circle *x p d* which will make the Angle *p x* B equal to 42 Degrees, as was required. See the Figure following.

Note, That the Point *p* muſt be ſo far from the Center *q*, that the Tangent from *q*, and the Secant from *p*, may interſect each other, elſe it is impoſſible.



PROB. 4.

To draw a great Circle perpendicular to a given great Circle.

GENERAL RULE.

DRAW a great Circle thro' the Pole of the given great Circle, and it will be perpendicular thereto.

IN this Problem there are Four Caſes, either,

1. PERPENDICULAR to the primitive Circle.
2. A right Circle perpendicular to a right Circle.
3. AN oblique Circle perpendicular to a right Circle.
4. AN oblique Circle perpendicular to an oblique Circle.

Caſe 1.

To draw a great Circle perpendicular to the primitive Circle.

RULE.

THRO' the Center of the primitive Circle (which is its Pole) draw the Diameter D B; and it is done.

Caſe 2.

To draw a right Circle perpendicular to a right Circle.

RULE.

THIS is done by drawing a Diameter, as D B, perpendicular to the Diameter or right Circle given C E. See the next Figure.

Case 3.

To draw an oblique Circle perpendicular to a right Circle.

RULE.

1. FIRST find the Poles of the given right Circle.
2. DRAW an oblique Circle thro' thoſe Poles.

EXAMPLE.

B C D E is the primitive Circle
A the Center thereof } Given.
B A D is a right Circle

To draw the oblique Circle C *q* E, *perpendicular to the right Circle* B A D.

1. TAKE the Chord of 90 Degrees, and lay it from B or D both ways to C and E, which are the two Poles of B A D.
2. WITH any Diſtance, and one Foot of your Compaſſes in C, and then in E, draw Arches to cut each other in *x*, ſo is *x* the Center of the oblique Circle C*q* E required to be drawn.

BUT here obſerve, that if the oblique Circle required to be drawn, was ſo limited as to make a given Angle with the primitive, ſuppoſing of 40 Degrees, then ſee *Caſe* 2d of *Prob.* 2d, or take A *o*, the Tangent of the given Arch in your Compaſſes, and ſet that from *t* to *r*, ſo will *r* be the Center as before. See the Figure for the ſaid Caſe.

Case 4.

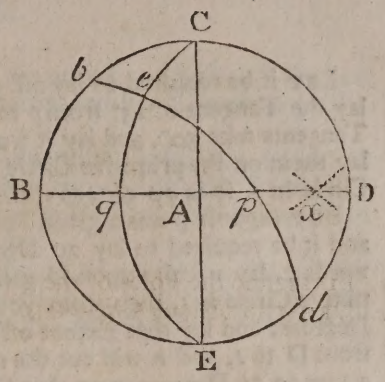
To draw an oblique Circle, ſuppoſe b p d, *perpendicular to a given oblique Circle* C *q* E.

RULE.

1. FIND the Pole *p* of the given oblique Circle C *q* E, by *Caſe* 3 of *Prob.* 1.
2. DRAW ſuch an Arch as may paſs thro' the Pole ſo found; and alſo interſect the primitive Circle in Points diametrically oppoſite, as at *b* and *d*, ſo will the oblique Circle *b p d* be perpendicular to the oblique Circle C*q* E.

BUT if the oblique Circle be ſo limited as to paſs thro' a given Point, ſuppoſe *e* in the given oblique C *q* E, then you have two Points, *viz.* the Pole at *p*, and Point *e*, thro which you may draw the oblique Circle *b e p d*, thro' the Pole, as at *Prob.* 13 of *Chap.* 2.

AGAIN



AGAIN, if it had been required to draw the oblique Circle thro' the Pole *p*, or perpendicular to the oblique Circle C *q* E, ſo as to make an Angle given at the primitive Circle; it may be done by *Prob.* 3, when poſſible.

PROB. 5.

To lay off any Number of Degrees reckoned upon a great Circle.

IN this *Prob.* there are three Caſes.

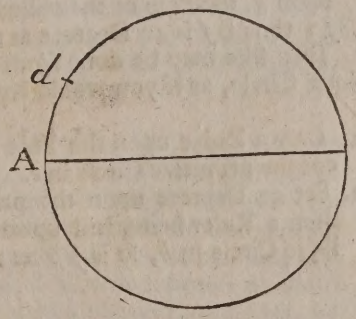
As 1. ON the primitive Circle.
2. ON a right Circle.
3. ON an oblique Circle.

Caſe 1.

To lay off any Number of Degrees upon the primitive Circle.

RULE.

TAKE from the Line of Chords (by which you did draw the primitive Circle) the Degrees given, ſuppoſe 32, and lay that Extent on the primitive Circle from A to *d*, and it is



Caſe 2.

To lay off any Number of Degrees upon a right Circle A C.

RULE.

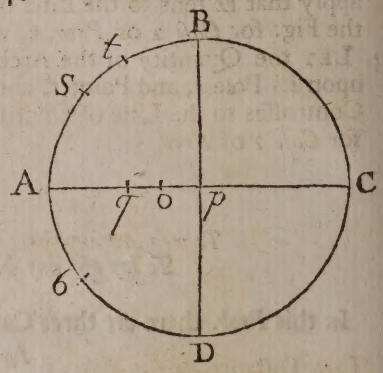
TAKE the Tangent of half the Degrees given, and lay them from the Center at *p*, upon the right Circle A C, and it is done.

EXAMPLE

EXAMPLE.

LET it be required to lay off 50 Degrees upon the right Circle A C, from *p* towards A, then lay the Tangent of 25° from *p* to *q*, ſo is the Part *p q* = to 50 Degrees, or from the Line of half Tangents take 50°, and lay it from *p* to *q*, or laſtly, take 50° from the Line of Chords, and lay them on the primitive Circle from D to *b*, then a Ruler laid from B to *b* will cut the right Circle in *q*, ſo is *q p* 50°, as above, and A *b* is equal to A *q* = 40.

BUT ſuppoſe *q* was a given Point in the right Circle A C, and it be required to lay 20 Degrees from that Point towards *p*, lay a Ruler upon D and *q*, and it will cut the primitive Circle in *s*, then from your Line of Chords take 20 Degrees, and lay that Extent off from *s* to *t*, lay a Ruler from D to *t*, and it will cut the right Circle A C in *o*, ſo is *q o* = to 20 Degrees upon that Circle.



Caſe 3.

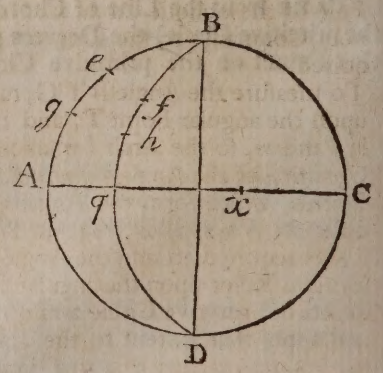
To lay off any Number of Degrees (ſuppoſe 42) upon an oblique Circle, as B *q* D, *from any Point given, as from* B *towards* q, &c.

RULE.

1. FIND the Pole of the oblique Circle (by *Caſe* 3 of *Prob.* 1.) which let be at at *x*.
2. FROM the Line of Chords, take 42 Degrees, and ſet them off from B to *e*, then lay a Ruler upon *x*, the Pole of the oblique Circle, and the Point *e*, and it will cut the oblique Circle in *f*; thus B *f* is 42 Degrees as required.

THE like may be done from any other Point in the oblique Circle, as if you would lay off 22° from *f* towrrds *q*.

1. LAY a Ruler upon the Pole *x*, and Point *f*, and it will cut the primitive Circle in *e*.
2. Set 22 Degrees upon the primitive Circle from *e* to *g*, then a Ruler being laid upon *x*, and *g* will cut the oblique Circle in *h*, ſo is *f h* = 22 Degrees.



PROB. 6.

To meaſure any Part of a great Circle.

IN this *Prob.* there are three Caſes which are but the Inverſe of the former.

EX-

EXAMPLES.

1. LET it be required to find the Quantity of the Arch A *d* of the primitive Circle. Take A *d* in your Compaſſes, and apply that Extent to the Line of Chords, and it will reach to 32 Degrees, the Anſwer. See the Figure for *Caſe* 1 of *Prob.* 5.
2. LET it be required to find the Quantity of the Arch *p q* of the right Circle A C, lay a Ruler upon B and *q*, and it will cut the primitive Circle in *b*, then take D *b* in your Compaſſes apply that Extent to the Line of Chords, and it will fall upon 50 Degrees, the Anſwer. See the Fig. for *Caſe* 2 of *Prob.* 5, where A *b* is the Meaſure of A *q*.
3. LET the Quantity of the Arch B *f* of the oblique Circle B *q* D be required; lay a Ruler upon its Pole *x*, and Point *f*, and it will cut the primitive Circle in *e*; then take B *e* in your Compaſſes to the Line of Chords, and it will reach to 42 Degrees, the Anſwer. See the Fig. for *Caſe* 2 of *Prob.* 5.

PROB. 7.

To meaſure any ſpherical Angle. In this Problem there are four Caſes.

GENERAL RULE.

THE Diſtance of the Poles of the containing Sides is the Meaſure of the contained Angle.

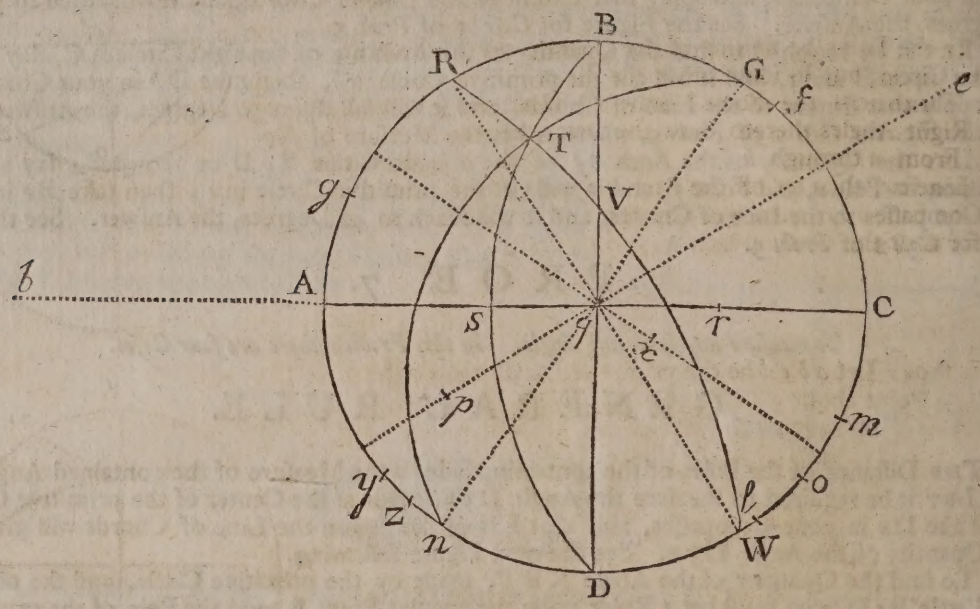
1. LET it be reguired to meaſure the Angle D *q n*, made at the Center of the primitive Circle. Take D *n* in your Compaſſes, and that Extent laid upon the Line of Chords will give the Quantity of the Angle D *q n*. See the next Figure following.
2. To find the Quantity of the Angle R B T, made by the primitive Circle, and the oblique Circle B *s* D, you muſt lay a Ruler upon the angular Point B, and the Pole of the primitive Circle *q*, and it will cut the primitive Circle in D.

AGAIN, Lay a Ruler on the angular Point B, and the Pole of the oblique Circle at *r*, and it will cut the primitive Circle in *o*; take the Arch D *o* in your Compaſſes, and apply that Extent to the Line of Chords and it will there ſhew the Quantity of the Angle R B T.

3. To meaſure the Angle R V B, made by the right Circle B V D and oblique Circle R V W, here A being the Pole of the right Circle, you need only lay a Ruler on the Point V, and Pole of the oblique Circle at *p*, and it will cut the primitive Circle in *y*, ſo the Arch A *y*, taken your Compaſſes to the Line of Chords, will give the Quantity of the Angle R V B, as was required.
4. To meaſure the Angle B T G, made by the two oblique Circles B *s* D and G T *n*, lay a Ruler upon the angular Point T, and their reſpective Poles at *r* and *x*, and it will cut the primitive in *l* and *m*, ſo the Arch *l m* taken in your Compaſſes to the Line of Chords will give the Quantity of the Angle required. Thus you work when the Convex Parts of the oblique Circles, which form the Angle as B *s* D and G T *n* are the ſame Way, but when they are different Ways, as B T D and R V W, which form the Angle S T V, it is otherways; for if it was required to find the Angle S T V, perhaps the Learner would expect the Anſwer, by laying a Ruler upon the angular Point T, and the reſpective Poles *p* and *r*, in order to find Points in primitive Circle *z* and *m*, which being found, take the Arch *m z* in your Compaſſes, and apply that Extent to the Line of Chords, will give a certain Angle above 90 Degrees; but you may be ſure that that is more than the Angle S T V, becauſe the oblique Circle R V W is drawn between the oblique Circle B *s* D, and its Pole at *r*; however, tho' this is not the Angle ſought, yet it is its Supplement; ſo then the Angle S T V is known. But to come at it directly, you muſt make uſe of two ſuch Poles of theſe oblique Circles R T W and B T D, as are on the ſame Side of the Center of the primitive Circle, ſuch as *r*, the upper Pole of the oblique Circle B *s* D and *e*, the lower Pole of the Circle R V W, ſo a Ruler laid on the angular Point T and *r*, will cut the primitive Circle in *m*, and being laid upon T and *e*, will cut the primitive Circle in *f*, ſo *m f* is the Meaſure of the Angle S T V.

Or

Or if you had laid the Ruler upon the angular Point T and *p*, the Pole of the oblique Circle R T W, to find the Point z, and upon T and *b* (which is the Under-Pole of the Circle B *s* D), it would cut the primitive Circle in *g*, then the Arch z *g*, is alſo the Meaſure of the Angle S T V, as before.



PROB. 8.

About any Point as a Pole, to deſcribe a great Circle.

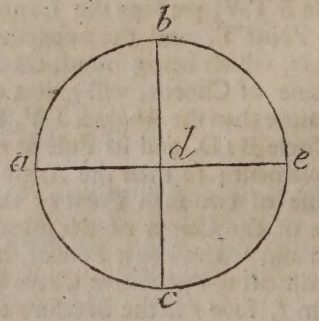
THIS has Three Caſes.

Caſe 1.

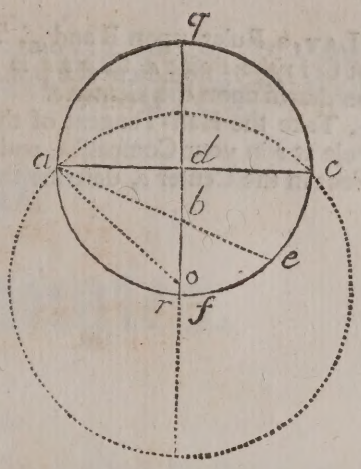
IF the Point be in the Center, then the primitive Circle is the great Circle required.

Caſe 2.

IF the Point given be in the primitive Circle, as at *b*, through *b*, and the Center of the primitive Circle *d*, draw the Diameter *b c*, then draw the Line *a e* through the Center *d*, and at Right-Angles to *b c*, ſo will *a e* be the great Circle required.



Case 3.



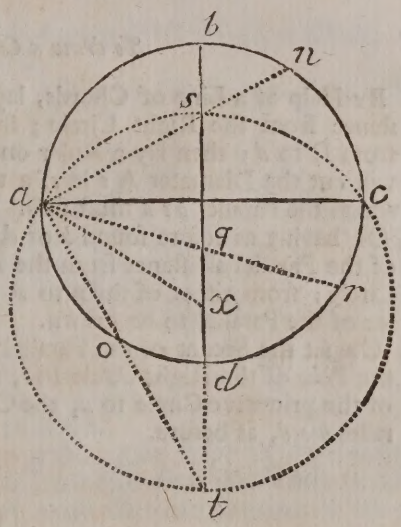
If the Point be neither in the Center, nor in the primitive Circle, but in some other Place, as *b*;

1. Through the Point *b* draw the Diameter *q r*, and at Right-Angles thereto draw the Diameter *a c*.
2. From *a* through *b* . . . . . draw the Line *a e*, then set off the Arch *c e* from *e* to *f*, and draw the Line *a o f*, then will *o* be the Center, and *a o* the Radius of the Circle which was to be drawn.

Or thus: Let *a b c d* be the primitive Circle, and *q* the the Point about which a great Circle is to be drawn.

1. From *a* through *q* draw the Line *a r*, and taking 90 Degrees from a Line of Chords set that Extent off from *r* to *n* and *o*.
2. Lay a Ruler from *a* to *n*, and it will cut *b d* in *s*. Again, lay the Ruler from *a* to *o*, and draw the Line *a t* through *o*.
3. Bisect *s t* in *x*, so *x* will be the Center of the Circle required, and its Radius *a x*.

Draw the Line *b d* through the Point *q*, and at Right-Angles thereto draw *a c*; then from *a* through *q* draw the Line *a r*, &c.



PROB. 9.

To draw a lesser Circle, parallel to any great Circle.

In this Problem there are Three Cases.

1. A Parallel to the primitive Circle.
2. Parallel to a Right Circle.
3. Parallel to an oblique Circle.

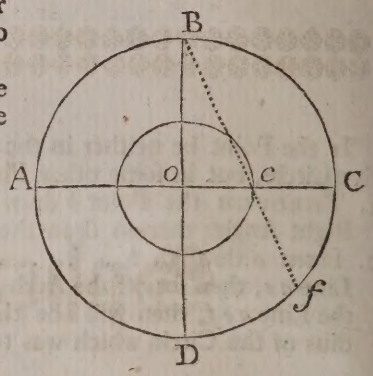
Case 1.

To draw a Circle parallel to the primitive Circle.

1. Take the Distance of the Parallel from the Pole or Center of the primitive Circle in your Compasses out of the Line of Chords, and set that Extent off from D to *f*.

2. LAY a Ruler upon B and *f*, and it will cut the Diameter A C in the Point *c*, then *o c* is the Radius of the Parallel to be drawn upon the Center *o*.

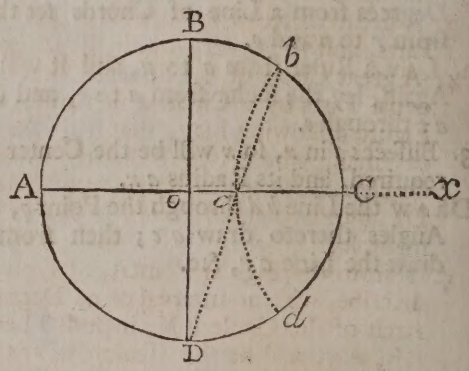
OR, Take the Half-Tangent of the Parallel's Distance from the Pole at *o* in your Compasses, and with that Extent, setting one Foot in the Center *o*, describe the Parallel.



Case 2.

To draw a Circle parallel to the Right Circle D B.

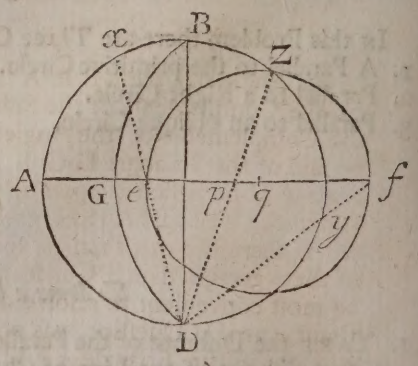
1. BY Help of a Line of Chords, lay the parallel Distance from the Right Circle; from B to *b*, and from D to *d*; then lay a Ruler on D and *b*, and it will cut the Diameter A *c* in *c*, a third Point thro' which the Parallel *b c d* must pass.
2. OR having as before found *b* or *d*, set the Tangent of the Parallel's Distance from the Pole of the Right Circle; from either of them to *x*, so is *x* the Center of the Parallel to be drawn.
3. OR set the Secant of the Parallel's Distance from the Pole of the Right Circle BD, from the Center of the primitive Circle to *x*, the Center of the Parallel *b c d*, as before.



Case 3.

To draw a Circle parallel to an oblique Circle B G D.

Find the Pole *p* of the oblique Circle B G D, then lay a Ruler upon D and *p*, and it will cut the primitive Circle in *z*; from *z* set off the Distance of the Parallel from the Pole of the oblique Circle both Ways to *x* and *y*; then lay a Ruler upon D and *x*, and D and *y*, and it will cut the Right Circle in *e* and *f*, bisect *e f* in the Point *q*; so is *q* the Center of the parallel Circle required to be drawn, and *e f* its Diameter.



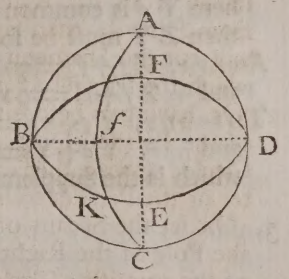
CHAP.

CHAP. XII.

Spherical Triangles.

DEFINITIONS.

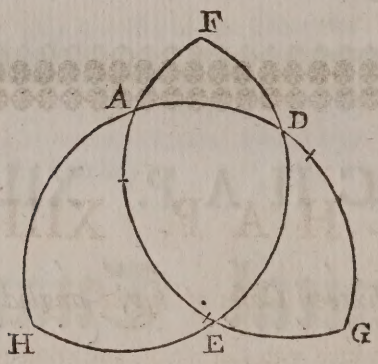
1. A Spherical Triangle, as well as a right-lined Triangle, doth conſiſt of Six Parts; namely, Three Sides and Three Angles.
2. THE Three Angles of every ſpherical Triangle, are the Arches of Three great Circles of the Sphere, every one of them being leſs than a Semicircle.
3. A great Circle of the Sphere is ſuch as divides the whole Sphere or Globe into Two equal Parts, and ſo is every-where diſtant from its Poles 90 Degrees.
4. A great Circle of the Sphere is cut at Right-Angles, and in Two equal Parts by a Circle which paſſes through its Poles; ſo in the Figure following I ſay, that if in the Circle BEDF, whether great or ſmall, whoſe two Poles are A and C, thro' which the great Circle ABCD paſſes, this great Circle ABCD, cuts the Circle BE DF, into two equal Parts; ſo that each Part BED, BFD, is a Semicircle.



5. IF from the angular Point A, of a ſpherical Triangle as a Pole, you deſcribe, with an Interval of 90 Degrees, a great Circle B K D, the Arch of this Circle B K, included between the Sides of the Angle AB, AK, will be the Meaſure of the ſame Angle, *viz.* of B A K.
6. EVERY ſpherical Triangle as A D E, hath, oppoſite to each angular Point (if you continue the Sides until they meet), another Triangle having the ſame Baſe with the former as A D, and the Angle oppoſite thereto equal, the other Parts of it are the Complements of the ſeveral Parts of the former to a Semicircle.

LET ADE be a ſpherical Triangle, then extend the Sides thereof D A and D E, until they meet at H, alſo A D and A E, until they meet at G, and laſtly E A and E D, until they meet at F, then are the Arches D A H, D E H, A E G, A D G, E D F, and E A F, Semicircles; and thus to each angular Point of the Triangle A D E, there is oppoſite another Triangle, having the ſame Baſe with the former, *&c.* As to the angular Point E, there is oppoſite the Triangle A F D, whoſe Angle at F is equal to the Angle at E, and Baſe A D, common to both Triangles, and the Sides F A, and F D, are the Complements of the Sides A E and D E. And the Angles F A D, and F D A, are the Complements of the Angles E A D, and and E D A, to a Semicircle, and the like may be ſaid of the Triangle D G E, which is oppoſite to the angular Point A, and of the Triangle A H E, which is oppoſite to the angular Point D; ſo that if there be three Things given in the Triangle A D E, there are the like given in every one of theſe Triangles.

Note, If therefore the Triangle to be reſolved be obtuſe angled, or have two of its Sides either of them greater than a Quadrant, tho' you might find the Thing required in that, yet it will be moſt convenient to reſolve one of the leaſt of the three Triangles oppoſite to its angular Point; as if a Queſtion was propoſed in the Triangle A D E, it may more conveniently be wrought in the Triangle A F D.

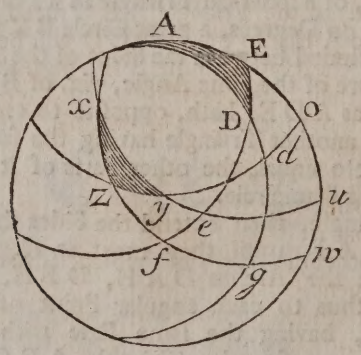


7. THE Sides of a ſpherical Triangle X Y Z, may be reduced into the Angles of the Triangle A E D, and the Angles of the Triangle A E D, into the Sides of the Triangle X Y Z, *&c.* Thus, Let A be the Pole of the Circle Z Y *d o*, E the Pole of the Circle *x y u*, and D that of the Circle *x z w*; alſo let *y* be the Pole of the primitive Circle, Z the Pole of the Circle A D *dg*, and *x* that of the Circle E D *f*.

BUT ——— ——— ——— ——— ——— Y *d*+*do*=90° }
and ——— ——— ——— ——— ——— Y*d*+ZY=90 }
here Y *d* is common, therefore *do*=ZY, now *do*, being the Meaſure of the Angle *o* A *d*, then Z Y muſt be ſo too.

AGAIN, Z*f*+*fg*=90° } here Z*f* is common, therefore Z *x* = *fg*, but *fg* is the Meaſure of
and Z*f*+Z*x*=90 } the Angle A D E, then Z *x* is equal to the ſame Angle.

THIRDLY, Y *e*+*eu*=90° } Y *e* is common, therefore *eu*=*yx*, but *eu* is the Meaſure of the
and Y *e*+Y*x*=90 } Angle *e* E *u*, then is Y *x*, the Meaſure of the ſame Angle, which is the Supplement of the Angle A E D.



8. THE three Angles of any ſpherical Triangle are more than two Right-Angles, and leſs than ſix.

9. IN every ſpherical Triangle, the external Angle is leſs than the two oppoſite internal Angles.

10. THE Sum of any two Sides of a ſpherical Triangle is greater than the third.

11. A ſpherical Triangle having one or more of its Sides Quadrants, is called a quadrantal Triangle.

12. Two oblique Angles of a ſpherical Triangle, are either of them of the ſame kind with their oppoſite Sides.

13. WHAT any Angle or Side wants of 90 Degrees, is called the Complement of that Angle or Side, and what they want of 180 Degrees, is uſually called the Supplement of that Side or Angle.

CHAP.

CHAP. XIII.

The Solution of the ſixteen Caſes of right-angled Spherical Triangles.

THAT the Learner may advance with Certainty, and conſequently with Pleaſure, it will be neceſſary to demonſtrate ſome of the moſt uſeful Theorems or Axioms by which the ſeveral Caſes in right-angled ſpherical Triangles are reſolved.

Axiom 1.

IN right-angled Triangles having the ſame acute Angle at the Baſe,

THE Sines of the Hypothenuſes are in direct Proportion to the Sines of the Perpendiculars, and the contrary.

1. The Demonſtration of this Axiom and the next will beſt appear from Paper or Paſte-board, cut as in the following Figure; in which ſuppoſe two Arches AB, and AC, and that ABN, the Plane of the greater Arch, was turned round A N, until that a Right-Line falling from B, perpendicular to the Plane ACN, may fall upon ſome Point of the Line H C, ſuppoſe on F, for in that Poſition BCA will be a ſpherical Triangle right-angled at C, of which the Side AB is the Hypothenuſe, AC the Baſe, and BC the Perpendicular.
2. AND ſuppoſe B C (Numb. 2d) of the Triangle A B C fitted according to its Letters; draw C D and B E perpendicular to A H; ſo the Line C D will be the Sine of the Baſe A C; the Line B E will be the Sine of the Hypothenuſe A B; and the Line B F will be the Sine of the Perpendicular B C; and their Coſines will be the Lines D H, E H, and F H. Theſe things being done, or a true Idea formed, the two firſt Axioms of ſpherical Triangles will preſently appear; to which end, let the Arch I L (Numb. 3.) be alſo fitted in the Figure, according to its Letters; then in the two right-angled Triangles B A C, and I A L, having the ſame acute Angle at the Baſe, *viz.* B A C, and I A L, it will be,

As B E : B F :: I H : I K, that is,

As B E, the Sine of the Hypothenuſe } in the little Triangle
Is to B F, the Sine of the Perpendicular } A B C,
So is I H, the Sine of the Hypothenuſe } in the great Triangle
To I K, the Sine of the Perpendicular } A I L.

Axiom 2.

IN right-angled ſpherical Triangles having the ſame acute Angle at the Baſe,

THE Sines of the Baſes are proportional to the Tangents of the Perpendiculars, and the contrary, C D : C G :: L H : L M; that is,

As C D, the Sine of the Baſe } in the little Triangle
Is to C G, the Tangent of the Perpendicnlar } A B C,
So is L H, the Sine of the Baſe } in the great Triangle
To L M, the Tangent of the Perpendicular } A I L.

For

For the two right-angled Triangles E B F and H I K are ſimilar, and ſo are the Triangles D C G and H L M.

To apply theſe two Axioms to the actual Solution of the ſixteen Caſes of right-angled ſpherical Triangles.

1. LET A B C (ſee the Figure next following) be a right-angled Triangle, which you may call the original Triangle, becauſe its Parts, or their Complements, do ſometimes (if I may uſe the Expreſſion) branch and ſpring up in other Triangles, where they appear otherwiſe than at firſt, having its Sides produced to A E, A D, C F, *&c.* Alſo let B K, B L, and L G, be Quadrants; and moreover, let F I and F H be Quadrants; then becauſe A E and B L are Quadrants, and B E common to both, therefore A B = E L, in like manner E D = F G = ∠ E A D, B C = F K, B F = I K, and F E = G H, alſo the Complement of A C is C D = ∠ C F D = ∠ H F I = H I, *&c.* and the Angles M, C, D, E, L, K, I, are Right-Angles.
2. Now, in this Figure you have twelve right-angled Triangles or ſix Pair of ſimilar Triangles, each Pair, *viz.*

   1. A B C and A E D
   2. F E B and F D C
   3. B E F and B L K
   4. G K F and G L E
   5. F K G and F I H
   6. B C A and B M N

   } having the ſame acute Angle at the Baſe.

3. THE Things given and required, or their Complements, will always be found in one or other of theſe Pair. But then it muſt be in that Pair where they ſhall be either two Hypothenuſes and two Perpendiculars, that they may fall under the firſt Axiom, or where they ſhall be two Baſes and two Perpendiculars, and then they will fall under the ſecond Axiom.

Note, That if the two Parts given, and that required, lie together, then the Queſtion falls under the ſecond Axiom, and ſo you muſt reduce them to one of the ſix Pair of Triangles, where

where they (and Radius, which is always one of the four Terms concerned) may be Baſes and Perpendiculars.

EXAMPLE. 1.

SUPPOSE, that in the right-angled Triangle A B C, you have given the Baſe A C, and Angle at the Baſe B A C, to find the Perpendicular B C.

HERE you ſee, that the two Parts given, and that required, lie all together (for though the Right-Angle at C is between the Baſe and Perpendicular, yet 'tis ſaid never to part them; and ſo the Queſtion falls under the ſecond Axiom, in which Baſes and Perpendiculars are employed. Now when you have conſidered the Figure, you'll preſently know that the Queſtion may be anſwered by the firſt Pair of Triangles, *viz.* A B C, and A E D; for in the great Triangle A E D, you have the Baſe A D=90° (or Radius) and the Perpendicular D E equal to the Angle D A E, being the Meaſure thereof, to compare with A C the Baſe, and BC the Perpendicular in the little Triangle; whence, according to the ſecond Axiom, you may form the following Analogy.

As A D, the Baſe of the great Triangle,
Is to A C, the Baſe of the little Triangle,
So is D E, the Perpendicular of the great Triangle,
To B C, the Perpendicular of the little Triangle.

BUT to give thoſe Sides their Names, you muſt ſay thus:

As Radius —— —— —— ——		A D
Is to the Sine of the Baſe —— ——		A C
So is the Tangent of the Angle at the Baſe (E A D) ——	=	E D
To the Tangent of the Perpendicular —— —		B C

EXAMPLE 2.

SUPPOSE that in the ſame right angled Triangle A B C, you have the Baſe A C, and Angle at the Perpendicular A B C, to find the Perpendicular B C.

HERE again, the two Parts given, and that required, lie all together, ſo the Queſtion falls under the ſecond Axiom, and may be anſwered by the fifth Pair of Triangles F K G, and F I H; for the Complement of A C is C D = ∠ C F D = ∠ H F I = H I, and the Angle A B C is equal to K B L = L K, whoſe Complement is G K; ſo then, in the great Triangle F I H, you have the Baſe F I = 90° (or Radius) and Perpendicular H I, to compare with F K, the Baſe of the little Triangle, and G K its Perpendicular. Then the Analogy will run thus:

As the Tangent of H I, the Perpendicular in the great Triangle,
Is to the Tangent of G K, the Perpendicular of the little Triangle,
So is Radius, or Side F I, in the great Triangle,
To the Sine of the Baſe F K, in the little Triangle; but F K is equal to B C, the Side required.

EXAMPLE 3.

SUPPOSE that in the ſame original Triangle A B C, you have given the Angle at the Baſe B A C, and the Angle at the Perpendicular A B C, to find the Perpendicular B C.

IN this Caſe you have two of the three Parts, *viz.* the Perpendicular B C, and Angle at the Perpendicular A B C, lying together, and the third, *viz.* the Angle at the Baſe B A C, lying by itſelf, as being ſeparated from the Perpendicular by the Baſe A C, and from the Angle at the Perpendicular A B C, by the Hypothenuſe A B.

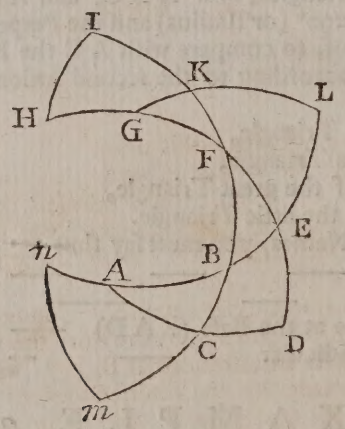
Now, whenever the Parts given and required lie in this manner, and not altogether as in the two former Examples, then the Queſtion falls under the firſt Axiom, and ſo you muſt reduce it to one of the ſix Pair of Triangles, where they (and Radius) may be Hypothenuſes and Perpendiculars,

pendiculars, which, in this Caſe, will be in the third Pair, *viz.* B E F, and B L K, where F E is the Complement of D E, equal to the Angle B A C, L K is equal to K B L=A B C, B F is the Complement of B C, the Side required, and B K is Radius = 90°.

In the great Triangle B K L, you have the Perpendicular K L, and Hypothenuſe B K, to compare with F E, the Perpendicular of the little Triangle B F E, and B F its Hypothenuſe. Then, according to the firſt Axiom, ſay,

As the Sine of the Perpendicular in the great Triangle K L
Is to the Sine of the Perpendicular in the little Triangle F E
So is Radius, or the Sine of the Hypothenuſe in the great Triangle B K
To the Sine of the Hypothenuſe of the little Triangle B F

Whoſe Complement to 90 Degrees is B C, the Side required.



Having now premiſed what I thought would give the Learner a clear Notion of this Method, the next thing propoſed was immediately to give Examples of the ſeveral Caſes; but that nothing might be wanting, we did reſolve to make him acquainted with the Univerſal Axiom invented by the Honourable *John Napier* Baron of *Marchiſton* in *Scotland*, under which all the Caſes of ſpherical Triangles, but two, will fall, and thoſe are the two laſt Caſes of oblique angled Triangles.

Axiom 3.

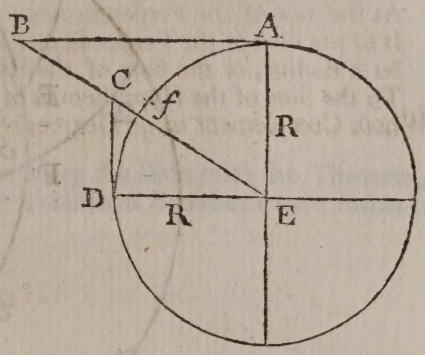
1. The Sine of the middle Part, multiplied by Radius, is equal to the Rectangle, or Product of the Tangents of the adjacent Extreams, or To the Rectangle of the Sines Complements of the oppoſite Extremes.

Thus you may work by the natural Sines and Tangents; but to perform the ſame by the Logarithms, it is thus:

2. The Sine of the middle Part more (or added to) Radius is equal to the Tangents of the adjacent Extremes, or To the Sines Complements of the oppoſite Extremes.

The Truth of the two firſt Axioms having been ſufficiently demonſtrated already, we may ſafely uſe them, and the following Theorem, as it were Teſts to prove the Truth of both Parts of the third Axiom.

A Theorem.



RADIUS is a Mean Proportional between the Tangent of an Arch and the Tangent Complement of the ſame Arch.

Let the Arch be D *f*, whoſe Complement is *f* A, its Tangent is D C, and Tangent of its Complement A B, A E, or D E, is Radius, which you may call R; now the Triangles A B E, and C D E, are ſimilar; therefore it will be,

As A E (= R) : A B :: D C : D E (= R).

THAT is, As R : A B :: D C : R. Therefore A B × D E = R². *Q. E. D.*

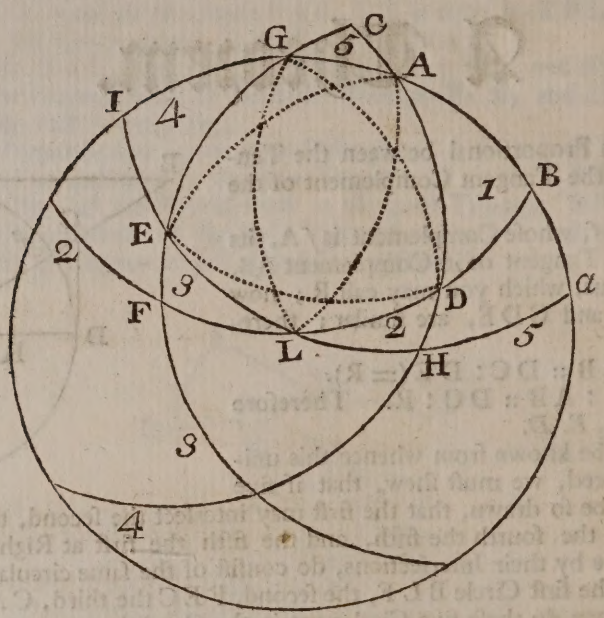
Now that it might be known from whence this univerſal Axiom is deduced, we muſt ſhew, that if five Circles of the Sphere be ſo drawn, that the firſt may interſect the ſecond, the ſecond the third, the third the fourth, the fourth the fifth, and the fifth the firſt at Right-Angles, the right-angled Triangles made by their Interſections, do conſiſt of the ſame circular Parts.

LET I G A B, be the firſt Circle B L F, the ſecond, F E C the third, C A D the fourth, and H L E I the fifth: Then do theſe five Circles retain the Conditions required, the firſt interſecting the ſecond in B, the ſecond the third in F, the third the fourth in C, the fourth the fifth in H, the fifth the firſt in I, and theſe Interſections at B, F, C, H, I, are at Right-angles; therefore the right-angled Triangles, made by the Interſections of the Circles, namely, A B D, D H L, L F E, E G I, and G C A, do all conſiſt of the ſame circular Parts; for the circular Parts in every one of theſe Triangles, are as follows:

1	ABD	AB,	BD	*c* BDA	*c* AD	*c* DAB.
2	DHL	*c* HLD,	*c* LD	*c* LDH	DH	HL
3	LFE	*c* ELF,	LF	FE	*c* FEL	*c* EL
4	EGI	IG.	*c* IGE	*c* GE	*c* GEI.	IE
5	GCA	*c* GA.	*c* AGC.	GC	CA	CAG.

Where you muſt obſerve, that the Side A B in the firſt Triangle is the Complement of H L D in the ſecond, or Complement of E L F, in the third, or I G in the fourth, or Complement of G A in the fifth. See *Norwood*'s Doctrine of Triangles.

IN this Fig. G from D, D from E, E from A, A from L, L from G, are diſtinguiſhed by Arches equal to a Quadrant; ſo there are five quadrantal Triangles, *viz.* G C D, E D H, E C A, A H L, and G L E, all under the ſame Axiom.



THESE things being well conſidered, we may proceed to demonſtrate the Truth of the third Axiom, which conſiſts of two Parts, each of them admitting of three Caſes; and firſt we are to prove, that the Sine of the middle Part more Radius, is equal to the Tangent Complements of the adjacent Extremes, or the Tangent of one, and T. *c.* of the other.

Caſe I.

GIVEN, in the Triangle A B D in the laſt Fig. the Perpendicular A B, the Baſe D B, and Angle at the Perpendicular A, here A B is the middle Part and the Angle at A, and Baſe B D, adjacent Extremes, then by the third Axiom it will be

	1	S. AB×R=*t*DB× *t. c.* A; or by the Logarithms, S.AB+R=*t* DB+*t. c.* A.
By Axiom 2d	2	S.AB : R :: *t.* DB : *t.* A (S.A B : R (=A*a*) :: *t* DB : *t* A (=H*a*=DAB.)
2 Alterned	3	S.A B : *t.* D B :: R : *t* A.
But ———	4	R : *t.* A :: *t. c.* A : R, by the laſt Theorem.
3 By Subſtit.	5	S.A B : *t.* D B :: *t. c.* A : R.
5 ∴	6	S.AB×R=*t.*DB×*t.c.*A. or by the Logarithms, S.AB+R=*t.*DB+*t.c.*A.

THIS 6th Step being the ſame as the firſt, demonſtrates the Truth of the 3d Axiom.

Note, THAT the two laſt Terms of the third Step being R : *t.* A. and :: *t. c.* A : R. at the 4th Step, being in the ſame Proportion as R : *t.* A. therefore *t. c.* A : R is ſubſtituted, as at the 5th Step, inſtead of R : *t.* A in the third.

THE Truth of the ſecond Part of the third Axiom may be proved thus:

GIVEN the Baſe D B, and Perpendicular A B, to find the Angle at the Perpendicular A.

By

By Axiom 3d	1	$\frac{S.AB \times R}{t.DB} = t.c.A$
By Axiom 2d	2	$\frac{S.AB \times t.c.DB}{R} = t.c.A.$
2 = 1	3	$\frac{S.AB \times t.c.DB}{R} = \frac{S.AB \times R}{t.DB}$
3 ÷ S. AB.	4	$\frac{t.c.DB}{R} = \frac{R}{t.DB}$
4 × R ——	5	$t.c.DB = \frac{R^2}{t.DB}$
5 in Analog.	6	$t.DB : R :: R : t.c.DB.$

This being the ſame with the Theorem, may prove the Truth of both the Axioms whence the firſt and ſecond Equations were formed.

Caſe 2.

GIVEN in the Triangle D L H, the Hypothenuſe L D, the Baſe L H, and L the Angle at the Baſe. Here, the Angle at the Baſe L, is the middle Part, L D, and L H, are adjacent Extremes.

IT was demonſtrated at the 6th Step of the firſt Caſe, from the ſecond Axiom and the Theorem, that $S.AB \times R = t.DB \times t.c.A.$

AND the Equation for this Caſe by the third Axiom is $S.c.L \times R = t.c.LD \times t.LH$

Now comparing theſe two Equations with the Scheme, you'll find that each Member of the ſecond is equal to the reſpective Member of the firſt; which demonſtrates the Truth of the ſaid Axiom in this Caſe alſo.

Caſe 3.

GIVEN, in the Triangle G C A, the Hypothenuſe G A, together with the Angles G and A. Here the Hypothenuſe G A is the middle Part, and the Angles at G and A are adjacent Extremes.

FROM the firſt Step you have —— —— $S.AB \times R = t.DB \times t.c.A$

AND, according to the third Axiom, the Equation for this Caſe is —— —— $S.c.GA \times R = t.c.G \times t.c.A$

Now, again, if you compare theſe two Equations and the Scheme together, you'll find that each Member in the ſecond is equal to its correſponding Member in the firſt. *Q. E. D.*

THERE remains now to prove the Truth of the ſecond Part of the third Axiom, *viz.* That the Sine of the Middle Part multiplied by Radius, is equal to the Rectangle, or Product of the oppoſite Extremes. And this admits of three Caſes, as aforeſaid.

Caſe 1.

GIVEN, in the Triangle A B D, the Baſe D B, the Hypothenuſe A D, and Angle at the Perpendicular A. Here D B is the middle Part, and A D, and the Angle at A the oppoſite Extremes.

THEN, by the first Axiom, it will be as R : S. A D :: S. A : S. D B
THEREFORE, —— S. D B × R = S. A D × S. A
AND, by Axiom the third, it is —— S. D B × R = S. A D × S. A
THESE two Equations being the same, demonstrates the Truth of the second Part of the third Axiom in this Case.

Case 2.

GIVEN, in the Triangle D H L, the Hypothenuse D L, the Perpendicular D H, and Base H L. Here the Hypothenuse D L is the middle Part, and D H, and L H, are opposite Extremes.

NOW, by the first Axiom, you have, as R : S. *c.* D H :: S. *c.* L H : S. *c.* L D
THEREFORE, — — S. *c.* L D × R = S. *c.* L H × S. *c.* D H
AND, by the third Axiom, it will be — S. *c.* L D × R = S. *c.* L H × S. *c.* D H

Case 3.

GIVEN, in the Triangle I E G, the Angle at G, the Angle at E, and the Side I E. Here the Angle at G, is the middle Part, and the Angle at E, and Side I E, are opposite Extremes.

BY the first Axiom it is as — R : *s.* E :: S. *c.* E I : S. *c.* G
THEREFORE, — — S. *c.* G × R = S. *c.* E I × S. E
AND, by the third Axiom, it is — S. *c.* G × R = S. *c.* E I × S. E

THE same Axiom may be applied to quadrantal Triangles, as A D L (whose Side A L is a Quadrant, and they have five Parts besides the quadrantal Side, *viz.* the Side L D, the Angle at L, the Angle at *A*, the Side *A D*; but three of these which are furthest from the quadrantal Side, *viz.* L D, Angle at D, and Side *A D*, are noted by their Complements, and ordered as a right-angled Triangle.

IN using the third Axiom (as well as the first and second) we must consider every spherical Triangle as consisting of five Parts besides the Right-angle; that is to say, three Sides, *viz.*

1. THE Hypothenuse, *A B*
2. THE Base, — *A C*
3. THE Perpendicular *B C*, and two Angles, *viz.*
4. THE Angle at the Perpendicular *A B C*
5. The Angle at the Base — *B A C.*

OF these five Parts, the Hypothenuse, the Angle at the Perpendicular, and Angle at the Base, are always noted to be Complements; so that whatever the Axiom directs you to do with any of these three Parts, you must just do the contrary. My Meaning is this, that though the Axiom says the Sine of the middle Part more Radius, *&c.* yet if either of them happens to be middle Part, you must then say the Co-sine of the middle Part more Radius, *&c.*

AGAIN, the Axiom says, that the Sine of the middle Part more Radius, is equal to the Co-sines of the opposite Extremes; but if one or two of these happens to be opposite Extremes, then you must say the Sine of the middle Part (if that is either the Base or Perpendicular, for these two will always obey the Axiom) more Radius is equal to the Sines of the opposite Extremes, *&c.* as you may observe throughout the following Examples.

NOW, if any two Parts be given in a right-angled spherical Triangle, besides the Right-angle, you may find a Third; but of these three one will be the middle Part, and the other two will be Extremes, either adjacent or opposite; and to know of which sort they are, you must observe (See the Fig. next following),

1. THAT

1. THAT if the three Things concerned in the Queſtion lie altogether, as if they were,
   1. B A C
   2. A C
   3. B H

   } Then *A C* is the middle Part, and the Side *B C*, and Angle *B A C*, Extremes adjacent.

2. Or if they were,
   1. A C
   2. B A C
   3. A B

   } Here the Angle *B A C* is the middle Part, and the Sides *A C* and *A B*, adjacent Extremes.

3. Or if they were,
   1. A B
   2. A B C
   3. B C

   } Here the Angle *A B C* is the middle Part, and the Sides *A B* and *B C* adjacent Extremes.

   BUT when two of the three Parts lie together, and the third by itſelf, then that, that lies by itſelf, is the middle Part, and the other two are oppoſite Extremes, as if the three Parts concerned were,

4. 1. B A C
   2. A B C
   3. B C

   } Here the Angle at *A* is the middle Part, and the Angle A B C, and Side B C, oppoſite Extremes.

5. Or if they were,
   1. A B
   2. A B C
   3. A C

   } Here the Side A C is the middle Part, and the Side A B, and Angle A B C, are oppoſite Extremes.

6. If they were,
   1. A B C
   2. B C
   3. B A C

   } Here the Angle B A C is the middle Part, and the Side B C, and Angle A B C, are oppoſite Extremes.

7. If they were,
   1. B A C
   2. A C
   3. A B C

   } Here the Angle A B C is the middle Part, and the Side A C, and Angle B A C, are oppoſite Extremes.

8. Or laſtly, if they were,
   1. A C
   2. B C
   3. A B

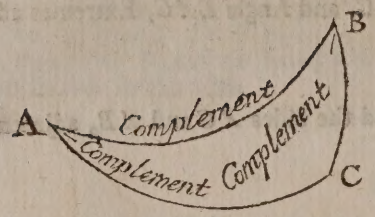
   } Here the Side A B is the middle Part, and the Sides A C and B C are oppoſite Extremes.

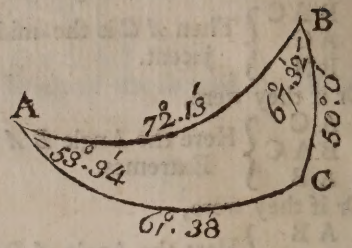
WHEN you are to form an Equation according to the third Axiom, you will always have two Things upon each Side, of which R or Radius, will be one; then obſerve upon which Side of the Equation the Part of that you want is, and what is placed with it; for whatever that is, it muſt be on the other Side with the negative Sign (—) before it, that ſo the Part required may ſtand upon one Side alone, and the other three Parts upon the other Side.

EXAMPLE.

SUPPOSE you have the Angle at *A*, and the Baſe *A C*, given to find the Perpendicular B C.

HERE *A C* is the middle Part, and the Angle at *A*, and Side B C, are adjacent Extremes; then, by the firſt Part of the Axiom, you'll ſay the Sine of A C + R = *t.* A + *t.* B C. But becauſe B C is wanting, the Equation will ſtand thus: S. A C + R — *t.* A = *t.* B C. Which ſhews, that unto the Sine of A C, you muſt add Radius, and from the Sum you muſt ſubſtract the Tangent of the Complement of *A*, and the Remainder will be the Tangent of B C. Theſe things being premiſed, we ſhall give Examples of all the ſixteen Caſes; in order to which, let the Numbers that are in the ſecond Figure be uſed in all the Caſes, and let the Triangle A B C (Fig. Caſe 1.) be called the original Triangle.





The Ambiguities of right-angled ſpherical Triangles may be ſolved by the Figure following.

1. THE Sides including the Right-angle, are of the ſame kind with the Angles oppoſite to them.

   So in the Triangle B D A, becauſe D A is greater than the Quadrant D P, the Angle DBA is greater than the Right-angle D B P, and in the Triangle B C A, becauſe A C is leſs than the Quadrant P C, the Angle C B A is leſs than the Right-angle C B P.

2. IF the Legs, and conſequently the Angles, are of the ſame or of different Kinds, the Hypothenuſe is accordingly greater or leſs than a Quadrant.

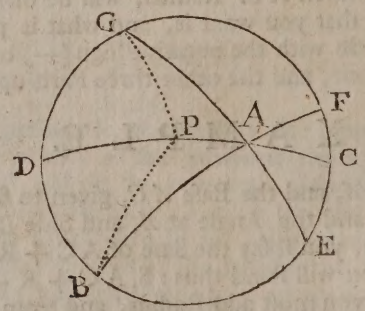
   So in the Triangles E D A, and E C A, the Hypothenuſe A E is leſs than a Quadrant; but in the Triangle B D A, the Hypothenuſe A B is greater than the Quadrant B P.

3. IF the Hypothenuſe be leſs than a Quadrant (or 90 Degrees), the Legs are ſimilar, and ſo are the two oblique Angles; but if the Hypothenuſe be greater than 90 Degrees, the Legs are diſſimilar, and the oblique Angles diſſimilar.

4. IF the Hypothenuſe be { leſs / greater } than 90 Deg. each Leg ſhall be { ſimilar / diſſimilar } to its adjacent Angle, this follows from the firſt and ſecond.

5. AND, on the contrary, if the Leg be { ſimilar / diſſimilar } to its adjacent Angle, the Hypothenuſe will be { leſs / greater } than 90 Degrees.

MUCH more may be ſaid about the Ambiguities of ſpherical Triangles, but of little Uſe to thoſe that are well acquainted with the Sphere or Globe, and know how to project ſuch Circles thereof, as they may have occaſion for in any Caſe.



For

For the Resolution of the Sixteen Cases of right-angled spherical Triangles, the Data, or Things given, can admit of no more than six Sorts or Varieties, as follow.

						Cas.
1	Dat.	A B and B C	Quere	1 B 2 A C 3 A		1 2 3
2	Dat	A B and A	Quere	1 B C 2 A C 3 B		4 5 6
3	Dat.	B C and A	Quere	1 A B 2 B 3 A C		7 8 9
4	Dat.	A C and A	Quere	1 B 2 B C 3 A B		10 11 12
5	Dat.	A C and B C	Quere	1 A B 2 A		13 14
6	Dat.	A and B	Quere	1 A B 2 B C		15 16

Case

Case I.

GIVEN the Hypothenuſe AB=72° : 13′ and Perpendicular BC=50° : 00′, to find the Angle at the Perpendicular B.

WHEN the Parts given, and that required, are reduced, they will fall in the fourth Pair of Triangles ; G K F, and G L E, there making two Perpendiculars, *viz.* L E and K F, and two Baſes G K, and G L= Radius.

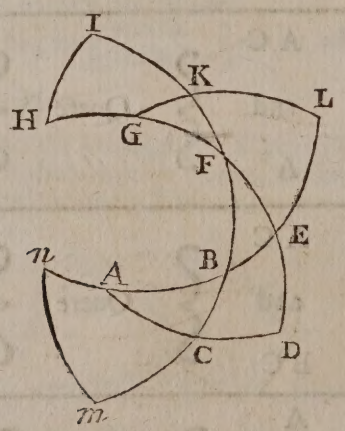
Now, ſay by the ſecond Axiom,

As the Tangent of L E (=A B the Hypothenuſe) = 72° : 13′ ———	10.4938414
Is to the Tangent of K F (=B C) the Perpendicular = 50° : 00′ ———	10.0761865
So is Radius L G = 90° ——— ——— —	10.
To the Sine of G K = 22° : 28′, whoſe Comp. is K L = K B L=67° : 32′	9.5823451

IF you would do this by the third Axiom, then the Angle at B is the middle Part, and the Sides A B and B C are adjacent Extremes ; ſo the Equation will ſtand thus : S. *c.* B+R=T. *c.* AB+T. B C, but the Angle at B is required, then S. *c.* B=T. *c.* A B+T. B C — R, will give the Anſwer.

THUS, T. *c.* A B=72° : 13′, is ——— ———	9.5061586
+T. B C=50° : 00′, is ——— — — —	10.0761865
THEIR Sum is S. *c.* B=T. *c.* A B+T. B C—R=22° : 28′ ———	9.5823451

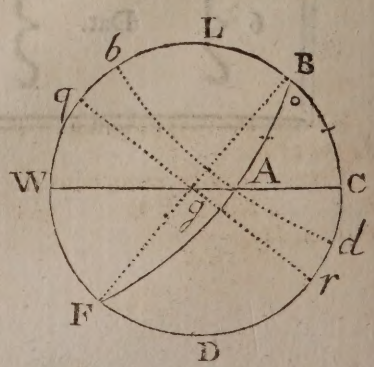
THE Complement of 22° : 28′ is 76° : 32′=B, the Angle required as above.



The Stereographic Projection.

THE primitive Circle W L C D being drawn, then,

1. TAKE 50° : 00′ in your Compaſſes from the Line of Chords, and ſet that from C to B, and from B, thro' the Centre at *g*, draw B *g* F, and *q r*, at Right-angles to each other.
2. ABOUT B as a Pole deſcribe *b d*, parallel to the right Circle *q r*, at the Diſtance of 17° : 47′ from it, or 72° : 13′ from its Pole at B, and this will cut the right Circle W C in A.
3. THRO' the Points B A F draw the oblique Circle B A F, and that will compleat the Triangle A B C ; then you may meaſure the Angle A B C.



Case 2.

GIVEN the Hypothenuse A B = 72° : 13′, and Perpendicular B C = 50° : 0′, to find the Base A C.

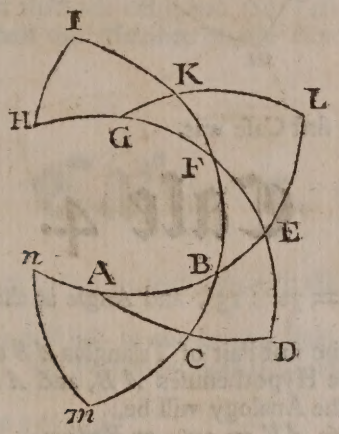
WHEN the Parts concerned are reduced, they will fall in the second Pair of Triangles FEB, and F D C, where the Perpendicular B E is the Complement of the Hypothenuse A B, the Perpendicular D C is the Complement of the Base A C, and the Hypothenuse F B is the Complement of the Perpendicular B C, and F C is Radius. Say then, by Axiom 1st,

As the Sine of F B = Co-sine of B C = 50° : 0′	9.8080675
Is to Radius F C = 90°	10.
So is the Sine of B E = S. *c.* of A B = 72° : 13′	9.4848951
To the Sine of D C = S. *c.* of A C = 61° : 38′	9.6768276

THIS, by the third Axiom, A B being the middle Part, and A C and B C oppofite Extremes, will ſtand thus: S. *c.* A B + R = S. *c.* A C + S. *c.* B C. But A C is required, therefore the Equation will be S. *c.* A B + R — S. *c.* B C = S. *c.* A C.

So, S. *c.* A B = 72° : 13′ + R =	19.4848951
— S. *c.* B C = 50° : 0′	9.8080675
Remains the S. *c.* of A C = 61° : 38′	9.6768276

THE Stereographic Projections is the ſame as the laſt.



Case 3.

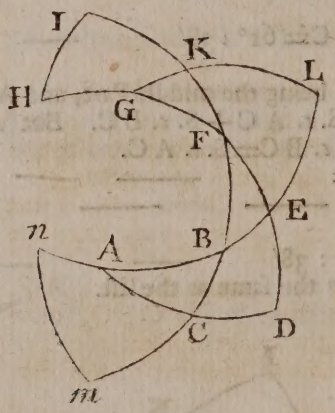
GIVEN the Hypothenuse A B = 72° : 13′, and Perpendicular B C = 50° : 00′, to find the Angle of A.

THESE may be reduced to the fourth Pair of Triangles G K F, and G L E. Here L E = A B . . . K F = B C . . . G F = E D = B A C, and G E is Radius. Now, by the firſt Axiom, ſay,

As the Sine of L E = A B = 72° : 13′	9.9787365
Is to the Sine of K F = B C = 50° : 0′	9.8842540
So is Radius = G E	10.
To the Sine of G F = E D = E A D = 53° : 34′	9.9055175

THIS, by the third Axiom, the Perpendicular B C being the middle Part, and the Hypothenuſe A B, and Angle at the Baſe A, the oppoſite Extremes will ſtand thus: S. B C + R = S. A B + S. A. But A is required, then it will be S. B C + R — S. A B = S. A.

S. B C + R =	19.8842540
— S. A B	9.9787365
Remains S. A = 53° : 34′	9.9055175



THIS alſo is conſtructed as the firſt Caſe was.

Caſe 4.

GIVEN the Hypothenuſe *A B* = 72° : 13′, and Angle at the Baſe *A* = 53° : 34, to find the Perpendicular B C.

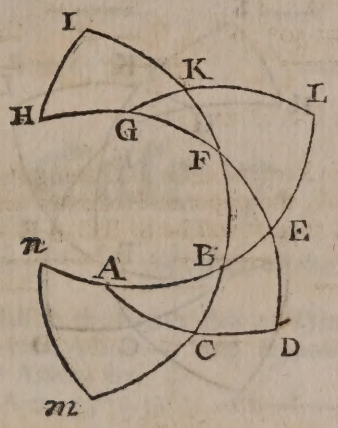
THIS may be anſwered from the firſt Pair of Triangles *A B C*, and *A E D*, where the Things given, and that required, will, be Hypothenuſes *A B*, and *A E*, and Perpendiculars *B C*, and *E D*; then, by the firſt Axiom, the Analogy will be,

As the Sine of the Hypothenuſe *A E* = 90°, or Radius	10.
Is to the Sine of the Hypothenuſe *A B* = 72° : 13′	9.9787365
So is the Sine of the Perpendicular *E D* = *E A D* = 53° : 34′	9.9055522
To the Sine of *B C* = 50° : 00′	9.8842887

To do this by the third Axiom, *B C* is the middle Part, and *A B* and *A* oppoſite Extremes, ſo it will be S. *B C* + *R* = *S. A* + *S. A B*; but the Perpendicular *B C* is wanting, then it muſt be *S. B C* = *S. A.* + *S A B* — *R.*

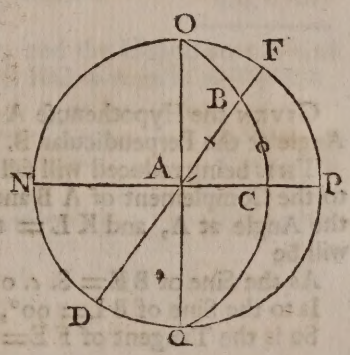
THE Sine of *A* = 53° : 34′	9.9055522
THE Sine of *A* B 72° : 13′	9.9787365
THEIR Sum want Radius is the Sine of B C = 50° : 00′	9.8842887

The



The Stereographic Projection.

1. DRAW the primitive Circle NOPQ, and quarter it with the Lines N P and O Q; then take the Angle at the Baſe 53° : 24′ in your Compaſſes from the Line of Chords, and ſet that from P to F, and from F thro' the Center at *A*, draw the Diameter F D.
2. SET the Half-Tangent of 72° : 13′ upon the Line A F, from *A* to B, and then thro' the three Points O B Q, draw the oblique Circle O B Q, and that will compleat the Triangle A B C; then you may find the Meaſure of the Perpendicular as required.



Caſe 5.

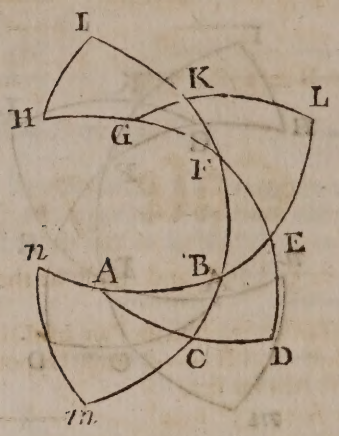
GIVEN the Hypothenuſe A B = 72° : 13′, and Angle at the Baſe *A* = 53° : 34′, to find the Baſe A C.

WHEN theſe are reduced, they will fall in the ſecond Pair of Triangles F B E and F C D, as Baſes F E and F D (=Radius) and Perpendiculars B E and C D; hence we ſhall have this Analogy (by the ſecond Axiom)

As the Sine of F E = S. *c.* A = 53° : 34′	9·7737039
Is to the Sine of F D, or Radius	10.
So is the Tangent of B E = T. *c.* A B = 72° : 13′	9·5061586
To the Tangent of C D = *t. c.* A C = 61° : 38′	9·7324547

THIS, by the third Axiom, the Angle at A being the middle Part, and A B and A C, adjacent Extremes, will ſtand thus .... S. *c.* A + R = T. *c.* A B + T. A C; but A C is wanting, then it muſt be S. *c.* A + R — T. *c.* A B = T. A C.

S. *c.* A = 53° : 34′ + R	19·7737039
— T. *c.* A B = 72° : 13′	95061586
Remains the Tangent of AC = 61° : 38′	10.2675453



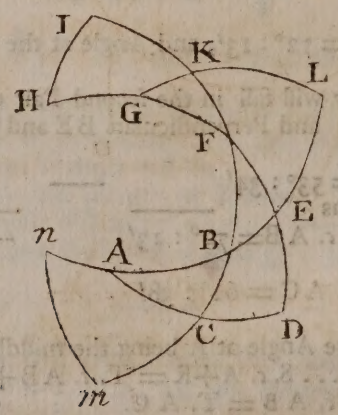
THIS conſtructed, as was the laſt Caſe. See that Fig. for this and the next Caſe following.

Caſe 6.

GIVEN the Hypothenuſe A B = 72° : 13′, and Angle at the Baſe A = 53° : 34′, to find the Angle at the Perpendicular B.

THIS being reduced will fall in the third Pair of Triangles B E F and B L K, as Baſes B E = to the Complement of A B and B L = Radius, and Perpendiculars F E = the Complement of the Angle at A, and K L = to the Angle at B; then, by the ſecond Axiom, the Analogy will be

As the Sine of B E = S. *c.* of A B = 72° : 13′	9.4848951
Is to the Sine of B L = 90°, or Radius	10.
So is the Tangent of F E = T. *c.* of A = 53° : 34′	9.8681517
To the Tangent of K L = the Angle at B = 67° : 32′	10.3832566



IF you chuſe to do this by the third Axiom, A B is the middle Part, and the Angles at A and B are adjacent Extremes; ſo the Equation will be thus .... S. *c.* A B + R = *t. c.* A + *t. c.* B. but B is wanting, then it muſt ſtand thus: S. *c.* A B + R — *t. c.* A = *t. c.* B.

S. *c.*

S. of AB = 72° : 13′ + R is	19.4848951
— *t. c.* A = 53°. 34′.	9.8681517
Remains the *t. c.* of the Angle at B = 67° : 32′	9.6167434

Case 7.

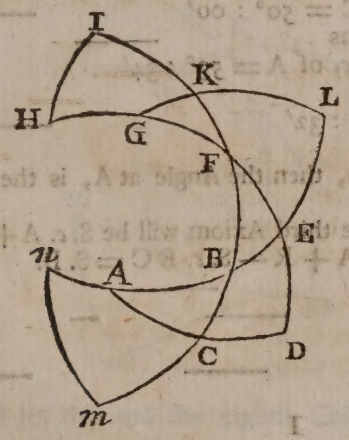
GIVEN the Perpendicular BC = 50° : 0′. and Angle at the Base A = 53° : 34′ to find the Hypothenuse AB.

THIS Case when reduced, will fall in the fourth Pair of Triangles GKF, and GLE.—— As Perpendiculars KF = BC. LE = AB.—— And Hypothenuses GF = ED = A. and GE = Radius. Now by the first Axiom say,

As the Sine of GF. = ED = A = 53° : 34′	9.9055522
Is to GE = Radius	10.
So is the Sine KF = BC = 50° : 0′	9.8842540
To the Sine of LE = AB = 72° : 13′	9.9787018

BY the third Axiom, the Perpendicular BC is the middle Part, and the Hypotheneuse and Angle at the Base, are opposite Extremes; so the Equation will be S. BC + R = S. AB + S. A. But AB is requir'd, so it must be S. BC + R — S. A = S. AB.

The Sine of BC = 50° : 0′ + R	19.8842540
— S. A = 53° : 34′	9.9055522
Remains the Sine of AB = 72° : 13′.	9.9787018



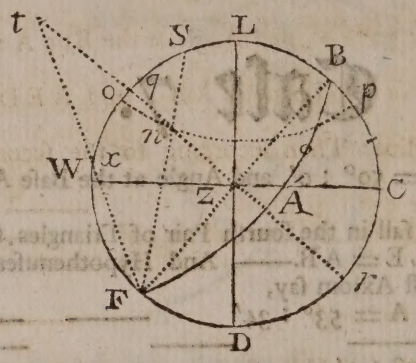
The Stereographic Projection.

1. DRAW the Primitive Circle W L C D, and quarter it with the Right Circles W C and L D, and from C set 50° : 0′ to B, and draw B F, and at Right Angles thereto the Line *r q*. continued as Occasion shall require.

2. ABOUT L, as a Pole, draw the small Circle *o p*, at the Distance of 53° : 34′ the Angle at the Base, and that will cut *r q*, in *n*, which is the Pole of the Oblique Circle B A F.

3. FROM F, and thro' the Pole at *n*, draw F *s*. then take B *s*. in your Compasses and set that from *s*. to *x*. Lay a Ruler upon F, and *x*, and it will cut *r q* produced, in the Point *t*. the

the Centre of the oblique Circle B A F. upon which the Radius F *t*. deſcribe it, and that will compleat the Triangle A B C. This being done you may meaſure A B, or any other Part that you want by the Rules given in *Chap.* 11*th*.



Caſe 8.

GIVEN the Perpendicular B C = 50° : 0′ and Angle at the Baſe A = 53° : 34′ to find the Angle at the Perpendicular B.

THIS Caſe, when reduced will fall in the third Pair of Triangles B E F and B L K, as Hypothenuſes B F, equal to the Complement of B C and B K Radius.

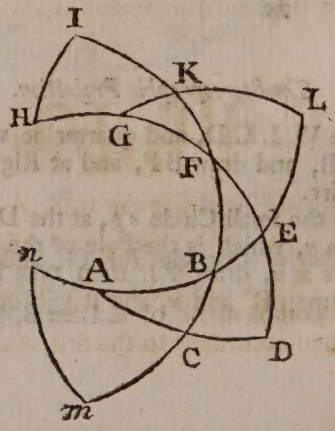
AND Perpendiculars F E, equal to the Complement of E D, or the Angle at A. and K L. equal to the Angle at B. Now by the firſt Axiom ſay,

As the Sine of B F. or S *t* of B C = 50° : 00′	9.8080675
Is to the Sine of B K, or Radius	10.
So is the Sine of F E, or the S. *t*. of A = 53° : 34′	9.7737039
To the Sine of K L = B = 67° : 32′	9.9656364

IF you work by the third Axiom, then the Angle at A, is the middle Part, and B C and B, are oppoſite Extremes,

So the Equation, according to the third Axiom will be S. *t*. A + R = S. *t*. B C + S. B. but B is wanting, then it muſt be S. *t*. A + R — S. *t*. B C = S. B.

S. *t*. A = 53° : 34′ + R	19.7737039
— S. *t*. B C = 50° : 0′	9.8080675
Remains the Sine of B = 67° : 32′	9.9656364



Caſe

Case 9.

GIVEN the Perpendicular BC = 50° : 0′, and Angle at the Base A = 53° : 34′, to find the Base AC.

THIS Case will fall in the first Pair of Triangles ABC and AED as Perpendiculars BC and ED.

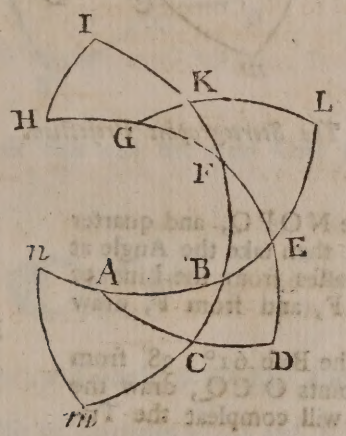
AND Bases AC and AD = Radius: Then according to the second Axiom the Analogy will be,

As the Tangent of ED = A = 53° : 34′	——	10.1318483
Is to the Tangent of the Perpendicular BC = 50° : 0′	——	10.0761865
So is the Sine of AD = Radius	—— ——	10.
To the Sine of the Base AC = 61° : 38′	——	9.9443382

BY the third Axiom, the Base AC is middle Part, and A and BC are adjacent Extremes, so the Equation will be S. AC + R = *t. c.* A + *t* BC.

BUT AC is wanting, then it must be *t. c.* A + *t.* BC — R = S. AC.

t c. A = 53° : 34′	— —	9.8681517
+ *t.* BC = 50° : 00′	—— ——	10.0761865
The Sum want. Radius is the Sine of AC = 61° : 38′	— —	9.9443382



THE Stereographic Projection for this and the eighth Case is the same with that of the seventh.

Case 10.

GIVEN the Base AC = 61° : 38′ and Angle at the Base. A = 53° : 34′ to find the Angle at the Perpendicular B.

THIS Case will fall in the fifth Pair of Triangles FKG, and FIH, as Hypotheneuses FG, and FH = Radius.

AND Perpendiculars GK, the Complement of KL = B, and HI = HFI = CD, or Complement of the Base AC. Then according to the first Axiom say,

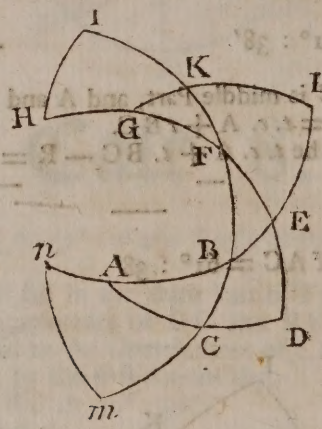
As

As Radius F H = 90° — — — —	10.
Is to the Sine of G F = E D = A = 53° : 34′ — —	9.9055522
So is the Sine of H I = C D. or the S. *c.* of A C = 61° : 38′ —	9.6767963
To the Sine of G K, or the S. *c.* of K L = S. *c.* of B = 67° : 32′ —	9.5823485

By the third Axiom, the Angle at B is the middle Part, and the Baſe A C, and Angle at the Baſe A, are oppoſite Extremes, ſo the Equation will be S. *c.* B + R = S. A + S. *c.* A C.

But B, is wanting, then it will be S. A + S. *c.* A C — R = S. *c.* B.

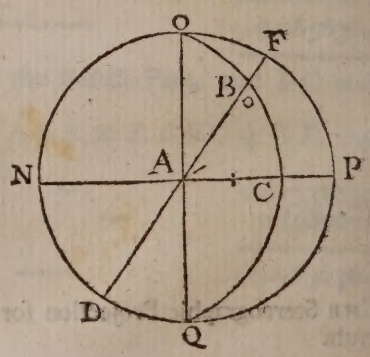
The Sine of A = 53° : 34′ — —	9.9055522
S. *c.* A C. = 61° : 38′ — —	9.6767963
The Sum want. Rad. is the S. *c.* of B = 67° : 32′ — —	9.5823485



The Stereographic Projection.

1. Draw the Primitive Circle N O P Q, and quarter it with the Lines O Q, and N P, then take the Angle at the Baſe 53° : 34′ in your Compaſſes from the Line of Chords, and ſet that from P to F, and from F, draw F D, thro' the Center at A.

2. Set the Half-Tangent of the Baſe 61° : 38′ from A to C, and thro' the three Points O C Q, draw the Oblique Circle O C Q, and that will compleat the Triangle A B C.



Caſe II.

Given the Baſe A C = 61° : 38′, and Angle at the Baſe A = 55° : 34′ to find the Perpendicular B C.

This Caſe will fall in the firſt Pair of Triangles, A B C, and A E D,, as Perpendiculars B C, and E D.

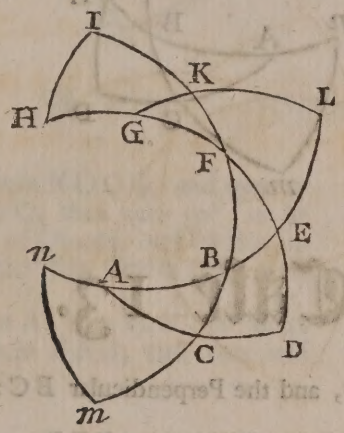
And

And Baſes A C, and A D = Radius. Now according to Axiom 2d, ſay,

As Radius A D = 90°		10.
Is to the Sine of the Baſe A C = 61° : 38′		9·9444457
So is the Tangent of E D = A = 53° : 34′		10.1318483
To the Tangent of the Perpendicular B C = 50° : 00′		10.0762940

By the third Axiom, the Baſe A C being the middle Part and Angle at the Baſe, and the Perpendicular adjacent Extremes, the Equation will be S. A C + R = T. *c.* A + T. B C. but B C is required; ſo it muſt be S. A C + R — T. *c.* A = T. BC.

S. A C = 61° : 38′ + Rad.		19.9444457
— T. *c.* A = 53° : 34′		9.8681517
Remains the Tangent of B C = 50° : 00′		10.0762940



The Stereographic Projection of this and the next Caſe following is the ſame as the 10th Caſe.

Caſe 12.

Given the Baſe A C = 61° : 38′, and Angle at the Baſe A = 53° : 34′, to find the Hypothenuſe A B.

This Caſe falls in the ſecond Pair of Triangles F E B and F D C, as Perpendiculars B E = the Complement of A B, and D C = the Complement of A C.

And Baſes F E = the Complement of the Angle at A, and F D = Radius; then, by the ſecond Axiom, ſay,

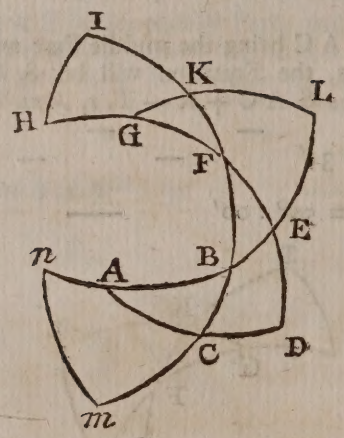
As Radius F D = 90°		10.
Is to the Sine of F E, or the S. *c.* of A = 53° : 34′		9·7737039
So is the Tangent of D C, or the T. *c.* of A C = 61° : 38′		9·7323506
To the Tangent of B E, or the T. *c.* of A B = 72° : 13′		9·5060545

By the third Axiom, the Angle at the Baſe A, is the middle Part, and the Hypothenuſe A B, and Baſe A C, adjacent Extremes; ſo the Equation will be

S. *c.* A + R = T. *c.* A B + T. A C; but A B is required, then it muſt be

S. *c.* A + R — T. A C = T. *c.* A B.

S. *c.* A = 53° : 34′ + R. is	19.7737039
— T. A C = 61° : 38′	10.2676494
Remains the T. *c.* A B = 72° : 13′	9.5060545



Case 13.

GIVEN the Base A C = 61° : 38′, and the Perpendicular B C = 50° : 0′, to find the Hypothenuse A B.

THIS Case will fall in the second Pair of Triangles F E B, and F D C, as Perpendiculars C D = the Complement of A C and B E = the Complement of A B.

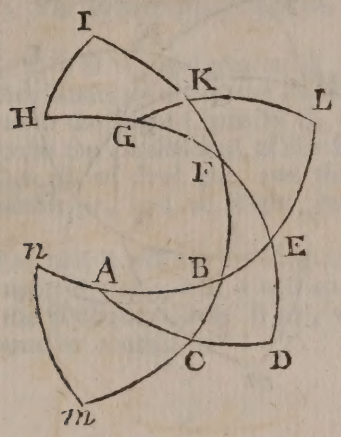
AND Hypothenuses F B = Complement of B C and F C = Radius. Now, by the first Axiom say

As Radius	10.
Is to the Sine F B, or S. *c.* of B C = 50° : 0′	9.8080675
So is the Sine of the Perpendicular C D or S. *c.* A C = 61° : 38′	9.6767963
To the Sine of B E or S. *c.* of A B = 72° : 13′	9.4848638

BY the third Axiom, the Hypothenuse is the middle Part, and the Base and Perpendicular are opposite Extremes, then the Equation will be,

S. *c.* A B + R = S. *c.* A C + S. *c.* B C. But A B is wanting, so it must be S. *c.* A C + S. *c.* B C — R = S. *c.* A B.

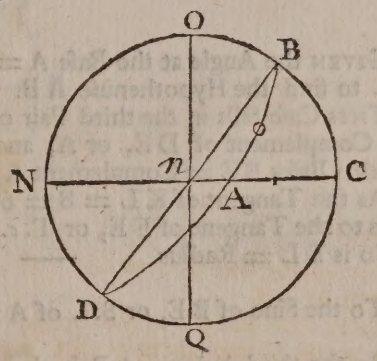
S. *c.* A C = 61° : 38′	9.6767963
+ S. *c.* B C = 50° : 0′	9.8080675
Their Sum want. Radius is the S. *c.* of A B = 72° : 13′	9.4848638



The Stereographic Projection.

1. DRAW the Primitive Circle N O C Q, and quarter it with the Lines N C, and O Q, then take 50° : 0′ in your Compaſſes from a Line of Chords, and ſet that up from C to B, and from B, thro' the Center at *n*, draw B D.

2. SET 61° : 38′ from C to A, then thro' the Points B A D, draw the Oblique Circle B A D, and that will compleat the Triangle B A C.



Caſe 14.

GIVEN the Baſe A C = 61° : 38′, and Perpendicular B C = 50° : 0′, to find the Angle at the Baſe A.

THIS Caſe falls in the firſt Pair of Triangles A B C, and A E D, as Perpendiculars B C and E D.

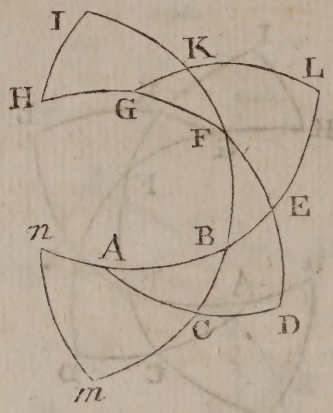
AND Baſes A C, and A D = Radius; then by the ſecond Axiom ſay,

As the Sine of the Baſe A C = 61° : 38′ — — —	9.9444457
Is to Radius A D = 90° — — — —	10.
So is the Tangent of the Perpendicular B C = 50° : 0′ —	10.0761865
To the Tangent of E D = the Angle at the Baſe, A = 53° : 34′ —	10.1317408

BY the third Axiom, the Baſe A C is the middle Part, and the Angle at the Baſe A, and Perpendicular B C, are adjacent Extremes, ſo the Equation will be S. A C + R = T. *c* A + T. B C. But the Angle at A is wanting, then it will be S. A C + R — T. B C = T. *c*. A.

The Sine of A C = 61° : 38′ + R is — — —	19.9444457
— T. B C = 50° : 0′ — — —	10.0761865
Remains T. *c*. of A = 53° : 34′ — — —	9.8682592

THE Stereographic Projection of this Caſe is as the laſt.



Case 15.

GIVEN the Angle at the Base A = 53° : 34′ and the Angle at the Perpendicular B = 67° : 32′. to find the Hypothenuse A B.

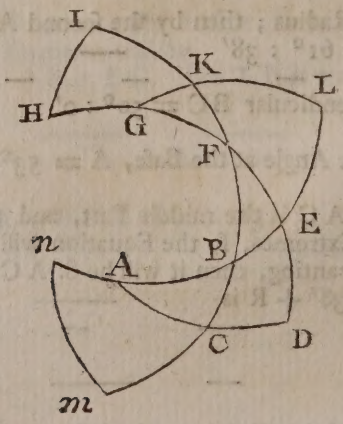
THIS Case falls in the third Pair of Triangles B E F, and B K L, as Perpendiculars F E = the Complement of D E, or A, and K L = B.

AND Bases B E = Complement A B and B L = Radius. Now by the second Axiom say,

As the Tangent of K L = B = 67° : 32′	10.3834907
Is to the Tangent of F E, or T. *c.* of E D = A = 53° : 34′	9.8681517
So is B L = Radius	10.
To the Sine of B E, or S. *c.* of A B = 72° : 13′	9.4846610

BY the third Axiom, A B is the middle Part, and the Angle at the Base A, and that at the Perpendicular B, are adjacent Extremes; so the Equation will be S. *c.* A B + R = T. *c.* A + T. *c.* B. But A B is wanting, then it must be T. *c* A + T. *c.* B — R = S. *c.* A B.

T. *c.* A = 53° : 34′	9.8681517
T. *c.* B = 67° : 32′	9.6165093
Their Sum want. Rad. is the S. *c.* of A B = 72° : 13′.	9.4846610



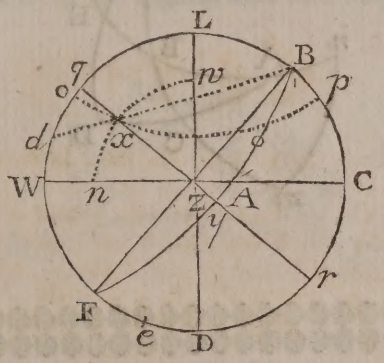
The

The Stereographic Projection.

1. DRAW the Primitive Circle L C D W, and quarter it with the Right Circles L D, and W C; this being done, draw the Parallel *o p* about the Point L, as a Pole, at the Diſtance of 53° : 34′ from it, or 36° : 26′ from the Right Circle W C, by *Caſe* 2d. of *Prob.* 9. *Chap.* 11.

2. ABOUT the Center *z*, deſcribe the Parallel *n w*, at the Diſtance of 67° : 32′ (the Angle at the Perpendicular) from it by *Caſe* 1ſt, of *Prob.* 9th, and this will interſect the former in *x*, thro' which Point draw the Diameter *q r*, and at Right Angles thereto draw the Diameter B *z* F.

3. LAY a Ruler upon B and *x*, and it will cut the Primitive Circle in *d*; from whence ſet 90 Deg. to *e*, then lay a Ruler upon B and *e*, and it will cut the Line *q r* in *y*, ſo you have three Points, B. *y*. F, to draw the Oblique Circle B *y* F, which will compleat the Triangle A B C, of which A B will be found to contain 72° : 13′.



Caſe 16.

GIVEN the Angle at the Baſe A = 53° : 34′ and Angle at the Perpendicular B = 67° : 32′, to find the Perpendicular B C.

THIS Caſe falls in the third Pair of Triangles B E F and B K L, as Perpendiculars F E = Complement of E D, or A, and K L = the Angle at B.

AND as Hypothenuſes F B = Complement of B C, and B K = Radius. Then ſay,

As the Sine of K L = B = 67° : 32′		
Is to the Sine of F E, or S. *c.* of E D = A = 53°. 34′		9·9657199
So is the Sine of K B = 90 Deg. or Radius		9·7737039
		10.
To the Sine of F B, or the S *c.* of B C = 50° : 0′		9.8080840

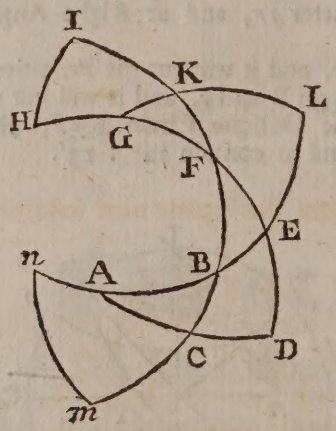
BY the third Axiom, A is the middle Part, and B C and B, are oppoſite Extremes; then the Equation will be

S. *c.* A + R = S. *c.* B C + S. B. But C is required, ſo it muſt be
S. *c.* A + R — S. B. = S. *c.* B C.

S. *c.*

S. *c.* of A = 53° : 34′ + R is	19.7737039
— S. B = 67° : 32′	9.9657199
Remains the S. *c.* of B C = 50° : 0′	9.8079840

THE Stereographic Projection is the ſame as the laſt.



CHAP. XIV.

Of Oblique-angled Spherical Triangles.

THE Axiom by which the two firſt Caſes of Oblique-angled Spherical Triangles are ſolv'd, is what we ſhall call

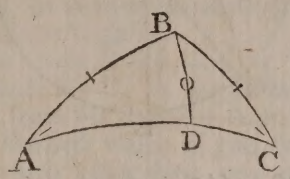
Axiom 4.

In all Spherical Triangles the Sine of any Angle is proportional to the Sine of its oppoſite Side, or the Sine of any Side is proportional to the Sine of its oppoſite Angle.

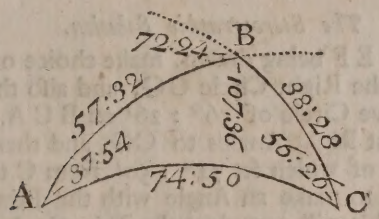
THE Truth of this will appear, if we divide the Oblique-angled Triangle A B C, into two Right-angled Triangles, as A B D, and C B D, and then take the two Hypotheneuſes A B and C B, and Angles at the Baſes, A and C to find the Perpendicular B D: Thus,

Again

	1	S. B D = S. B C + S. C — R	by Axiom the 3d.
Again	2	S. B D = S. A B + S. A — R	
1=2	3	S. B C + S. C. — R = S. A B + S. A — R	
3+R.	4	S. B C + S. C = S. A B + S. A.	by the Logarithms
that is	5	S. B C × S. C = S. A B × S. A.	by the natural Numbers
Consequently	6	S. A : S. B C :: S. C : S. A B.	
6 inverted	7	S. B C : S. A :: S. A B : S. C.	*Q. E. D.*



LET the Examples be taken from the Figures next following.



Case I.

GIVEN the two Angles with a Side oppoſite to one of them to find the Side oppoſite to the other.

Examp. 1. Dat. $\left\{\begin{array}{l} A B = 57° : 32' \\ B = 107 : 36 \\ C = 56 : 26 \end{array}\right\}$ to find A C.

The Stereographic Solution.

1. DRAW the Primitive Circle B N E F, and alſo the Diameters B E and F N, at Right Angles to each other. Then

2. BY *Caſe* 2d, of *Prob.* 2d, *Chap.* 11. draw B A E, ſo as to make the Angle F B A = 72° : 24′ (the Supplement of 107° : 36′.)

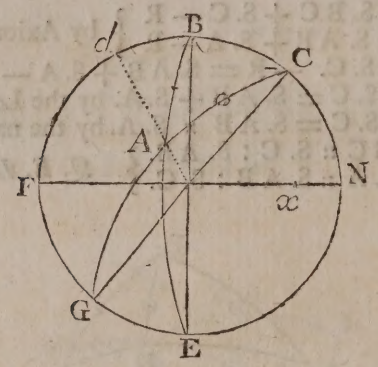
3. FIND the Pole *x*. of this Oblique Circle, by Help of which ſet 57° : 32′ from B to A.

4. THROUGH the Point A, draw the great Circle G A G, to make an Angle with the Primitive Circle of 56° : 26′, and this will compleat the Triangle of A B C; then you may meaſure A C, by the 3d *Caſe* of *Prob.* 3d, *Chap.* 11.

BY Calculation it will be,

As the Sine of the Angle at C = 56° : 26′. *co. ar.*	0.0792283
Is to the Sine of . . . . A B = 57° : 32′	9.9261901
So is the Sine of the External Angle F B A = 72° : 24′	9.9791790
To the Sine of A C = 74° : 50′	9.9845984

Examp.

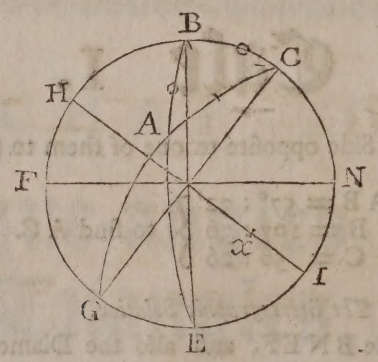


Examp. 2. Dat. $\left\{\begin{array}{l} B = 107^\circ : 36' \\ C = 56 : 26 \\ CA = 74 : 50 \end{array}\right\}$ to find A B.

The Stereographic Solution.

1. THE Primitive Circle B N E F being drawn, make choice of any convenient Point therein, as at C, from whence draw the Right Circle C G, and alſo the Oblique Circle C A G, to make an Angle with the Primitive Circle of $56^\circ : 26' = B C A$.

2. DRAW the Diameter H I at Right Angles to C G, and therein find the Pole *x*. of the Oblique Circle C A G, by Help of which ſet $74^\circ : 50'$ from C to A, and thro' the Point A, draw the Oblique Circle B A E, to make an Angle with the Primitive Circle of $72^\circ : 24' = F B A$. (by *Prob.* the 3d.) and this will compleat the Triangle A B C, then you may meaſure the Side A B.



BY Calculation it will be,

As the Sine of the Angle H B A $= 72^\circ : 24'$. *co. ar*	———	0.0208207
Is to the Sine of A C $= 74^\circ : 50'$	———	9.9846033
So is the Sine of the Angle at C $= 56^\circ : 26'$	———	9.9207717
To the Sine of A B $= 57^\circ : 32'$	———	9.9261957

Caſe 2.

GIVEN two Sides with an Angle oppoſite to one of them, to find the Angle oppoſite to the other Angle.

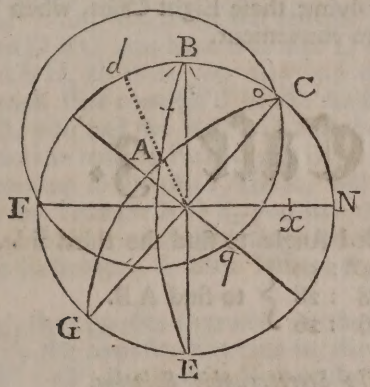
Examp.

Examp. 1. Dat. $\left\{\begin{array}{l} AB = 57° : 30' \\ AC = 74 : 50 \\ B = 107 : 36 \end{array}\right\}$ to find the Angle C.

The Stereographic Solution.

1. DRAW the Primitive Circle B N E F, and the Diameters B E and F N, at Right Angles to each other, and having made choice of a Point in the Primitive Circle as at B, draw from thence the Oblique Circle B A E, ſo as to make an Angle with the Primitive Circle of 72° : 24′ = F B A (the Supplement of 107° : 36′.)

2. FIND *x*, the Pole of the Oblique Circle B A E, and lay 57° : 32′ from B to A, and about A, as a Pole, draw the ſmall Circle F *q* C, at the Diſtance of 74° : 50′ from it, and that will cut the Primitive Circle in C, from which Point draw the Diameter C G; ſo you will have three Points, *viz.* C, A and G‘ through which if you draw the Oblique Circle C A G, it will compleat the Triangle B A C, which being done you may meaſure the Angle B C A, (by *Caſe* the 2d. of *Prob.* 7.)



BY Calculation, thus :

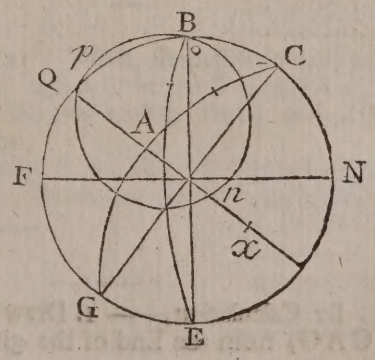
As the Sine of A C = 74° : 50. *co. ar.*	——— ———	0.0153967
Is to the Sine of the Angle F B A = 72° : 24′	——— ———	9.9791798
So is the Sine of A B = 57° : 32′	——— ———	9.9261901
To the Sine of B C A = 56 : 26′	——— ———	9.9207666

Examp. 2. Dat. $\left\{\begin{array}{l} AB = 57° : 32' \\ AC = 74 : 50 \\ C = 56 : 26 \end{array}\right\}$ to find the Angle A B C.

The Stereographic Solution.

1. DRAW the Primitive Circle B N E F, and having made choice of a Point therein, as at C, draw from thence the Diameter C G, and alſo the Oblique Circle C A G, to make an Angle with the Primitive Circle of 56° : 26′ = B C A.

2. FIND the Pole *x*, of the Oblique Circle C A G, and by the Help thereof ſet 74° : 50′ from C to A, then about A as a Pole, deſcribe the ſmall Circle *p* B *n*, at the Diſtance of 57° : 32′ from it, and that will cut the Primitive Circle in B, from which draw the Diameter BE, then you will have the three Points B. A. E. thro' which the Oblique Circle B A E. muſt paſs, which being drawn will complete the Triangle A B C, then you may meaſure the Angle *p* B A, or the Angle A B C.



By Calculation, thus :

As the Sine of A B = 57° : 32′ *c o. ar.*	———	0.0738099
Is to the Sine of the Angle at C = 56° : 26′	———	9.9207717
So is the Sine of A C = 74° : 50′	———	9.9846033
To the Sine of Q B A = 72° : 24′	———	9.9791849

Its Supplement is 107° : 36′ = A B C, the Angle required.

In order to ſolve the Eight Caſes next following at two Operations, you muſt know that a Perpendicular is to be drawn, ſo as to divide the given Oblique-angled Triangle into two Right-angled Triangles, as the Triangle A B C, in the next *Fig.* following is divided by the Oblique Circle B D E, paſſing thro' *p*, the Pole of the Circle C A G.

Or, ſo that two Sides of one of the Right-angled Triangles may fall out of the given Triangle.

The Univerſal Propoſition, or what we make the Third Axiom of Right-angled Spherical Triangles, is what we ſhall uſe in ſolving theſe Eight Caſes, when a Perpendicular is drawn, or elſe by other Axioms when more convenient.

Caſe 3.

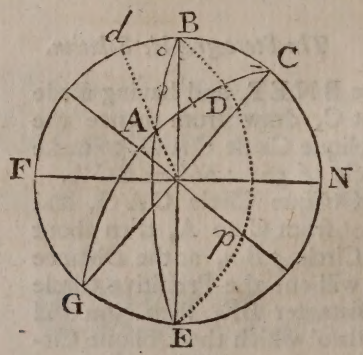
Given two Sides and the included Angle to find the third Side.

Examp. 1. Dat. $\left\{\begin{array}{l} A\,C = 74° : 50' \\ B\,C = 38 : 28 \\ \angle\, C = 56 : 26 \end{array}\right\}$ to find A B.

The Stereographic Solution.

1. Draw the Primitive Circle B N E F, and Diameter B E and F N at Right Angles to each other, then take 38° : 28′ in your Compaſſes from a Line of Chords, and lay that Extent upon the Primitive Circle from B to C, and from C draw the Diameter C G, and alſo the Oblique Circle C A G, ſo as to make an Angle with the Primitive Circle of 56° : 26′.

2. Set 74° : 50′ upon the Oblique Circle from C to A, and having thus found the Point A, you have two other Points, *viz.* B and E ; then by theſe three you may draw the Oblique Circle B A E, which will compleat the Triangle A B C ; this being done, you may meaſure the Side A B required.



By Calculation,—— 1. Draw the great Circle B D E, thro' *p* the Pole of the Oblique Circle C A G, from the End of the given Side B C, *&c* according to the following

RULE

RULE.

FROM the End of a Side given, being adjacent to an Angle given, let fall the Perpendicular oppoſite to that Angle, and touching in ſome Part the Side requir'd, or if it may be oppoſite to the Angle requir'd; or ſo, that in one of the Right-angled Triangles you may have two of the given Things entire, and the other (whenever you can) entire in the other; or elſe that they be Part in one and Part in the other. According to this the Perpendicular B D is drawn, which reduces the Oblique-angled Triangle A B C into two Right-angled Triangles B D A and B D C.

2. BY the Help of B C = 38° : 28′ and Angle at C = 56° : 26′ find the Part C D, by the third Axiom, thus: The Angle at C being the middle Part.

S. *c.* C = 56° : 26′ + R	—•—	19.7426520
— T. *c.* B C = 38° : 28′	———	10.0999135
Remains the Tangent D C = 23° : 43′	———	9.6427385

Take this out of A C = 74° : 50′, and there will remain A D = 51° : 7′.

Now, in the Right-angled D B C you have B C and D C to compare with the Side B A, and A D of the Triangle A B D, by theſe you may find the Side A B, but firſt you muſt know what Name to give each Part concern'd in the Analogy; in order to which, ſuppoſe that in the Triangle B C D, you had the Side B C and the Part C D to find the Perpendicular B D, then B C would be the middle Part, and B D and D C would be oppoſite Extremes; therefore their Names, according to the third Axiom, will be S. *c.* B C, and S. *c.* D C.

AGAIN, ſuppoſe that in the Triangle A D B, you had the Side A B, and Part A D, to find the ſaid Perpendicular B D; then, according to the ſame Axiom, A B is the middle Part, and A D and B D, are oppoſite Extremes; ſo their Names are S. *c.* A B, and S. *c.* A D: Then to find A B, ſay,

As S. *c.* D C = 23° : 43′, the oppoſite Extreme in the firſt Triang. *co. ar.*		0.0383200
Is to S. *c.* A D = 51° : 7′, the oppoſite Extreme in the ſecond Triang.	——	9.7977775
So is the S. *c.* B C = 38° : 28′, the middle Part in the firſt Triang.	———	9.8937452
To the S. *c.* of A B = 57° : 32′, the middle Part in the ſecond Triang.	——	9.7298427

Examp. 2. Dat. $\left\{\begin{array}{l} A C = 74° : 50' \\ B C = 38 \ : 28 \\ \angle C = 56 \ : 26 \end{array}\right\}$ to find A B, as before.

1. DRAW A *d*, perpendicular to C B produced, and then you'll have two Right-angled Triangles A *d* B and A *d* C, having the Right Angle at *d*, common to both; now with the Side A C, and Angle A C B find the Side *d* C. (*See the laſt Fig.*)

S. *c.* A C *d* = 56° : 26′ + R	———	——	19.7426520
— T. *c* A C = 74° : 50′	———	——	9.4330804
Remains the Tangent of C *d* = 63° : 53′	——	——	10.3095716

2. FROM *d* C, take B C, and there will remain B *d* = 25° : 25′. Now, ſuppoſe you had, in the Right-angle Triangle A *d* B, the Sides A B and B *d*, or *d* C and A C in the Triangle A *d* C to find the Perpendicular A *d*; then A B and A C would be the two middle Parts, and *d* B and *d* C the oppoſite Extremes; therefore the Analogy will be,

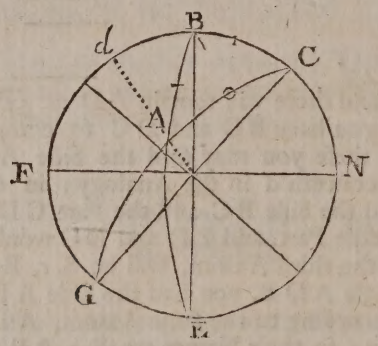
As the S. *c.* of the oppoſite Extreme *d* C = 63° : 53′ *co. ar.*	———	0.3563496
Is to the S. *c.* of the oppoſite Extreme *d* B = 25°. 25′	———	9.9557890
So is the S. *c.* of the middle Part A C = 74° : 50′	——	9.4176837
To the S *c.* of the middle Part A B = 57° : 32′	———	9.7298223

Examp. 3. Dat. $\left\{\begin{array}{l} AB = 57^\circ : 32' \\ BC = 38 : 28 \\ \angle B = 107 : 36 \end{array}\right\}$ to find AC.

The Stereographic Solution.

1. THE Primitive Circle FBNE being drawn, and alſo the Diameters BE and FN, at Right Angles to each other, then draw the Oblique Circle BAE, to make an Angle *d*BA with the Primitive Circle of 72° : 24′, (the Supplement of 107° : 36′,) upon which ſet 57° : 32′ from B to A.

2. FROM B, ſet 38° : 28′ to C, and from C draw the Diameter CG, then thro' the three Points C.A.G. draw the Oblique Circle CAG, and that will compleat the Triangle ABC; then meaſure AC, the Side required.



BY Calculation, firſt draw the Perpendicular A*d* to the Primitive Circle, then in the Right-angled Triangle B*d* A, you have the Angle *d*BA = 72°2 : 24′, and Hypothenuſe AB = 57° : 32′, to find *d*B. Here the Angle *d*BA is the middle Part, and *d*B and AB, are adjacent Extremes; therefore it will be

S. *c.* *d*BA = 72° : 24′ + R	19.4805385
— T. *c.* AB 57° : 32′	9.8036296
Remains the Tangent of *d*B = 25° : 25′	9.6769089
To which add BC = 38° : 28′.	
The Sum is *d*C = 63° : 53′.	

Now, if AB and *d*B, in the Triangle *d*BA, or AC and *d*C, in the Triangle A*d*C, were given to find the Perpendicular A*d*, then AB and AC would be two middle Parts, and *d*B and *d*C two oppoſite Extremes. Hence the Analogy will be,

As the S. *c.* of the oppoſite Extreme *d*B = 25° : 25′, *co. ar.*	0.0442110
Is to the S. *c.* of the oppoſite Extreme *d*C = 63° : 53′.	9.6436504
So is the S. *c.* of the middle Part AB = 57° : 32′	9.7298197
To the S. *c.* of the middle Part AC = 74° : 50′	9.4176811

THERE are other Ways to ſolve this Caſe; one is from the very learned and ingenious Mr. FACIO, as he has it in his *Navigation Improv'd*, which we will call

Axiom

Axiom 5.

To the Logarithmic verſed Sine of the given Angle add the Logarithmic Sines of the two Sides, and from the Sum cut off twice the Radius, then find the natural Number of the remaining Logarithm, to which add the natural verſed Sine of the Difference of the Sides, the Sum is the natural verſed Sine of the third Side.

To make this plain, we ſhall uſe it in ſolving the two laſt Examples.

GIVEN { A C = 74° : 50′ / B C = 38 : 28 / ∠ C = 56 : 26 } to find the Side A B.

The Logarithmic verſed Sine of the ∠ C = 56° : 26′, is	9.6503980
The Logaritmic Sine of A C = 74° : 50′, is	9.9846033
And the Logarithmic Sine of B C = 38° : 28′, is	9.7938317
The Sum is (cutting of twice Radius)	9.4288330
	ch.
The natural Number anſwering to 9.4288330, is	26843 (9)
The natural verſed Sine of the Difference of the Sides, *viz.* 36° : 22′	19476 (9)
The Sum is the natural verſed Sine of A B = 57° : 32′	46319 (9)

AGAIN, given, { A B = 57° : 32′ / B C = 38 : 28 / ∠ B = 107 : 36 } to find A C.

The natural verſed Sine of 107° : 36′ is 130237, its Logarithm is	10.1147343
The Logarithmic Sine of A B = 57° : 32′, is	9.9261901
The Logarithmic Sine of B C = 38° : 28′, is	9.7938317
Their Sum is	29.8347561
	ch.
The natural Number anſwering to 9.8347561, is	68353 (9)
The natural verſed Sine of the Difference of the Sides 19° : 4′	5486 (8)
The Sum is the natural verſed Sine of A C = 74° : 50′	73839 (9)

IF a Table of verſed Sines is not at hand, a Solution to the third Caſe may be given according to the following

RULE.

1. As Radius
Is to the Sine Complement of the contain'd Angle,
So is the Tangent of the leſſer of the given Sides
To the Tangent of a fourth Arch;
Then if the contain'd Angle be Acute, ſubſtract the fourth Arch from the greater of the given Sides; but when it is Obtuſe, from the Supplement thereof, and call the Remainder the Reſidual Arch.

2. As

2. As the S. *c.* of the fourth Arch
Is to the S. *c.* of the Reſidual Arch,
So is the S. *c.* of the leſſer given Side
To the S. *c.* of the Side required.

EXAMPLE 1.

GIVEN, $\left\{\begin{matrix} AC = 74^\circ : 50' \\ BC = 38 : 28 \\ \angle C = 56 : 26 \end{matrix}\right\}$ as before, to find AB. (*See the laſt Fig.*)

1. As Radius		10.
Is to the S. *c.* of C = 56° : 26′		9·7426520
So is the Tangent of BC = 38° : 28′		9·9000865
To the Tangent of a fourth Arch = 23° : 43′		9·6427385
The greater Side is		74° : 50′
The fourth Arch ſubſtract		23 : 43
Reſidual Arch is		51 : 07
2. As the S. *c.* of the fourth Arch = 23° : 43′, *co. ar.*		0·0383200
Is to the S. *c.* of the Reſidual Arch 51° : 7′		9·7977775
So is the S. *c.* of BC = 38° : 28′		9·8937452
To the S. *c.* of the Side requir'd, AB = 57° : 32′, (as before)		9·7298427

Note, That when the Contain'd Angle, and Reſidual Arch, are each leſs or more than 90 Deg. the Side ſought is leſs than 90 Degrees; but when one is more, and the other leſs, 'tis more than 90 Degrees.

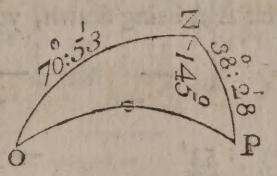
EXAMPLE 2.

GIVEN, $\left\{\begin{matrix} \odot z = 70^\circ : 53' \\ P z = 38 : 28 \\ \odot z P = 145 : 00 \end{matrix}\right\}$ to find $\odot$P.

1. As Radius		10.
Is to the S. *c.* of 145°, or its Supplement 35° : 0′		9·9133645
So is the Tangent of 38° : 28′		9·9000865
To the Tangent of a fourth Arch 33° : 3′		9·8134510
The Contain'd Angle is Obtuſe, therefore 180°—70° : 53′ = 109° : 7′		
The fourth Arch ſubſtract	33 : 3	
The Reſidual Arch	76 : 4	
2. As the S. *c.* of the fourth Arch = 33° : 3′ *co. ar.*		0·0766550
Is to the S. *c.* of the Reſidual Arch 76° : 4′		9·3816434
So is the S. *c.* of the leſſer Side zP = 38° : 28′		9·8937452
To the S. *c.* of 77° : 0′		9·3520436

Note,

Note, That becauſe the Contain'd Angle ⊙ ≈ P is more than 90 Deg. and the Reſidual Arch leſs than 90 Deg. therefore the Side ſought is more than 90 Deg. then 180° — 77° = 103° : 0′ = ⊙ P, the Side required.



Caſe 4.

GIVEN two Sides and the included Angle, to find one of the other Angles.

Examp. 1. Dat. $\left\{\begin{array}{l} AB = 57° : 32' \\ BC = 38 : 28 \\ \angle B = 107 : 36 \end{array}\right\}$ to find the Angle at C.

THE Stereographic Solution of this Example, as to the Figure, is the ſame with the third Example of the third Caſe; ſo if you meaſure the Angle BCA, you'll find it to be 56° : 26′.

BY Calculation, having conſider'd the Queſtion, you'll find that the Perpendicular muſt fall from A, to the Primitive Circle at *d*; then you have two Right-angled Triangles, *viz.* A*d*B and A*d*C, by the Help of AB, and Angle A B*d*, find *d*B, the Angle *d*B A being the middle Part, and *d*B and AB adjacent Extremes. Thus,

S. *c.* AB*d* = 72° : 24′ + R =	—	—	19.4805385
— T. *c.* AB = 57° : 32′	—	—	9.8036296
Remains the Tangent of DB = 25° : 25′	—	—	9.6769089
To which add BC = 38 : 28			
The Sum is *d*C = 63 : 53			

FOR the ſecond Operation; ſuppoſe that if *d*B and AB*d*, in the Triangle AB*d*, or *d*C, and AC*d* in the Triangle A*d*C, were given to find the Perpendicular A*d*, then *d*B and *d*C would be the middle Parts, and the Angles *d*BA and *d*CA adjacent Extremes; then the Analogy will run thus:

As the Sine of the middle Part *d*B = 25° : 25′. *co. ar.*		—	0.3673424
Is to the Sine of the middle Part *d*C = 63° : 53′	—	—	9.9532278
So is the T. *c.* of the adjacent Extreme *d*BA = 72° : 24		—	9.5013588
To the T. *c.* of the adjacent Extreme *d*CA = 56° : 26′		—	9.8219290

EXAMPLE 2.

DAT. $\left\{\begin{array}{l} AC = 74° : 50' \\ BC = 38 : 28 \\ \angle C = 56 : 26 \end{array}\right\}$ to find the Angle at B.

THE

THE Stereographic Projection is the ſame with that of the firſt Example of the third Caſe, in that you was to find the Side A B. But here you muſt find the Angle *d* B A, the Supplement of which is the Angle A B C required. (*See that Fig.*)

BY Calculation, the Perpendicular A *d*, being drawn, you muſt find *d* C, by the given Parts A C, and A C B: Thus,

S. *c.* C = 56° : 26′ + R	—	—	—	19.7426520
— T. *c.* A C = 74° : 50	—	—	—	9.4330804
Remains the Tangent of *d* C = 63° : 53′	——	——		10.3095716
From which take B C = ——38 : 28				
And there will remain *d* B = 25 : 25				

FOR the ſecond Operation, ſuppoſe you had *d* B, and the Angle A B *d*, of the Triangle A *d* B, or the Side *d* C and Angle A C *d* of the Triangle A *d* C to find the Perpendicular A *d*, then *d* B, and *d* C would be the two middle Parts, and the Angle A B *d*, and A C *d*, adjacent Extremes. So the Analogy will run thus,

As the Sine of the middle Part *d* C = 63° : 53′. *co. ar.*	—	—	0.0467728
Is to the Sine of the middle Part *d* B = 25° : 25′	—	—	9.6326576
So is the T. *c.* of the adjacent Extreme A C *d* = 56° : 26′		——	9.8218803
To the T. *c.* of the adjacent Extreme A B *d* = 72° : 24′		—	9.5013107

Having thus found the Angle A B *d*, its Supplement is 107° : 36′ the Angle A B C required.

ANOTHER Way to do the ſame, without drawing a Perpendicular

RULE.

(1.) As the Co-ſine of half the Sum of the Sides
Is to the Co-ſine of half their Difference,
So is the Co-tangent of half the contain'd Angle
To the Tangent of half the Sum of the other two Angles.

(2.) As the Sine of half the Sum of the two Sides
Is to the Sine of half their Difference,
So is the Co-tangent of half the contain'd Angle
To the Tangent of half the Difference of the other two Angles.

(3.) Add the half Difference thus found to half the Sum of the other two Angles, and you'll have the greater of them, and ſubſtract the ſaid half Difference from the half Sum, and you'll have the leſſer.

EXAMPLE 3.

LET the Side A C, BC and Angle at C be given, to fide the Angle at B, as at the ſecond Example of the fourth Caſe.

A C

A C = 74° : 50′
B C = 38 : 28

Sum is 113 : 18 half this Sum } is { 56° : 39′ }. Then
Differ. 36 : 22 half the Differ. } { 18 : 11 }
∠ C is 56 : 26 half . . . { 28 : 13 }

As the Co-fine of 56° : 39′. *co. ar.* — 0.2598332
Is to the S. *c.* of 18 : 11 — 9.9777523
So is T. *c.* of . . 28 : 13 — 10.2703737

To the Tangent of half the Sum of the other two Angles 72° : 45′ — 10.5079592

AGAIN, As the Sine of 56° : 39′. *co. ar.* — 0.0781430
Is to the Sine of . . 18 : 11 — 9.4942361
So is the T. *c.* of . . 28 : 13 — 10.2703737

To the Tangent of half the Difference of the Angles, which is 34° : 51′ — 9.8427528
To which add half the other two Angles . . . . . 72 : 45

And the Sum is . . . . . 107 : 36 = the Angle at B.

And the Difference is . . . . . 37 : 54 = the Angle at A.

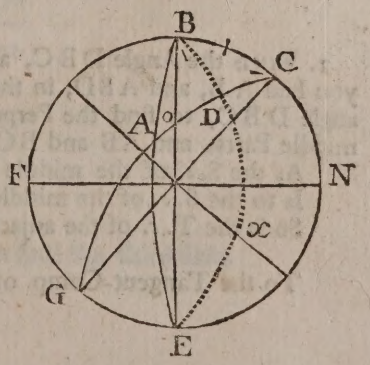
Case 5.

GIVEN two Angles and the Side between them, to find the third Angle.

Examp. Dat. { A B C = 107° : 36′ / A C B = 56 : 26 / B C = 38 : 28 } to find the Angle B A C.

The Stereographic Projection.

1. DRAW the Primitive Circle B N E F, and Diameters B E, and F N, at right Angles to each other.
2. DRAW the Oblique Circle B A E, ſo as to make the Angle F B A, with the Primitive Circle of 72° : 24′.
3. FROM B, ſet off 38° : 28′ to C, and from C draw the Diameter C G, and alſo the Oblique Circle C G A, to make an Angle B C A, with the Primitive Circle of 56° : 26′, and that will cut the Oblique Circle B A E in A, and compleat the Triangle A B C, which being done you may meaſure the Angle BAC, and find it to be 37° : 54′.



By Calculation,

1. Draw the Oblique Circle B D E, thro' x the Pole of the Oblique Circle C A G, and it will make Right Angles with the Oblique Circle C A G at the Point D, and divide the Triangle A B C into two Right-angled Triangles B C D, and B A D.
2. Find the Angle D B C, by Help of the Side B C and Angle B C D, B C being the middle Part, and B C D, and D B C adjacent Extremes: Thus,

S. *c.* B C = 38° : 28′ + R	—		—	19.8937452
— T. *c.* A C B = 56° : 26′	—	—	—	9.8218803
Remains the T. *c.* of D B C = 40° : 17′	——	——		10.0718649
Which take from the Angle A B C = 107 : 36				
And there will remain the Angle A B D = 67 : 19				

For the fecond Operation, fuppofe that you had the Angle D A B and A B D, in the Triangle A B D, or D B C and D C B in the Triangle D B C, to find the Perpendicular B D, then would D A B and D C B be the two middle Parts, and A B D, and D B C two oppofite Extremes; then the Analogy will be,

As the Sine of the oppofite Extreme D B C = 40° : 17. *co. ar.*	—	0.1893859
Is to the Sine of the oppofite Extreme A B D = 67° : 19′	——	9.9650371
So is the S. *c.* of the middle Part D C B = 56° : 26′	——	9.7426520
To the S. *c.* of the middle Part D A B = 37° : 54′	——	9.8970750

Cafe 6.

Given two Angles and a Side between them, to find one of the other Sides.

Examp. Dat. $\left\{\begin{array}{l} \text{A B C} = 107° : 36' \\ \text{A C B} = 56 : 26 \\ \text{B C} = 38 : 28 \end{array}\right\}$ to find the Side A B.

The Parts given here being the fame as the 5th *Cafe*, the Conftruction will be the fame.

By Calculation.

1. Find the Angle D B C, and alfo A B D, as you did at the 5th *Cafe*; and fuppofing that you had A B, and A B D, in the Triangle A B D, or the Side B C, and D B C in the Triangle D B C, to find the Perpendicular D B; then A B D, and D B C would be the two middle Parts, and A B and B C, the adjacent Extremes. Hence you have this Analogy,

As the S. *c.* of the middle Part D B C = 40° : 17′, *co. ar.*	——	0.1175572
Is to the S. *c.* of the middle Part A B D = 67° : 19′.	——	9.5861795
So is the T. *c.* of the adjacent Extreme B C = 38° : 28′	——	10.0999135
To the Tangent-Comp. of the adjacent Extreme A B = 57° : 32′	——	9.8036508

Another

Another Way to do the ſame without a Perpendicular.

RULE.

1. As the Co-ſine of half the Sum of the Angles, is to the Co-ſine of half their Difference, ſo is the Tangent of half the included Side, to the Tangent of half the Sum of the other two Sides.

2. As the Sine of half the Sum of the two Angles, is to the Sine of half their Difference, ſo is the Tangent of half the included Side, to the Tangent of half the Difference of the other two Sides.

Now, if you add this half Difference of the Sides to half their Sum, you will have the greater Side; or ſubſtract it from the ſaid half Sum, and the Remainder will be the other Side.

1. LET $\left\{\begin{array}{rl} ABC = & 107° : 36' \\ ACB = & 56 : 26 \\ BC = & 38 : 28 \end{array}\right\}$ be given, to find AB, or AC.

$ABC =$	$107° : 36'$					
$ACB =$	$56 : 26$					
Sum	$164 : 02$					
Difference	$51 : 10$	half each is	$82° : 1'$			
Side BC	$38 : 28$		$25 : 35$			
			$19 : 14$			

Now ſay,

As the S. *c.* of	$82° : 1'$	*co. ar.*	—	—	—	0.8573445
Is to the S. *c.* of	$25° : 35'$	—	—	—	—	9.9551864
So is the Tangent of	$19° : 14'$	—	—	—	—	9.5426877
To the Tangent of half the Sum of the other two Sides			$66° : 11'$	—		10.3552186
2. As the Sine of	$82° : 1'$	*co. ar.*	—	—	—	0.0042295
Is to the Sine of	$25° : 35'$	—	—	—	—	9.6353062
So is the Tangent of	$19° : 14'$	—	—	—	—	9.5426877
To the Tangent of half the Difference of the Sides			$8° : 39'$	—		9.1822234
Half the Sum of the Sides		—	$66 : 11$			
The Sum is the greater Side AC =		—	$74 : 50$			
And Difference the leſſer Side AB =			$57 : 32$			

Caſe 7.

GIVEN two Sides with an Angle oppoſite to one of them, to find the third Side.

Examp. 1. Dat. $\left\{\begin{array}{rl} AB = & 57° : 32' \\ AC = & 74 : 50 \\ \angle B = & 107 : 36 \end{array}\right\}$ to find BC.

THE Conſtruction of this Example is the ſame with the firſt Example of the ſecond Caſe. *See the Fig. belonging thereto.*

By Calculation.

DRAW A *d* perpendicular to the Primitive Circle, and then with the given Side A B, and Angle A B *d*, find the Part *d* B. Thus,

S. *c.* A B *d* = 72° : 24′ + R	19.4805385
— T. *c.* A B = 57° : 32′	9.8036296
Remains the Tangent of *d* B = 25° : 25′	9.6769089

NOW if *d* B and A B were given to find the Perpendicular A *d*, or *d* C, and A C to find the ſame; then A B and A C would be the two middle Parts, and *d* B and *d* C two oppoſite Extremes. So the Analogy will be,

As the S. *c.* of the middle Part A B = 57° : 32′ *co. ar.*	0.2701803
Is to the S. *c.* of the middle Part A C = 74°. 53′	9.4176837
So is the S. *c.* of the oppoſite Extreme B *d* = 25° : 25′	9.9557890
To the S *c.* of the oppoſite Extreme *d* C = 63° : 53′	9.6436530
From which take *d* B = 25 : 25	
And there will remain B C = 38 : 28	

Examp. 2. Dat. { A C = 74° : 50′ ; B C = 38 : 28 ; ∠C = 107 : 36 } to find A B.

The Stereographic Projection.

1. DRAW the Primitive Circle B N E F, and the Diameters B E and F N at Right Angles to each other, then from B draw the Oblique Circle B A E ſo as to make an Angle F B A with the Primitive Circle of 72° : 24′.

2. FROM B ſet off 38° : 28′ to C, and from C draw the Diameter C G; then about C, as a Pole, draw the ſmall Circle *q* A *n* at the Diſtance of 74° : 50′ from it, and this will cut the Oblique Circle B A E in the Point A; ſo now you have three Points, *viz.* C. A. G. through which draw the Oblique Circle C A G, and that will compleat the Triangle A B C; then you may meaſure A B, the Side required.

By Calculation.

1. THROUGH the Pole *x* of the Oblique Circle B A E, draw the great Circle *d* C *x* G, to cut the Oblique Circle E A B produc'd in *d*, and there it will cut it at Right Angles; and thus you will have two Right-angle Triangles A *d* C, and B *d* C.

2. IN

2. In the Triangle B*d* C, you have the Hypothenuſe B C, and the Angle *d* B C, by the Help of theſe you muſt find B *d*.

Here the Angle *d* B C is the middle Part, and *d* B and B C adjacent Extremes. Then

S. *c*. B = 72° : 24′ + R —— —— ——	19.4805385
— T. *c* B C = 38° : 28′ —— ——	10.0999135
Remains the Tangent of B D = 13° : 31′ —— ——	9.3806250

Now, ſuppoſe you had B *d* and B C, in the Triangle B *d* C, or A *d* and A C in the Triangle A *d* C, to find the Perpendicular *d* C; then B C and A C would be the two middle Parts, and B *d* and A *d* oppoſite Extremes; ſo the Analogy will be,

As the S. *c*. of the middle Part B C = 38° : 28′, *co. ar*. ——	0.1062548
Is to S. *c*. of the middle Part A C = 74° : 50′ —— ——	9.4176837
So is the S. *c*. of the oppoſite Extreme B *d* = 13° : 31′ ——	9.9878012
To the S. *c*. of the oppoſite Extreme A *d* = 71° : 3′ ——	9.5117397
From which take B *d* = —— 13 : 31	
And there will remain A B = —— 57 : 32 the Side required.	

Caſe 8.

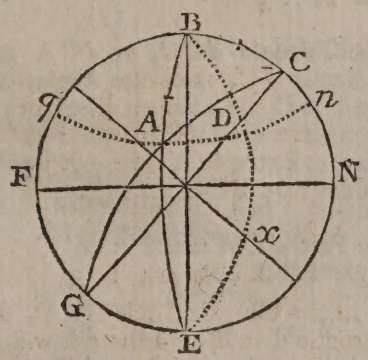
Given two Sides with an Angle oppoſite to one of them, to find the contain'd Angle.

Examp. 1. Dat. { A B = 57° : 32′ / B C = 38 : 28 / ∠ C = 56 : 26 } to find the Angle A B C.

The Stereographic Projection.

1. The Primitive Circle B C N E F being drawn, and alſo B E and F N at Right Angles to each other, ſet off 38° : 28′ from B to C, and from C draw the Diameter C G, and alſo the Oblique Circle C A G, to make an Angle with the Primitive of 56° : 26′.

2. About B, as a Pole, draw the ſmall Circle *q* A *n* at the Diſtance of 57° : 32′ from it, and that will cut the Oblique Circle C A G in A; ſo have you three Points B. A. and E. to draw the Oblique Circle B A E, and this will compleat the Triangle A B C.



By Calculation.

1. THROUGH *x*, the Pole of the Oblique Circle C A G, and the Points B and E, draw the Oblique Circle B D *x* E, and it will cut the Oblique Circle C A G in D, and there make Right Angles with it; and ſo divide the Triangle A B C into two Right-angled Triangles A B D and C B D, in the laſt you have the Hypotheneuſe B C and Angle B C A, by theſe to find the Angle D B C; here B C is the middle Part, and the Angles D B C, and D C B, adjacent Extremes. Then

S. *c.* B C = 38° : 28′ + R	——	——	19.8937452
— T. *c.* D C B = 56° : 26′	——	——	9.8218803
Remains the T. *c.* of D B C = 40° : 17′	—	—	10.0718649

HAVING found this Angle, ſuppoſe that it and the Hypotheneuſe B C in the Triangle B C D, or the Angle A B D and Hypotheneuſe A B in the Triangle A B D were given to find the Perpendicular B D, then the Angles D B C and D B A, would be the two middle Parts, and B C and B A the adjacent Extremes. So it will be

As the T. *c.* of the adjacent Extreme B C = 38° : 28′. *co. ar.*	—	9.9000865
Is to the T. *c.* of the adjacent Extreme A B = 57° : 32′	—	9.8036296
So is the S. *c.* of the middle Part D B C = 40° : 17′	—	9.8824428
To the S. *c.* the of middle Part A B D = 67° : 19′	——	9.5861589
To which add the Angle D B C = — 40 : 17		
And the Sum is the Angle A B C = — 107 : 36		

Examp. 2. Dat. { A B C = 107° : 36′ ; B C = 38 : 28 ; A C = 74 : 50 } to find the Angle A C B.

THE ſame Parts being given here as was at the ſecond Example of the ſeventh Caſe, the Stereographic Conſtruction muſt be the ſame. *See the Fig. belonging thereto.*

By Calculation.

1. IN the Triangle B *d* C you have *d* B C, and the Hypotheneuſe B C, to find the Angle *d* C B, the middle Part is B C, and the Angles C B *d* and *d* C B, adjacent Extremes. Then it will be,

S. *c.* B C = 38° : 28′ + R	——	——	19.8937452
— T. *c.* *d* B C = 72° : 24′		——	9.5013588
Remains the T. *c.* of *d* C B = 22° : 3′		——	10.3923864

2. IF *d* C B, and B C, in the Triangle *d* B C, or *d* C A and A C in the Triangle *d* C A, were given to find the Perpendicular *d* C, then the Angles *d* C B and *d* C A, would be the two middle Parts, and B C and A C the adjacent Extremes: So the Analogy will be

As the T. *c.* of the adjacent Extreme B C = 38° : 28′ *co. ar.*	——	9.9000865
Is to the T. *c.* of the adjacent Extreme A C = 74° : 50′	—	9.4330804
So is the S. *c.* of the middle Part *d* C B = 22° : 3′	——	9.9670125
To the S. *c.* of the middle Part *d* C A = 78° : 29′	——	9.3001794
From which take the Angle *d* C B = 22 : 3		
And there will remain A C B = — 56 : 26		

Caſe

Case 9.

GIVEN two Angles and a Side oppoſite to one of them, to find the third Angle.

Examp. Dat. $\left\{\begin{matrix} AB = 57^\circ : 32' \\ ABC = 107 : 36 \\ ACB = 56 : 26 \end{matrix}\right\}$ to find the Angle BAC.

THE Stereographic Projection is the ſame with that of the firſt Example of the firſt Caſe; but for the Calculative Part, you muſt draw the Perpendicular A *d*, ſo you will have two Right-angled Triangles A B *d*, and A C *d*. In the Triangle A B *d*, you have the Hypothenuſe A B and Angle A B *d*, to find the Angle *d* A B, the Side A B being the middle Part, and A B *d* and B A *d* adjacent Extremes. The firſt Operation will ſtand thus:

S. *c.* A B = 57° : 32′ + R	19.7298197
— T. *c.* A B *d* = 72° : 24′	9.5013588
Remains the T. *c.* of B A *d* = 30° : 35′	10.2284609

Now, if A B *d* and B A *d*, or A C *d* and C A *d*, were given to find the Perpendicular A *d*, then A B *d* and A C *d* would be the two middle Parts, and C A *d*, and *d* A B the oppoſite Extremes. Therefore ſay,

As the S. *c.* of the middle Part *d* B A = 72° : 24′ *co. ar.*	0.5194615
Is to the S. *c.* of the middle Part A C *d* = 56° : 26′	9.7426520
So is the Sine of the oppoſite Extreme *d* A B = 30° : 35′	9.7065394
To the Sine of the oppoſite Extreme C A *d* = 68° : 30′	9.9686529
From which take the Angle *d* A B = — 30 : 35	
And there will remain the Angle B A C = 37 : 55	

Case 10.

GIVEN two Angles, and a Side oppoſite to one of them, to find the Side between them.

Examp. Dat. $\left\{\begin{matrix} AB = 57^\circ : 32' \\ ABC = 107 : 36 \\ ACB = 56 : 26 \end{matrix}\right\}$ to find the Side B C.

THE Stereographic Projection for this will be the ſame alſo, with that of the firſt Example of the firſt Caſe; which ſee.

THE Perpendicular A *d*, being drawn, as in that Fig. then in the Right-angled Triangle A *d* B you have the Hypothenuſe A B, and the Angle A B *d*, to find B *d*; here A B *d* is the middle Part, and A B, and B *d*, the adjacent Extremes. Then

S. *c.* A B *d* = 72° : 24′ + R	19.4805385
— T. *c.* A B = 57° : 32′	9.8036296
Remains the Tangent of D B = 25° : 25′	9.6769089

Now,

Now, if you had dB and A B d in the Triangle A B d, or dC and A C d in the Triangle A C d to find the Perpendicular A d; then dB and dC would be the two middle Parts, and A B d and A C d adjacent Extremes. Now, ſay

2. As the T. *c.* of the adjacent Extreme A B d = 72° : 24′ *co. ar.*	—	0.4986412
Is to the T. *c.* of the adjacent Extreme A C D = 56° : 26′	—	9.8218803
So is the Sine of the middle Part dB = 25° : 25′	—	9.6326576
To the Sine of the middle Part dC = 63o : 53′	—	9.9531791
From which take dB = – 25 : 25		
And there will remain B C = – 38 : 28		

Caſe II.

GIVEN the three Sides to find an Angle.

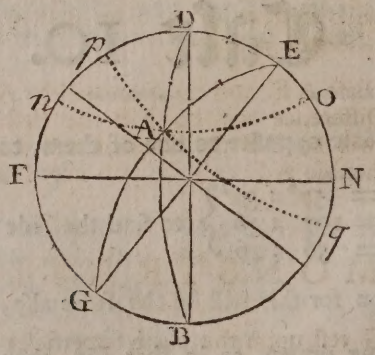
Examp. 1. Dtt. { A D = 57° : 32′ / D E = 38 : 28 / A E = 74 : 50 } to find the Angle at A.

The Stereographic Projection.

1. HAVING drawn the Primitive Circle F D N B, and the Diameters D B and F N, at Right Angles to each other, ſet 38° : 28′ from D to E, and from E draw the Diameter E G.

2. ABOUT D, as a Pole, draw the ſmall Circle *n o*, at the Diſtance of 57° : 32′ from it; and alſo about E, as a Pole, draw the ſmall Circle *p q*, at the Diſtance of 74° : 50′ from it; then theſe will interſect each other in the Point A.

3. THROUGH the Points E, A, G and D, A, E, draw the Oblique Circles E A G, and D A B, and they will compleat the Triangle A D E; which being done, you may meaſure any of the Angles by the Rules given in *Chap.* 11. for that Purpoſe.



By Calculation.]

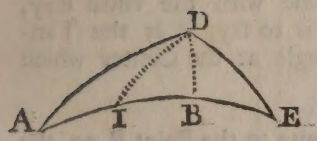
IN this Caſe let fall a Perpendicular oppoſite to the Angle requir'd, the Side upon which it falls is (for Diſtinction Sake) called the Baſe, and the other two are Sides.

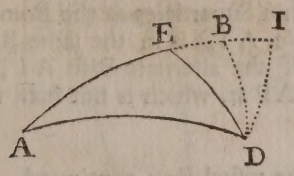
So in the three Triangles following A E is the true Baſe, A D and E D are Sides; D B, the Perpendicular B being placed at the Right, and B I always ſuppoſed equal to B E; ſo in every of them A E being the true Baſe, A I is the alternate Baſe.

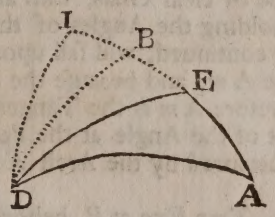
Now,

Now, whoever makes uſe of the next Axiom, will find it convenient to let the ſame Letters be placed to the Figures, as they are here; and becauſe Things of this Kind are apt to eſcape the Memory, the following Lines may afford ſome Aſſiſtance.

D, *is the Point, from whence the Lines proceed,*
Right Angles ſtill of B, *will ſtand in Need;*
A E *'s the Side, which we the true Baſe call,*
And is the Side on which the Lines do fall;
Alternate Baſe, A I *muſt bear that Name,*
B E, B I, *muſt always be the ſame.*







Axiom 6.

As the Tangent of half the true Baſe
Is to the Tangent of half the Sum of the Sides,
So is the Tangent of half the Difference of the Sides
To the Tangent of the alternate Semi-Baſe:
That is,
As the Tangent of half the true Baſe — A E
Is to the Tangent of half the Sum of the Sides — A D + E D
So is the Tangent of half the Difference of the Sides A D − E D
To the Tangent of half the alternate Baſe — A I

To which add half the true Baſe A E, and the Sum will be A B the Baſe of the Right-angled Triangle A B D, and their Difference will be E B the Baſe of the Right-angled Triangle A B D; and their Difference will be E B the Baſe of the Right-angled Triangle E B D; by either of theſe and the Hypothenuſe A D or E D, you may find the Angle requir'd.

DEMONSTRATION.

1. LET the Sphere A F P G reſt upon the plane Superficies H N K Q that they may touch each other in the Point A; from which erect the Perpendicular A P, which will alſo paſs thro' the Center of the Sphere at ⊙.

2. DRAW the two Vertical Circles P *e* D A, and P I E A, at any convenient Diſtance from each other; add from the Angle at A, let there be deſcribed upon the Superficies of the

Sphere the Triangle A D I, acute-angled at I, or the Triangle A D E, obtuſe-angled at E.

3. TAKE D for the Pole acoording to the Diſtance D I, or D E, which is equal thereto, and draw the Circle E *d e* I, cutting D P in *e*, D A in *d*, and A E I, in E and I.

4. FROM the Point D to the Arch A E I let fall the Perpendicular D B; here then A D ſhall be the greater Side, and D I or D E the leſs, A E is the true Baſe, and A I the alternate Baſe; A *e* is the Sum of the Sides, and A *d* their Difference, D *e* and D *d* by the Conſtruction are equal to the leaſt Side D E = D I.

5. THIS being done, and ſuppoſing the Sphere upon which thoſe black Lines are drawn to be of clear Glaſs, thin and concave, then if an Eye was placed in the vertical Point P, beholding the Angles of the Triangle before drawn, the viſual Ray, when directed to I, and continued, will fall upon the plain Superficies at the Point *m*; to which Point draw the Line A *m*, and becauſe the Semicircle A I P is in the ſame Right Line with the viſual Ray, therefore A *m* is the Tangent of half the alternate Baſe A I; that is to ſay, it is the Tangent of the Angle at the Periphery A P *m*, which is but half that Angle at the Center which is meaſured by the Arch A I,

6. THE Eye at P, beholding *e*, the viſual Ray continued, will come to the Point R on the Plane; from whence draw the Line A R, and that ſhall be the Tangent of the Angle A P R, which Angle is half the Quantity of the Arch A *e* equal to the Sum of the Sides A D + E D.

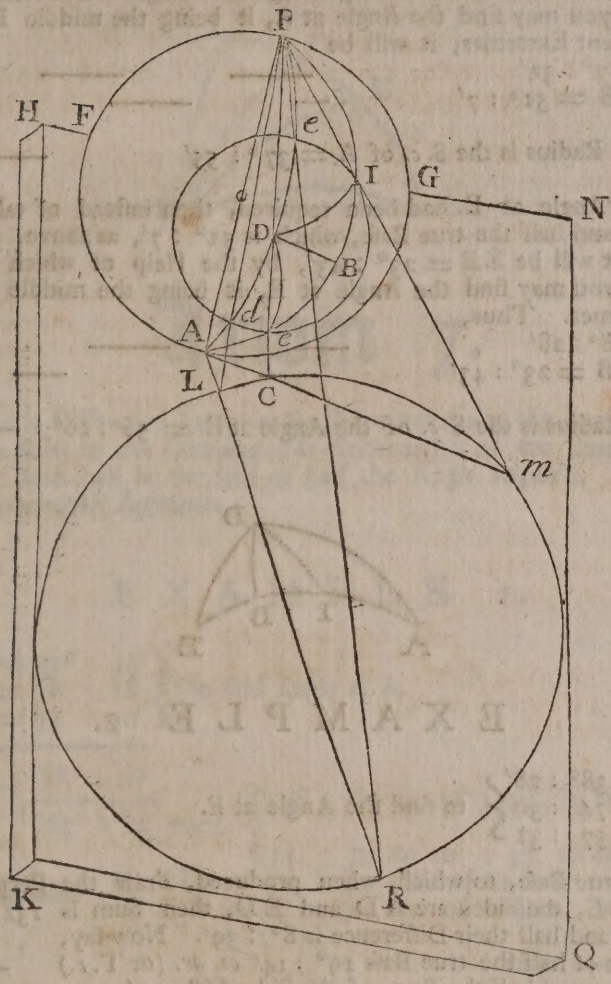
7. THE Eye at P, beholding E, the viſual Ray continued will fall upon the Plane at C, to which Point draw A C, and that will be the Tangent of the Angle A P C, which is half the Quantity of the true Baſe A E.

8. THE Eye at P, beholding *d*, the viſual Ray continued, will fall on the Plane at the Point L, to which draw the Line A L, and that will be the Tangent of the Angle A P L; but A P L is but half the Quantity of the Arch A *d*, therefore A L is the Tangent of half the Difference of the Sides.

Now, ſince it appears that the Points L, C, *m*, and R, are in a Circle (as is demonſtrated by *Prop.* 1. in Page 84, of *Haynes's Trigonometry*,) then by *Theorem* 21, Chap 3. it will be, A C × A *m* = A L × A R; and from the ſame the Analogy will be, A C : A R :: A L : A *m*. That is,

As A C the Tangent of half the true Baſe A E
Is to A R, the Tangent of half the Sum of the Sides A *e*;
So is A L, the Tangent of half the Difference of the Sides A *d*
To A *m*, the Tangent of half the alternate Baſe A I. *Q. E. D.*

Examp.



Examp. 1. Dat. $\left\{\begin{array}{l} AD = 57^\circ : 32' \\ DE = 38 : 28 \\ AE = 74 : 50 \end{array}\right\}$ to find the Angle at A.

MAKE AE the true Baſe, and let fall the Perpendicular DB, then make BI equal to BE, ſo will AI be the alternate Baſe.

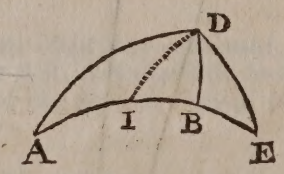
Now, As the Tangent of half the true Baſe $37^\circ : 25'$. *co. ar.*	—	10.1163279
Is to the Tangent of half the Sum of the Sides $= 48^\circ : 0'$	—	10.0455626
So is the Tangent of half the Difference of the Sides $9^\circ : 32'$	—	9.2251560
To the Tangent of half the alternate Baſe $AI = 13^\circ : 42'$	—	9.3870465
To which add half the true Baſe $AE =$ —— $37 : 25$		
And the Sum is $AB =$ — — $51 : 07$		

HAVING found A B. the Base of the Right angled Triangle A D B, and having the Hypothenuse A D given, you may find the Angle at A, it being the middle Part, and the Sides A B and D B adjacent Extremes, it will be

T. *c.* A D = 57° : 32′	——	——		9.8036296
+ Tangent A B = 51° : 7′	—	——		10.0934397
The Sum want. Radius is the S. *c.* of A = 37° : 55′			——	9.8970693

Note, That if the Angle at E had been required, then instead of taking the Sum of the alternate Semi-base and half the true Base, which is 51° : 7′, as above, you must take their Difference, and that will be E B = 23° : 43′, by the Help of which and the given Side D E = 38° : 28′ you may find the Angle at E, it being the middle Part, and E B and E D adjacent Extremes. Thus,

T. *c.* D E = 38° : 28′	——	——		10.0999135
+ Tangent E B = 23° : 43′	——		——	9.6427773
Their Sum want. Radius is the S. *c.* of the Angle at E = 56° : 26′			——	9.7426908



EXAMPLE 2.

Dat. { A E = 38° : 28′ / A D = 74 : 50 / D E = 57 : 32 } to find the Angle at E.

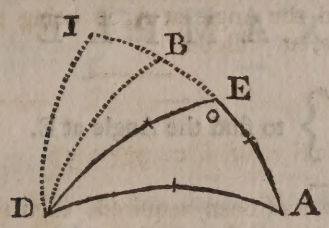
MAKE A E the true Base, to which, when produced, draw the Perpendicular D B, and make B I equal to B E, the Sides are A D and E D, their Sum is 132° : 22′, the half of which is 66° : 11′, and half their Difference is 8° : 39′. Now say,

As the Tangent of half the true Base 19° : 14′ *co. ar.* (or T. *c.*)		———	10.4573123
Is to the Tangent of half the Sum of the Sides 66° : 11′		——	10.3551676
So is the Tangent of half the Difference of the Sides 8° : 39′		——	9.1822106
To the Tangent of half A I the alternate Base = 44° : 39′		—	9.9946905
From which take half the true Base	— 19 : 14		
And there will remain B E =	—— 25 : 25		

Now, having found B E, D E being given, you may find the middle Part B E D. Thus,

Tangent B E = 25° : 25′	——		——	9.6768686
+ T. *c.* D E = 57° : 32′	——	——	——	9.8036296
The Sum want. Radius is the S. *c.* of B E D = 72° : 24′			—	9.4804982
Which take from	——	180 : 00		
And there will remain A E D =	—	107 : 36		

Axiom



Axiom 7.

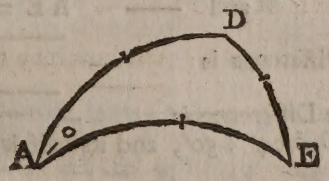
ADD the Sines of the Difference between the Sides containing the Angle requir'd, and half the Sum of the three Sides to the Complements-Arithmetical of the Sines of the ſaid two Sides, then half that Sum ſhall be the Sine of half the Angle requir'd. (See the Demonſtration in *Newton*'s *Mathematical Inſtitution.*)

EXAMPLE 1.

GIVEN { AD = 57° : 32′ / DE = 38 : 28 / AE = 74 : 50 } to find Angle at A.

Sum	170 : 50		
Half Sum	85 : 25	want. 57° : 32′ = 27° : 53′, its Sine	9.6699420
		74 : 50 = 10 : 35, its Sine	9.2640274
Containing Sides	AD = 57	: 30. *c. a.* of its Sine	0.0738099
	AE = 74	: 50. *c. a.* of its Sine	0.0153967
		Sum	19.0231760
Half the Sum of the Sine of	18° : 57′		9.5115880

Which being doubled is — 37 : 54, the Angle at A required.

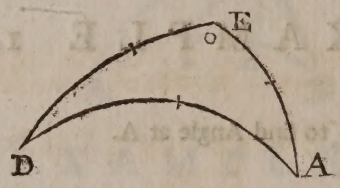


EXAMPLE

EXAMPLE 2.

GIVEN $\left\{\begin{array}{l} AE = 38° : 28' \\ AD = 74 : 50 \\ DE = 57 : 32 \end{array}\right\}$ to find the Angle at E.

Sum —	170 : 50		
Half the Sum is —	85 : 25	want. $57° : 30' = 27° : 53'$. S. —	9.6699420
		$38 : 28 = 46 : 57$. S. —	9.8637737
Containing Sides	$DE = 57 : 32$. *c. a.*		0.0738099
	$AE = 38 : 28$. *c. a.*		0.2061683
		Sum —	19.8136939
Half the Sum is the Sine of —	$53° : 48'$	—	9.9068469
Which being doubled is —	107 : 36	the Angle DEA.	



Another Way to find an Angle, having the three Sides given.

RULE.

ADD half the Difference of the two Sides containing the Angle required, to half the Side that is oppoſite to that Angle, and alſo ſubſtract it from the ſame, noting the Sum and Remainder. Then,

To the Complements-Arithmetical of the Sines of the two containing Sides, add the Sines of the ſaid Sum and Remainder, half that Sum will be the Sine of half the Angle required.

LET the Things given, and that required, be the ſame as at the laſt Example, where — — $DE = 57° : 32'$

and — $AE = 38 : 28$

Their Difference is —	19 : 04
Half the Difference is —	9 : 32
AD is 74° : 50', and its half is	37 : 25
The Sum of the two laſt is — — —	46 : 57
And their Difference — — — —	27 : 53

Co. ar. of Sine D E = 57° : 32′	0.0738090
Co. ar. of Sine A E = 38° : 28′	0.2061683
The Sine of the Sum = 46° : 57′	9.8637737
The Sine of the Difference = 27° : 53′	9.6699420
The Sum is	19.8136939
Half the Sum is the Sine of 53° : 48′	9.9068469

Twice this is —— 107 : 36 = the Angle D E A. The like for either of the other two Angles.

Caſe 12.

GIVEN the three Angles of an Oblique-angled ſpherical Triangle, to find a Side.

Dat. $\left\{\begin{matrix} x\,z\,y = 107° : 36' \\ z\,x\,y = 37 : 55 \\ z\,y\,x = 56 : 26 \end{matrix}\right\}$ to find the Side $x\,y$ of the Triangle $x\,y\,z$.

THIS Caſe may be ſolv'd by the 6th *Axiom*, the Angles of the Triangle $x\,y\,z$, being equal to the Sides of the Triangle A E D, according to *Definition* the 7th. As thus:

Dat. $\left\{\begin{matrix} x\,z\,y = 107° : 36', \text{ its Supplement is} \\ z\,x\,y = 37 : 55 \\ z\,y\,x = 56 : 26 \end{matrix}\right\} = \left\{\begin{matrix} g\,d = \text{A D} = 72° : 24' \\ f\,e = \text{D F} = 37 : 55 \\ u\,o = \text{A E} = 56 : 26 \end{matrix}\right\}$ to find $e\,u = x\,y = \text{B E D}$.

THE Geometrical Conſtruction of this Caſe, is the ſame with the laſt, when the Angles are turn'd into Sides, as above.

By Calculation.

MAKE A E the true Baſe, and draw the Perpendicular B D, then make B I equal B E, ſo will A I be the alternate Baſe, and A D and E D the Sides.

The true Baſe is A E = 56° : 26′, its half is			28° : 13′
The Sides	A D = 72 : 24		
	E D = 37 : 55		
Their Sum ——	110 : 19	half is —	55 : 9½
Their Difference —	34 : 29	half is —	17 : 14½

Now, As the Tangent of half the true Baſe 28° : 13′ *co. ar.* — — 0.2703737
Is to the Tangent of half the Sum of the Sides 55° : 9½′ —— 10.1573301
So is the Tangent of half their Difference 17° : 14½′ —— 9.4918500

To the Tangent of half the alternate Baſe (A I) = 39° : 43′ : 24″ — 9.9195538

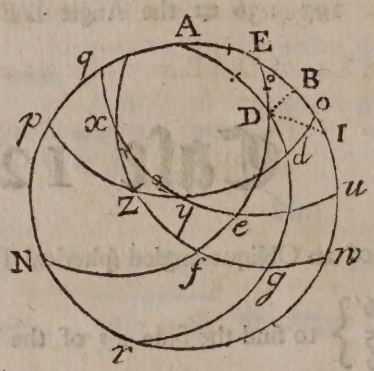
From half the alternate Baſe A I =	39° : 43′ : 24″
Take half the true Baſe ——	28 : 13 : 00
Their Difference is E B — —	11 : 30 : 24

IN

In the Right-angled Triangle BED you have DE and EB, to find the Angle BED.

Thus, T. *c.* DE = 37° : 55′	— — —	10.1084926
+ Tangent EB = 11° : 30′ : 24″	— — —	9.3087210
Their Sum want. Radius is the S. *c.* of BED = 74° : 50′	— —	9.4172136

But this Angle BED, is equal to $eu = xy$, which was required.



EXAMPLE 2.

DAT. $\left.\begin{matrix} xzy = 107° : 36', \text{ its Sup. is} \\ zxy = 37 : 55 \\ zyx = 56 : 26 \end{matrix}\right\} = \left\{\begin{matrix} dzg = dg = AE = 72° : 24' \\ fe = de = 37 : 55 \\ uo = AD = 56 : 26 \end{matrix}\right\}$ to find $zx = fg = AED$.

MAKE AE the true Base, AI the alternate Base, and AD and DE the Sides. (*See the next Figure.*)

The true Base 72° : 24′, half is 36° : 12′		
The Sides AD = 56° : 26′		
DE = 37 : 55		
Their Sum — — 94 : 21 half 47 : 10½		
Difference — — 18 : 31 half 9 : 15½		
As the Tangent of half the true Base 36° : 12′ *co. ar.*	— —	10.1355546
Is to the Tangent of half the Sum of the Sides 47° : 10½′	—	10.0330041
So is the Tangent of half the Difference of the Sides 9° : 15½′	—	9.2122131
To the Tangent of half the alternate Base 13° : 30′ : 45″	—	9.3807718
Which take from half the true Base — 36 : 12 : 00		
Remains EB = — — 22 : 41 : 15		

Now, in the Right-angled Triangle DEB you have the adjacent Extremes EB, and DE, to find the middle Part BED.

T. *c.* DE = 37° : 55′	— — —	10.1084926
+ Tangent BE = 22° : 41′ : 15″	— —	9.6212311
The Sum want. Rad. is the S. *c.* of BED = *fg* = *zx* = 57° : 32′	—	9.7297237

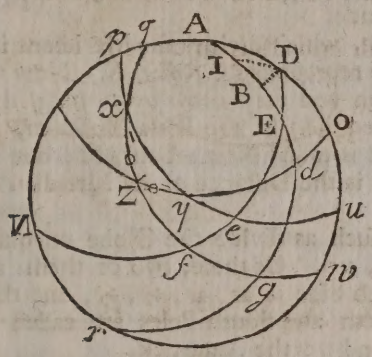
SUPPOSE

SUPPOSE you had the ſame Things given to find the Side $zy = do =$ DAE, then you may make AE the true Baſe, and find half the alternate Baſe as above, to be 13° : 30′ : 45″
To which add half the true Baſe —— —— 36 : 12 : 00

The Sum is the adjacent Extreme AB —— —— 49 : 42 : 45

By which, and the other adjacent Extreme AD = 56° : 26′ : 00″ you may find the middle Part DAD, thus, T. *c.* AD = 56° : 26′ —— —— 9.8218803
+ Tangent AB = 49° : 42′ : 45″ —— —— 10.0717646

The Sum want. Rad. is the S. *c.* of DAE = *do* = *zy* = 38° : 28′ —— 9.8936449



CHAP. XV.

HAVING now prepared the Learner, we ſhall next ſhew how to apply what, we hope, he underſtands.

1. To ASTRONOMY.
2. To NAVIGATION.
3. To DIALLING.

BUT, becauſe a true Idea of the ſeveral Lines, or Circles of the Sphere is abſolutely neceſſary to ſuch as would know what they are doing, and the beſt Way to attain ſuch Ideas being to be had from the Uſe of the Globes, which, perhaps our Reader cannot reach, we ſhall therefore endeavour to aſſiſt him by ſuch Figures and Definitions, as we think moſt uſeful.

LET the Figure next following repreſent a Globe. Then,

1. N. and S. are the two Poles; N. the North-Pole, and S. the South-Pole.

2. A Line imagin'd to paſs from the North to the South-Pole, and conſequently through the Center of the Globe, is called the Axis; and at the Ends there are fixed two Pieces of ſtrong Iron Wire, and ſet into two oppoſite Points of the Braſs Meridian, ſo as that the Globe itſelf may turn upon them.

3. THE Braſs Meridian is repreſented by the outward Circle *Æ N Æ S*, and is always ſet in two Notches *c* and *d*, at the North and South Parts of the Horizon, and in one in the Pedeſtal at *y*.

4. THE Horizon is a broad wooden Circle encompaſſing the Globe, mark'd with A C B, upon the upper Side of which are placed the Months and Days of the Month, and alſo the Signs with the Degrees of each Sign, by which you may count the Sun or Stars, Azimuth and Amplitude; the Poles of this Circle are, *x* the Zenith, and *y* the Nadir.

5. THE Quadrant of Altitude is a thin Braſs Plate, rivetted to a Braſs Nut, which ſlides upon the Meridian, mark'd in the Figure with *x n*, and divided into 90°, beginning at *n*; this ſerves to find the Altitude of the Sun, Moon and Stars above the Horizon, and is an univerſal Azimuth Line.

6. THE Braſs Meridian (before deſcribed) is an univerſal Meridian, and other great Circles paſſing through the Poles, ſuch as N *c* S, N *f* S, N *n* S, N *g* S, *&c.* are alſo called Meridians, Hour-Circles, or Circles of Longitude.

7. THE Equator is a great Circle, which divides the Meridians into two Parts at an equal Diſtance from each Pole, it is here repreſented by *Æ f Æ*. Now the Latitude of any Place is its Diſtance from the Equator, which you may know by turning the Globe about until the Place is brought under the graduated Side of the Braſs Meridian, and if the Place is on the North Side of the Equator, then it is called North Latitude; but if on the South Side, it is South Latitude, and its Longitude is the Diſtance of its Meridian counted upon the Equator from the firſt Meridian.

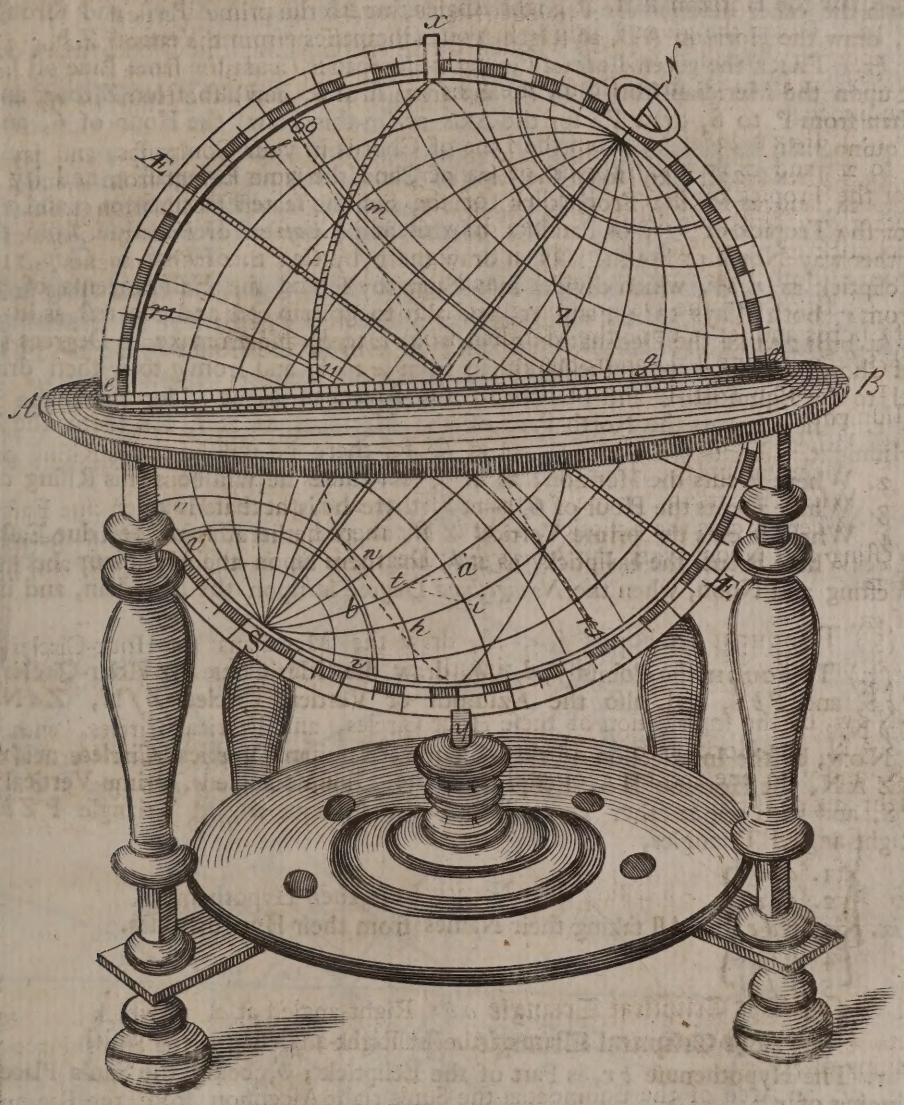
8. PARALLELS of Latitude are ſuch as divide the Globe unequally, and are all parallel to the Equator *ll.* ♋, ♋, ♑, ♑, *r r*, *v v*. Of theſe, two of them are the Tropicks, and are 23° : 29′ from the Equator, on each Side as ♋, ♋, ♑, ♑, and thoſe two Parallels which are at the ſame Diſtance from the North and South Poles are called the Artick and Antartick Circles; theſe are *r r* the Artick, and *v v* the Antartick.

9. THE Ecliptic, is a great Circle making an Angle of 23° : 29′ with the Equator, this is mark'd with ♋ C ♑.

10. AZIMUTHS, or Vertical Circles, are great Circles paſſing through the Zenith and Nadir, as *x z y* and *x n y*, the Altitude of the Sun, Moon, *&c.* are meaſured upon them, and counted from the Horizon upon the Quadrant of Altitude, as aforeſaid. So if the Sun was at *m*, his Altitude above the Horizon would be the Arch *m n*, and his Azimuth *n e*, equal to the Angle *e x n*.

RUMBS, are Points of the Compaſs ſailed upon, as if a Ship at *t* ſhould ſail thence to *a*, then her Courſe is directly North; if to *b*, 'tis South; to *w*, Weſt; if to *h*, 'tis Eaſt; if ſhe ſails from *t* upon any Point between *t a* and *t h*, the Courſe is North-Eaſterly; if between *t a* and *t w*, 'tis North-Weſterly; if between *t b* and *t h*, South-Eaſterly. Laſtly, if between *t b* and *t w*, it is then South-Weſterly, according to the Angle which the Line of the Ship's Motion makes with the Meridian *b a*; ſo if a Ship had ſailed directly from *t* to *i*, then her Courſe was the Angle *a t i*, the Difference of Latitude *a t*, her Diſtance run is *t i*, and her Meridional Diſtance *a i*.

THESE are the principal Points and Lines which relate to the Projection of the Sphere upon the Plane of the Meridian, of which it may be ſaid there are two Sorts; one is uſed in Aſtronomy, and drawn for a particular Latitude and Time, the other is univerſal and uſed in Geography and Navigation.



To project the Sphere upon the Plane of the Meridian, suitable to any Latitude and Declination.

EXAMPLE.

Let it be requir'd to project the Sphere upon the Plane of the Meridian for the Latitude of 52° : 16′, supposing the Sun to have 20° : 00′ of North Declination.

(1.) WITH 60 Degrees of the Line of Chords in your Compaſſes ſet one Foot in *r*, and with the other deſcribe the Primitive Circle, or Meridian A Z P N, and through the Center *r*, draw the Horizon A B, at Right Angles thereto the prime Vertical Z N.

(2.) TAKE the given Latitude in your Compaſſes (from the ſame Line of Chords) and ſet it upon the Meridian from B to P, and from A to S, and alſo from Z to ♈, and from N to *q*, then from P to S, draw P S for the Axis or (in this View) the Hour of 6, and ♈ *q*, for the Equinoctial.

(3.) TAKE 23° : 29′ from the Line of Chords in your Compaſſes, and lay it from ♈ and *q* to ♋, and ♋ for the Tropick of Cancer, and the ſame Extent from ♈ and *q* to ♑ and ♑, for the Tropick of ♑ ; or Parallels of the Sun's greateſt Declination from the Equinoctial either way North or South. Then draw them by *Caſe* 2. of *Prob.* 9. *Chap.* 11. and alſo the Ecliptick ♋, *r*, ♑, which divide into Signs by ſetting the Half-tangents of 30°, and 60°, from *r*, both Ways to ♉, and ♊, and alſo to ♎ and ♌, and the reſt as in the *Figure*.

(4.) BECAUSE the Declination given is 20 Degrees North, take 20 Degrees from your Line of Chords and ſet it upon the Meridian from ♈ to *h*, and from *q* to *i*, then draw *h d i*, parallel to the Equinoctial, and obſerve

1. Where it cuts the North Part of the Meridian as at *i*, for there the Sun will be at Midnight.
2. Where it cuts the Horizon, as at *f*, for there he will be at his Riſing or Setting.
3. Where it cuts the Hour of 6, as at *d*, there he is at that Hour.
4. Where it cuts the prime Vertical Z N, as at *c*, there he is when due Eaſt or Weſt.
5. Where it cuts the Ecliptick, as at *b*, for there he is at ſome Hour between Eaſting or Weſting and Noon, when the *Nonageſſima* Degree is upon the Meridian, and in the firſt Point of ♋.

(5.) THROUGH the Points *f*, *d*, *c*, *b*, draw the Meridians (or Hour-Circles) P *f* S, P *d* S, P *c* S, and P *b* S, and alſo the Azimuth or Vertical Circles Z *f* N, Z *d* N, Z *c* N, and Z *b* N.

NOW, by the Interſection of theſe Hour Circles, and Vertical Circles, with the Meridian B Z A N, the Horizon B A, Ecliptick ♋, ♑, Equinoctial ♈ *q*, prime Vertical Z N, the Axis P S, and with one another you will form the Oblique-angled Triangle P Z *b*, and alſo five Right-angled Triangles,

Viz. { 1. *a b r.* 2. *c l r.* 3. *r d e.* 4. *r f g.* 5. P *f b.* } All taking their Names from their Hypothenuſes.

(1.) **The Ecliptical Triangle** *a b r* Right-angled at *a*, of which

1. The Hypothenuſe *b r*, is Part of the Ecliptick ; *b*, being the Sun's Place from the Beginning of *r*.
2. *a r*, an Arch of the Equinoctial the Sun's right Aſcenſion from the Beginning of *r*.
3. *b a*, an Arch of the Meridian P *a* S, the Sun's preſent Declination.
4. The Angle *b r a*, made by the Ecliptick and Equinoctial, is the Sun's greateſt Declination, which, according to *Flamſhead* is 23° : 29′.
5. The Angle *a b r*, made by the Ecliptick and Meridian, is the Angle of the Sun's Poſition.

(2.) **The Azimuthal Triangle** *c l r*, Right-angled at *l*, whoſe Parts are,

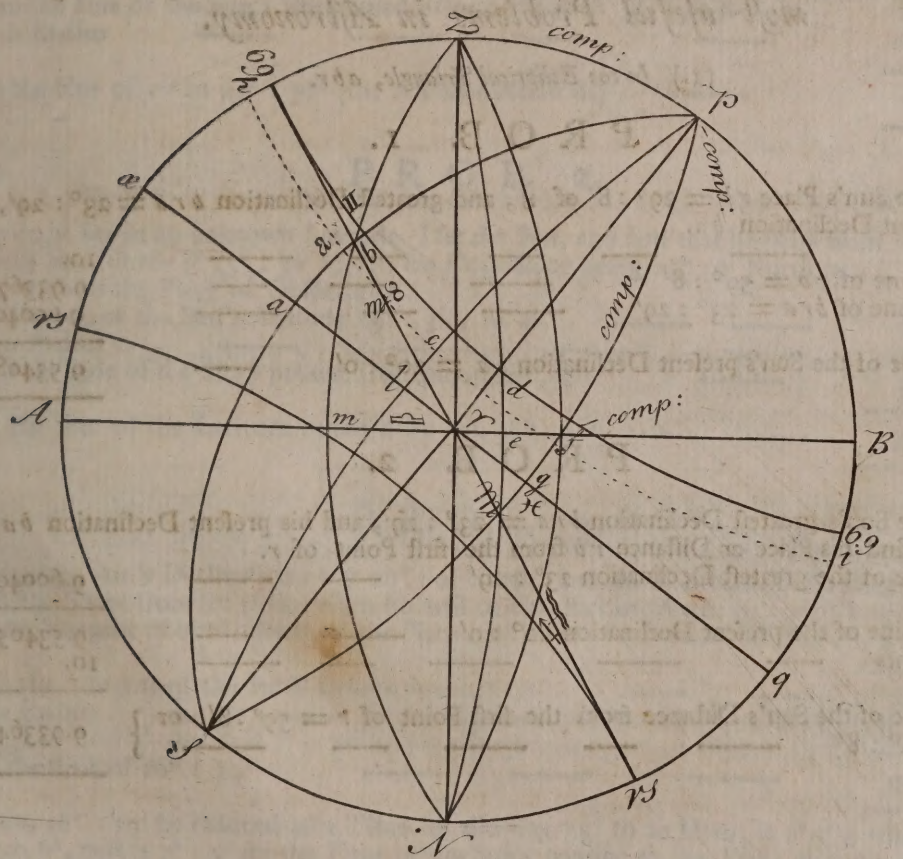
1. The Hypothenuſe *c r*, an Arch of the prime Vertical, is the Sun's Altitude, being due Eaſt or Weſt.
2. The Arch of an Hour Circle *c l*, the Sun's preſent Declination.
3. The Side *r l*, an Arch of the Equinoctial repreſenting the Time after ſix in the Morning, or before ſix in the Afternoon, when the Sun will be due Eaſt or Weſt.

4. The

4. The Angle *c r l*, is the Latitude of the Place, equal to Z ≏ = BP.
5. The Angle *r c l*, is the Angle of the Sun's Poſition, in reſpect of the Pole and Zenith.

(3.) **The Meridional Triangle** *r d e*, Right-angled at *e*, of which,
1. The Hypothenuſe *r d*, is the Sun's preſent Declination.
2. The Side *r e*, is the Sun's Azimuth from the Eaſt or Weſt at the Hour of ſix.
3. The Side *d e*, is the Sun's Height at the Hour of ſix.
4. The Angle *d r e*, is the Latitude of the Place. And,
5. The Angle *r d e*, is the Angle of the Sun's Poſition.

(4.) **The Horizontal Triangle** *r f g*, Right-angled at *g*, of which,
1. The Hypothenuſe *r f*, is the Sun's Amplitude at Riſing and Setting.
2. The Side *r g*, is the aſcenſional Difference or Time between his Riſing or Setting and Six o'Clock, or 'tis the Difference between the Right and Oblique Aſcenſion.
3. The Side *f g*, is the Sun's preſent Declination.
4. The Angle *f r g*, is the Complement of the Latitude of the Place.
5. The Angle *r f g*, is the Angle of the Sun's Poſition.



(5.) The

(5.) **The Meridional Triangle** P*f*B, Right-angled at B, in which,

1. The Hypothenuſe P*f*, is the Complement of the Sun's preſent Declination.
2. The Side *f*B, is the Sun's Azimuth of Riſing and Setting from the North Part of the Meridian.
3. The Side PB, is the Latitude of the Place.
4. The Angle *f*PB, is the Time from Midnight, at which the Sun riſes or ſets. And,
5. The Angle P*f*B, is the Complement of the Sun's Poſition, in reſpect of the Pole and Zenith.

The Parts o the Oblique-angled Triangle PZ*b*, are,

1. The Side Z*b*, is the Complement of the Sun's Altitude (*bm*).
2. The Side ZP, is the Complement of the Latitude of the Place.
3. P*b*, is the Complement of the Sun's preſent Declination.
4. The Angle ZP*b*, is the Time from Noon.
5. The Angle *b*ZP, is the Sun's Azimuth from the North, whoſe Supplement is the Angle *b*Z*h*, his Azimuth from the South.
6. The Angle P*b*Z, is the Angle of the Sun's Poſition.

The Uſe of the particular Meridional Projection in ſome of the moſt uſeful Problems in Aſtronomy.

(1.) *In the Ecliptical Triangle, a b r.*

PROB. 1.

GIVEN the Sun's Place $r\,b = 29° : 8'$ of ♉, and greateſt Declination $b\,r\,a = 23° : 29'$, to find his preſent Declination $b\,a$.

As Radius —— —— —— —— ——	10.
Is to the Sine of $r\,b = 59° : 8'$ —— —— ——	9.9336713
So is the Sine of $b\,r\,a = 23° : 29'$ —— —— ——	9.6004090
To the Sine of the Sun's preſent Declination $b\,a = 20° : 0'$ ——	9.5340803

PROB. 2.

GIVEN the Sun's greateſt Declination $b\,r\,a = 23° : 29'$, and his preſent Declination $b\,a = 20° : 0'$, to find his Place or Diſtance $r\,b$ from the firſt Point of r.

As the Sine of the greateſt Declination $23° : 29'$ —— ——	9.6004090
Is to the Sine of the preſent Declination $20° : 0'$ —— ——	9.5340517
So is Radius —— —— —— —— ——	10.
To the Sine of the Sun's Diſtance from the firſt Point of $r = 59° : 8'$, or ♉ $29° : 8'$ —— —— —— —— ——	9.9336427

PROB.

PROB. 3.

GIVEN the Sun's Place ♉, 29° : 8′, and the preſent Declination 20° : 00′ to find its right Aſcenſion *r a*. Say,

As the S. *c.* of the preſent Declination 20° : 0′	9·9729858
Is to Radius	10.
So is the S. *c.* of 59° : 8′	9·7101529
To the Sine-Complement of the Sun's R. A. *r a* = 56° : 55′	9·7371671

(2.) *In the Azimuth Triangle, c l r.*

PROB. 1.

GIVEN the Latitude of the Place 52° : 20′ = *c r l*, and preſent Declination of the Sun 20° : 0′ = *c l*, to find the Sun's Height when due Eaſt or Weſt *c r*.

As the Sine of the Angle *c r l* = 52° : 20′	9.8984944
Is to the Sine of the Sun's Declination *c l* = 20° : 0′	9·5340517
So is Radius	10.
To the Sine of *c r* = 25° : 36′ (the Altitude requir'd.)	9·6355573

PROB. 2.

BEING at Sea in an unknown Latitude, I ſet the Sun, and find that he bears from me directly Eaſt with an Altitude of 25° : 36′ = *c r*, his Declination being 20° : 0′ North = *c l*. I demand the Latitude of the Place of Obſervation.

As the Sine of the Sun's Altitude 25° : 36′, *co. ar.*	0.3644301
Is to Radius	10.
So is the Sine of the Sun's preſent Declination 20° : 0′	9·5340517
To the Sine of the Latitude requir'd *c r l* = 52° : 20′	9·8984818

PROB. 3.

GIVEN the Sun's Declination *c l* = 20° : 0′ North, and the Latitude of the Place 52° : 20′ to find the Time from ſix (*r l*), when he will be due Eaſt or Weſt.

As the Tangent of the Latitude of the Place 52° : 20′	10.1124058
Is to the Tangent of the Sun's Declination 20° : 0′	9·5610658
So is Radius	10.
To the Sine of 16° : 19′	9·4486600

THUS, 16° : 19′ be reduced into Time by allowing 15° to an Hour, is 1ʰ 5′; which being added to 6ʰ, makes 7ʰ : 5′ for the Time of the Sun's coming to the Eaſt; and the ſame being taken from 6ʰ, there will remain 4ʰ : 55′ for the Time of his coming to the Weſt.

(3.) *In*

(3.) *In the Meridional Triangle, r d e.*

PROB. 1.

GIVEN the Latitude of the Place 52° : 20′ = *d r e*, and the preſent Declination of the Sun 20° : 0′ = *r d*, to find *d e*, the Height of the Sun at the Hour of ſix.

As Radius	10.
Is to the Sine of the Declination 20° : 0′	9.5340517
So is the Sine of the Latitude of the Place 52° : 20′	9.8984944
To the Sine of the Sun's Height 15° : 42′	9.4325461

PROB. 2.

GIVEN the Latitude of the Place 52° : 20′, and the Sun's Declination 20° : 0′ North, to find (*r e*), the Sun's Azimuth, at the Hour of ſix.

As the S. *c.* of the Latitude 52° : 20′	9.7860886
Is to Radius	10.
So is the T. *c.* of the Sun's Declination 20° : 0′	10.4389341
To the T. *c.* of the Arch *r e* = 12° : 32′	10.6528455

THE Complement of this is 77° : 28′ = *e* B, the Sun's Azimuth from the North.

PROB. 3.

GIVEN the Sun's Declination (*r d*) 20° : 0′ North, and his Altitude (*d e*) at the Hour of ſix 15° : 42′, to find the Latitude of the Place (*d r e*).

As the Sine of the Sun's Declination 20° : 0′	9.5340517
Is to Radius	10.
So is the Sine of the Sun's Altitude at ſix = 15° : 42′	9.4323285
To the Sine of the Latitude requir'd 52° : 18′	9.8982768

HERE the Learner might expect that there ſhould come out 52° : 20′, becauſe the other two Parts of this Analogy are the ſame as at *Prob.* 1. but the Reaſon of the Difference is, that, for the Anſwer to that, we accepted of the next neareſt Minute, *viz.* 15° : 42′, without enquiring any further; but to be more exact you may do thus:

The Sine of 15° : 43′ is 9.4327777, Sine found was	9.4325461
of 15 : 42 is 9.4323285, that of 15° : 42′ is	9.4323285
Difference for 60 Min. is — 4492, Difference is	2176

Now

Now say, As 4492° : 60′ :: 2176
60

4492) 130560 (29
40720

.292

HENCE we see, that at *Prob.* 1. the Sun's Height should be 15° : 42′ : 29″.
Now, if in the Analogy of this third *Prob.* you take the Sine of 15 : 42 : 29.
INSTEAD of the Sine of 15° : 42′, 'twill be,

As S. of 20° : 0′	9·5340517
Is to Radius	10.
So is the Sine of 15° : 42′ : 29″	9·4325456
To the Sine of 52° : 20′, the Latitude requir'd	9·8984939

(4.) *In Horizonal Triangle, r f g.*

PROB. 1.

GIVEN the Latitude of the Place P*r* B = 52° : 20′, whose Complement is the Angle *f r g* = 37° : 40′, and also the Sun's Declination *f g* = 20° : 0′, to find the Side *r f* = the Sun's Amplitude at Rising or Setting.

As the Sine of *f e g* = 37° : 40′	9·7860886
Is to the Sine of the Sun's Declination *g f* = 20° : 0′	9·5340517
So is Radius	10.
To the Sine of the Sun's Amplitude *r f* = 34° : 2′	9·7479631

PROB. 2.

GIVEN the Latitude of the Place = 52° : 20′, or its Complement 37° : 40′ = *f r g*, and the Sun's Declination *f g* = 20° : 0′, to find the Ascensional Difference *r g*.

As the Tangent of 37° : 40′	9·8875942
Is to the Tangent of the Sun's Declination 20° : 0′	9·5610658
So is Radius	10.
To the Sine of *r g* = 28° : 8′	9·6734716

THIS reduced to Time is 1ʰ : 52′ : 32″, which being added to six Hours, gives the Time of Sun's Setting 7ʰ : 52′ : 32″, and being subtracted from six Leaves the Time of his Rising, if you'd find the Length of the Day double the Time of Setting, and that will be 15ʰ : 45′ : 4″, which being subtracted from 24 Leaves the Length of the Night 8ʰ : 14′ : 56″.

PROB. 3.

GIVEN the Sun's Right Aſcenſion, and his Aſcenſional Difference, to find his Oblique Aſcenſion and Deſcenſion.

RULE 1.

IF the Sun's Declination be North, ſubſtract the Aſcenſional Difference from the Right Aſcenſion, and the Remainder will be the Oblique Aſcenſion; but if you add them together, the Sum will be the Oblique Deſcenſion.

RULE 2.

IF the Sun's Declination be South, add the Aſcenſional Difference and Right Aſcenſion together, the Sum will be the Oblique Aſcenſion; but if you ſubſtract the Aſcenſional Difference from the Right Aſcenſion, the Remainder will be the Oblique Deſcenſion.

EXAMPLE 1.

AT the third Problem of the Ecliptical Triangle the Sun's Right Aſcenſion was found to be	56° : 55′
AND at the ſecond Problem of the Horizontal Triangle the Aſcenſional Difference was (ſubſtract)	28 : 08
REMAINS the Oblique Aſcenſion	28 : 47
THEIR Sum is the Oblique Deſcenſion	85 : 03

EXAMPLE 2.

LET the Sun's Declination be 20° : 0′ South, whence, and his greateſt Declination his Right Aſcenſion will be found to be	236° : 54′
AND let the Aſcenſional Difference be (as above)	28 : 08
THEIR Sum is, the Oblique Aſcenſion	265 : 02
AND Difference, the Oblique Deſcenſion	208 : 46

PROB. 4.

GIVEN the Latitude of the Place 52° : 20′ and Sun's Declination 20° : 0′, to find the Angle of the Sun's Poſition *g f r*.

As the S. *c.* of the Declination 20° : 0′	9.9729858
Is to Radius	10.
So is the Sine of the Latitude 52° : 20′	9.8984944
To the Sine of the Sun's Angle of Poſition 57° : 24′	9.9255086

(5.) *The*

(5.) *The Meridional Triangle*, P*f*B.

PROB. 1.

GIVEN the Latitude of the Place BP = 52° : 20′, and the Sun's Declination *g f* = 20° : 0′, whose Complement is *f*P = 70 : 00′, to find the Sun's Azimuth from the North at his Rising and Setting.

As the S. *c.* of the Latitude 52° : 20′	9.7860886
Is to Radius	10.
So is the Sine of the Declination 20° : 0′	9.5340517
To the S. *c.* of the Sun's Azimuth B*f* = 55° : 58′	9.7479631

PROB. 2.

GIVEN the Latitude of the Place 52° : 20′, and the Sun's Declination 20° : 0′, to find the Time of the Sun's Rising and Setting B P*f*.

As T. *c.* of the Sun's Declination 20° : 0′,	10.4389341
Is to the Tangent of the Latitude of the Place 52° : 20′	10.1124058
So is Radius	10.
To the Co-sine of the Hour from Midnight 61° : 52′	9.6734717

REDUCE this to Time, and it is $4^h : 7' : 28''$ the Time of Sun-rising, which being subtracted from 12^h, leaves $7^h : 52' : 32''$ for the Time of his setting.

PROB. 3.

GIVEN the same Things as above, to find the Angle of Position B*f*P. Say,

As the S. *c.* of the Sun's Declination 20° : 0′	9.9729858
Is to Radius	10.
So is the Sine of the Latitude of the Place 52° : 20′	9.8984944
To the Sine of the Angle of the Sun's Position, in respect of the Horizon and Meridian 57° : 23′ = B*f*P	9.9255086

AND the Complement thereof is 32° : 37′ = the Angle of the Sun's Position, in respect of the Pole and Zenith Z*f*P.

Of the Oblique-angled Triangle P Z *b*.

PROB. 1.

GIVEN the Latitude of the Place P B = 52° : 20′, whoſe Complement is P Z = 37° : 40′, the Sun's Declination *a b* = 20° : 0′, the Complement of which is P *b* = 70° : 00′, and alſo the Sun's Altitude above the Horizon *b m* = 50° : 00′, whoſe Complement is Z *b* = 40° : 0′, to find the Angle *b* P Z, the Time from Noon, when that Altitude was taken.

THIS, by the 7th *Axiom*, will ſtand thus :

Given { P *z* = 37° : 40′ ; P *b* = 70 : 0 ; *z b* = 40 : 0 } to find the Angle at P.

Sum	—	147 : 40		*b* P = 70° : 00′ = 3° : 50′ Sine	—	8.8251299
Half	—	73 : 50	want.	*z* P = 37 : 40 = 36 : 10 Sine	—	9.7709522
Containing Sides	—	—		*b* P = 70 : 00 *co. ar.*	— —	0.0270142
				z P = 37 : 40 *co ar.*	— —	0.2139114
				Sum	— —	18.8370077
				Half Sum = 15° : 12′	—	9.4185038

This doubled is 30° : 24′ = *b* P Z = 2ʰ : 1′ 36″, the Time requir'd.

PROB. 2.

GIVEN the Sun's Altitude 50° : 00′, the Latitude of the Place 52° : 20′, and the Time from Noon 2ʰ : 1′ : 36″ = 30° : 24′, to find the Sun's Azimuth from the North P *z b*, or its Supplement *b z h*, his Azimuth from the South, (ſee the laſt *Meridional Projection*) and let the Triangle *b z* P, in that be repreſented by the following.

THEN from *z*, let fall the Perpendicular *z* B, and find the Angle B *z* P. Thus,

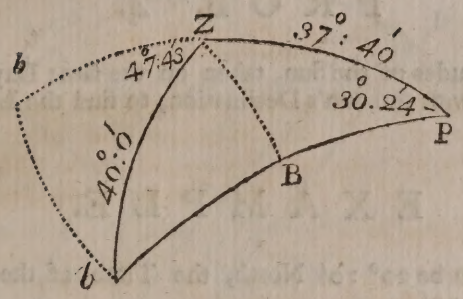
(1.)	S. *c.* *z* P = 37° : 40′ + R	—	—	—		19.8984944
	— *t. c.* B P *z* = 30° : 24′	—	—	—		10.2315865
	Is equal to the T. *c.* of B *z* P = 65° : 5′	—	—			9.6669079
(2.)	As T. *c.* *z* P = 37° : 40′. *co. ar.*	—	—	—		9.8875942
	Is to T. *c.* *b z* = 40° : 00′	—	—	—		10.0761865
	So S. *c.* B *z* P = 65° : 5′	—	—	—		9.6245911
	To S. *c.* *b z* B = 67° : 12′	—	—	—		9.5883718

Add B *z* P = 65 : 05

Sum is . . 132 : 17 the Sun's Aximuth from the North = *b z* P.

Its Supplement 47 : 43 is the Azimuth from the South = *b z h*.

PROB.



PROB. 3.

GIVEN the Complement of the Sun's Declination Pb = 70° : 0′ the Sun's Aximuth from the South bzh = 47° : 43′, and the Complement of the Sun's Altitude zb = 40° : 0′, to find the Complement of the Latitude of the Place of Obſervation Pz.

(1.) THEN, to find the Side bz.

S. *c.* bzh = 47° : 43′ + R	—	—		19.8278843
— T. *c* bz = 40° : 0′	—	—		10.0761865
Is equal to Tangent of bz = 29° : 27′		—		9·7516978
(2.) Say, As S. *c.* of bz = 40° : 0′ *co. ar.*	—		—	0.1157460
Is to S. *c.* bP = 70° : 0′	—		—	9·5340517
So is S. *c.* bz = 29° : 27′	—		—	9·9399110
To S. *c.* bP =	—	67° : 07′		9·5897087
From which take bz =	—	29 : 27		
There remains zP	—	37 : 40		the Comp. of the Lat.

Therefore the Lat. of the Place of Obſervation was 52 : 20.

THE moſt common Way to find the Latitude of a Place, is by the Sun's Meridian Altitude, and his Declination; ſo in the Meridional Projection, let $b\mathfrak{a}$ be the Sun's Declination = 20° : 0′ North, and Ab his Meridian Altitude = 57° : 40′. Then, if from the Meridian Altitude, you take the Sun's Declination (becauſe 'tis North) there will remain A$\mathfrak{a}$ = 37° : 40′, for the Complement of the Latitude; therefore the Latitude is 52° : 20′.

AGAIN, let the Sun's Declination be 23° : 29′ = $\mathfrak{a}\,\mathfrak{y}$, South; and his Meridian Altitude A$\mathfrak{y}$ = 14° : 11′; then to the Meridian Altitude add the Sun's Declination (becauſe 'tis South,) and the Sum is A$\mathfrak{a}$ = 37° : 40′, for the Complement of the Latitude; therefore the Latitude is 52° : 20′.

BUT it often happens, that when the Sun is upon the Meridian it is clouded, ſo that then the preceding Rules cannot take place; therefore the following may be very uſeful.

PROB.

PROB. 4.

GIVEN two unequal Altitudes of the Sun, taken on the ſame Day, and the Time between each Obſervation, together with the Sun's Declination, to find the Latitude of the Place.

EXAMPLE.

LET the Sun's Declination be 20° : 0′ North, the Time of the firſt Obſervation 7^h : 34′ A. M. and his Altitude 30° : 0′.

THE Time of the ſecond Obſervation 9^h : 58′ A. M. and the Sun's Altitude at that Time 50° : 0′; ſo the Time between each Obſervation is 2^h : 24′ equal to 36° : 0′. I demand the Latitude of the Place where theſe Obſervations were made?

IN the following Diagram, let *q a e n* be the Parallel of the Sun's Declination, *z e* the Complement of the Sun's Altitude at the Time of the firſt Obſervation, *z a* the Complement of his Altitude at the ſecond Obſervation, and the Angle *a p e* the Time between each Obſervation.

1. THROUGH the Points *p* and *s*, draw the Meridian *p a s*, ſo as to make the Angle *z p a*, with the primitive Circle of 30° : 30′, and alſo the Meridian *p e s*, to make an Angle with the Primitive Circle of 66° : 30′ = *z p e*.

2. DRAW the Parallel of Declination *q a e n*, and it will interſect the ſaid Meridians, or Hour-Circles, in *a* and *e*; thro' which Points draw the Vertical Circles *z a* N, and *z e n*.

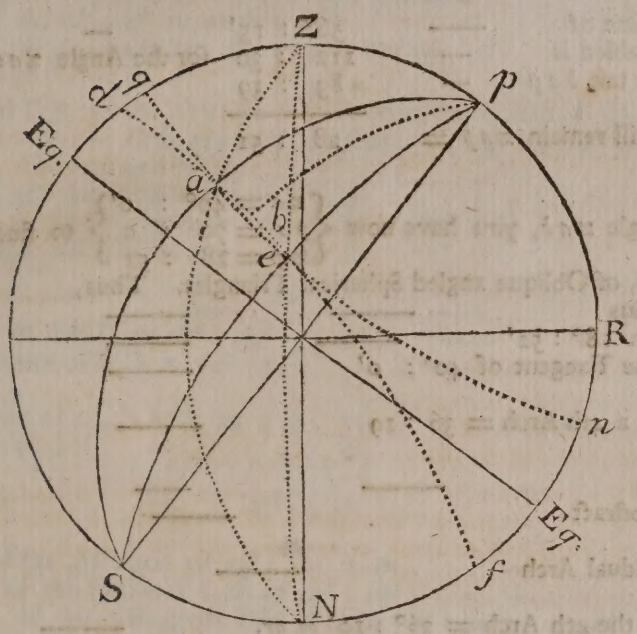
3. THROUGH the Points *a* and *e*, draw the great Circle *d a e f*, and draw *p b*, at Right Angles thereto; which will divide the Angle *a p e*, into two equal Parts, *viz.* *a p b* = 18°, and *b p e* = 18°; *a p e* being 36° : 0′. Now, in the Right-angled Triangle *a p b*, you have the Hypothenuſe *a p* = 70° : 0′, and Angle *a p b* = 18° : 00′, to find the Side *a b*; which, being doubled is *a e*, one Side of the Oblique-angled Triangle *a e z*, and alſo the Angle *b a p*.

THEN, in the Oblique-angled Triangle *z a e*, you will have

$\left\{\begin{matrix} a\,z \\ a\,e \\ e\,z \end{matrix}\right\}$ to find the Angle *z a e*, from which take *p a b*, and there will remain *z a p*.

AGAIN, in the Triangle *a z p*, you'll have *a p*, the Complement of the Sun's Declination, *a z* the Complement of the greateſt Altitude, and the Angle *p a z*, to find *p z*, the Complement of the Latitude; whence you'll have *p* R, the Latitude requir'd.

OPERATION.



OPERATION.

(1.) To find *ab*, S. *c.* of *ap* = 70° : 0′ + R — 19.5340517
— *t. c.* of *bpa* = 18° : 0′ — 10.4882249

Is equal to *t. c.* of *bap* = 83° : 39′ — 9.0458277

(2.) To find *ab*, say,
As Radius — 10.
Is to the Sine of *ap* = 70° : 0′ — 9.9729858
So is the Sine of *bpa* = 18° : 0′ — 9.4899824

To the Sine of *ba* = 16° : 53′ — 9.4629682
This doubled is *ae* = 33° : 46′.

(3.) In the Triangle *zae*, you have
az = 40° : 00′
ae = 33 : 46 } to find the Angle *zae*. (by *Axiom* 7.)
ze = 60 : 00

Sum 133 : 46
Half is 66° : 53′ want. { 40° : 0′ = 26° : 53′ its Sine — 9.6553068
33 : 46 = 33 : 07 its Sine — 9.7374675
Containing Sides { *az* = 40 : 0 *co. ar.* — 0.1919325
ae = 33 : 46 *co. ar.* — 0.2550720

Sum

Sum	——	——	19.8397788
Half is the Sine of	—— 56° : 15′	—	9.9198894
And this doubled is	— 112 : 30	for the Angle *zae*.	
From which tak *bap*	— 83 : 39		
And there will remain *zap* =	28 : 51		

(4. In the Triangle *zab*, you have now $\left\{\begin{matrix} za = 40° : 0' \\ pa = 70 : 0 \\ zap = 28 : 51 \end{matrix}\right\}$ to find *zp*, by a Rule given at *Case* the 3d, of Oblique angled Spherical Triangles. Thus,

As Radius	——	——	10.
To S. *c.* 28° : 51′	——	——	9.9424476
So is the Tangent of 40° : 0′		——	9.9238135
To the Tangent of a 4th Arch = 36 : 19		——	9.8662611
The greateſt Side	——	——	70° : 0′
The 4th Arch ſubſtract		——	36 : 19
Remains the Reſidual Arch		——	33 : 41
As the S. *c.* of the 4th Arch = 36° : 19′ *co. ar.*		——	0.0937964
Is to the S. *c.* of the Reſidual Arch 33° : 41′		——	9.9201836
So is the Sine-Comp. of *za* = 40° : 0′		——	9.8842540
To the S. *c.* of *zp*, the Comp. of the Latitude = 37 : 43′			9.8982340

And thence the Latitude requir'd is *p*R = 52° : 17′.

It may ſo happen, that you can have but one Altitude of the Sun upon that Day that you would find the Latitnde of the Place according to the Method propos'd above, by reaſon of the Sun's being clouded too ſoon after you have taken the firſt Altitude, before you can take a ſecond; and this may be the Caſe on the Morrow, or three or four Days together; however, if you can but take two Altitudes, tho' upon different Days, and the Time of each Day when you took thoſe Altitudes, you may find the Latitude of the Place, as below.

EXAMPLE.

Suppose, that upon *January* the 20th. 1728, at 30 Minutes paſt Two, P.M. I found the Altitude of the Sun to 13° : 5′, his Declination being that Day 17° : 31′ South, and not having an Opportunity of taking another Altitude (by Reaſon of Clouds) until *January* the 24th, at 18 Minutes paſt Nine in the Morning, I then found the Altitude of the Sun to be 13° : 0′, and Declination 16° : 22′ South, I demand the Latitude of the Place where theſe Obſervations were made.

In the following Figure, let B be the North Pole, C the Zenith, BC a Part of the Meridian, or Complement of the Latitude requir'd, AB and DB Arches of two Hour-Circles, making the Angle ABD = 5^h : 12′, or 78° : 0′; A and D, the Poſitions of the Sun at the Times of Obſervation, AB and DB the Sun's Diſtance from the North-Pole at thoſe Times, and AC and DC, the Complements of his Altitudes. Then,

1. In

In the Oblique-angled Triangle ABD, you have the Sides
DB = 90° + 17° : 31′ = 107° : 31′
AB = 90 + 16 : 22 = 106 : 22 } to find AD.
And Angle ABD = 78° : 00′

The Sides AB and DB, of the Triangle ABD, being more than a Quadrant, it will be more convenient (according to *Definition* 6th, of *Spherical Triangles*,) to find AD, as Part of the Triangle ADH; in which you have,
AH = 73° : 38′ the Supplement of AB,
DH = 72 : 29 the Supplement of BD, } to find AD.
And AHD = 78 : 00 = ABD,

(1.) As Radius	—	10.
Is to the S. *c.* of AHD = 78° : 00′	—	9.3178789
So is the Tangent of HD = 72° : 29′	—	10.5008374
To the Tangent of a 4th Arch = 33° : 22′	—	9.8187163
The greater Side AH — 73 : 38		
Residual Arch — 40 : 16		
(2.) As the S. *c.* of the 4th Arch = 33° : 22′ *co. ar.*	—	0.0782261
Is to the S. *c.* of the Residual Arch = 40° : 16′	—	9.8825499
So is the S. *c.* of the lesser given Side HD = 72° : 29′	—	9.4785423
To the S. *c.* of the Side required AD = 74° : 2′	—	9.4393183

(3.) Having found AD, you may find the Angle ADH. Thus,

As the Sine of AD = 74° : 2′ *co. ar.*	—	0.0170860
Is to the Sine of AHD = 78° : 0′	—	9.9904044
So is the Sine of AH = 73° : 38′	—	9.9820351
To the Sine of ADH = 77° : 28′	—	9.9895255

The Supplement of 77° : 28′ is 102° : 32′ equal to the Angle ADB.

(4.) Having AC = 77° : 00′
DC = 76 : 55 } find the Angle ADC.
And AD = 74 : 02

Sum — 227 : 57

Half is — 113 : 58	want.	74° : 2′ = 39° : 56′	Sine	9.8074646
		76 : 55 = 37 : 03	Sine	9.7799655
		AD = 74 : 2 *co. ar.*	—	0.0170860
		CD = 76 : 55 *co. ar.*	—	0.0114224
		Sum	—	19.6159385
Half is the Sine of half the Angle ADC = 39° : 59′ : 21″			—	9.8079692

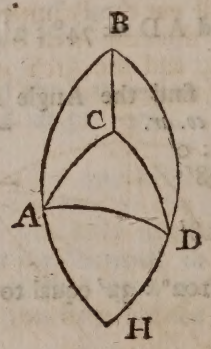
Which being doubled is 79° : 59′ *fere* = ADC.

From ADB =		102° : 32′
Take ADC =		79 : 59
And there will remain the Angle CDB =		22 : 33

(5.) By Help of BD, DC, and BDC, you may find BC. Saying,

As Radius		10.
Is to the Sine-Comp. BDC = 22° : 33′		9.9654582
So is the Tangent of CD = 76° : 55′		10.6337626
To the Tangent of a 4th Arch = 75° : 53′ subſt.		10.5992208
Greater Side — 107 : 31		
Remains the Reſidual Arch — 31 : 38		
Then, As S. *c.* of 75° : 53′. *co. ar.*		0.6127933
To S. *c.* 31 : 38		9.9301448
So S. *c.* 76 : 55		9.3548150
To the S. *c.* of BC = 37 : 47		9.8977531

Therefore the Latitude requir'd is 52° : 13′.



PROB. 5.

Some Time in the Spring Quarter, in 1739, in the Forenoon, an Obſervation was made of the Sun, his Altitude was found 33° : 41′ : 40″, and Azimuth from the North 102° : 40′ : 52″; and ſome Time after, on the ſame Forenoon, his Altitude was found 48° : 46′ : 53″, and Azimuth 134° : 39′ : 56″. From whence the Latitude of the Place of Obſervation, Month, Day, and Hours of Obſervation may be found, *&c.*

This was made the 214th Queſtion in the *Ladies Diary*, for the Year 1740, and anſwered in the Year 1741, by Mr. *Thacker*, who gave a general Theorem, as requir'd; but the Foundation thereof being of too high a Nature for our young Trigonometriſt, we have anſwered it according to the Courſe of this Work; where he will have the Satisfaction of ſeeing the great Harmony there is amongſt Things that are built upon Truth.

CONSTRUCTION.

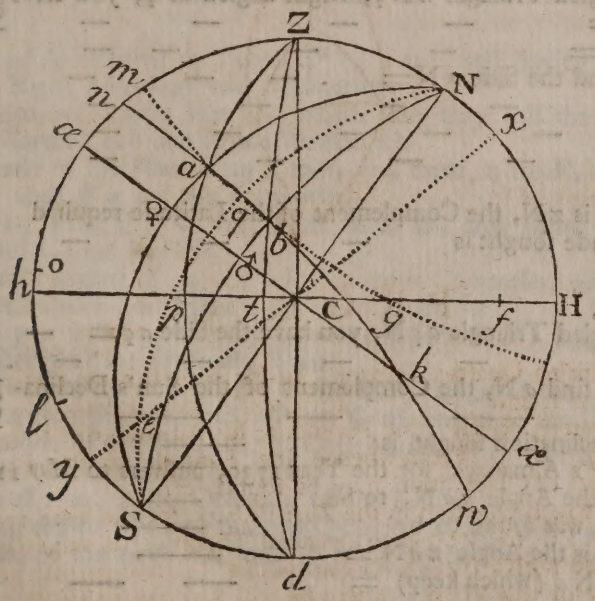
1. TAKE 60 Degrees in your Compaſſes from a Line of Chords, and ſetting one Foot in C, with the other draw the Primitive Circle z H d h, and quarter it with the Horizon h H, and prime Vertical z d.

2. FROM a Line of Half-Tangents take 12° : 40′ : 52″ = 102° : 40′ 52″—90°, and lay off that Extent of your Compaſſes from C to t, and from the ſame Line take 44° : 39′ : 56″ = 134° : 39′ : 56″—90°, and ſet that off from C to p, and thro' the Points t and p draw the Vertical Circles z t d and z p d, and find the Pole of z t d at f, by Help of which ſet off the Sun's Altitude 33° : 41′ : 40″ (found at the firſt Obſervation) from t to b; find alſo the Pole of the Vertical Circle z p d, as at g, by Help of which ſet off the Sun's Altitude 48° : 46′ : 53″ (found at the ſecond Obſervation) from p to a.

3. THROUGH the two Points a and b, draw the great Circle n a b w; which being done, biſect the Part a b in q, and find the Pole e; then thro' the two Points e and q, draw the great Circle N q e s, ſo will N. be the North-Pole, and S. the South-Pole; which being found, draw N C S for the Axis, and at Right Angles thereto the Line æ C æ, for the Equinoxial; ſo is H N the Latitude requir'd and z N or h æ, its Complement.

4. FIND k the Pole of the great Circle N b s, and laying a Ruler upon the Points k and b, it will cut the Meridian or Primitive Circle in m, then the Arch æ m is the Sun's Declination.

5. LAY a Ruler upon N ♂, and it will cut the Primitive Circle in l; lay it alſo upon N ♀, and it will cut the Primitive Circle in o; then the Arch æ l, when reduced to Time, will be that of the firſt Obſervation from Noon, and the Arch æ o the ſecond.



By Calculation.

1. In the Oblique-angled Triangle azb you have the Angle pzt = $134° : 39' : 56'' - 102° : 40' : 52'' =$ — — } $31° : 59' : 4''$
zb = the Complement of the Sun's Altitude at the Time of the firſt Obſervation = — — — } $56° : 18 : 20$
And az = Complement of the Sun's Altitude at the ſecond Obſervation — — — — } $41 : 13 : 07$
By which you will find the Angle baz = — — $110 : 33 : 25$
The Angle abz = — — — — $47 : 51 : 48$
And Side ab = — — — — $28 : 4 : 41$

2. In the Oblique-angled Triangle zan, you have the Angle zan = the Supplement of baz = — — } $69 : 26 : 35$
The Angle nza = the Supplement of $134° : 39' : 56''$ = — $45 : 20 : 04$
With the Side za = as above — — — $41 : 13 : 07$
By which you'll find the Side zn to be — — $39 : 37 : 59$
The Angle anz — — — — $75 : 18 : 16$
And Side an — — — — — $28 : 58 : 49$
The Side ab was found to be $28° : 04' : 41''$, the half of which is aq = — — — — } $14 : 02 : 20$

The Sum of the two laſt is — — — $43 : 01 : 09$

3. In the Right-angled Triangle nNq, Right-angled at q, you have the Side nq = — — — — } $43 : 01 : 9$
And Angle anz = — — — — $75 : 18 : 16$
By theſe you'll find the Side qN = — — — $68 : 56 : 59$
The Angle nNq — — — — $44 : 59 : 30$
And Side nN — — — — — $74 : 47 : 28$
The Side nz is — — — — $39 : 37 : 59$

Their Difference is zN, the Complement of the Latitude requir'd $35 : 09 : 29$
Hence the Latitude ſought is — — — $54 : 50 : 34$

4. In the Right-angled Triangle aqN, you have the Side aq = — $14 : 2 : 20$
And Side qN = — — — — $68 : 56 : 59$
By which you'll find aN, the Complement of the Sun's Declination to be — — — } $69 : 36 : 59$
Therefore the Declination ſought is — $20 : 23 : 34$
Which in *Weaver*'s Almanack, for the Year 1739, anſwers to *May* 11th.
You'll alſo find the Angle qaN, to be — $84 : 39 : 52$
The Angle zab, was found to be — $110 : 33 : 25$
Their Difference is the Angle zaN = — $25 : 53 : 33$
And the Angle qNa (which keep) = — — $14 : 59 : 56$

5. In

5. In the Oblique-angled Triangle $a z N$, you have the Side $a N$ = $69° : 36' : 26''$
The Side $a z$ = ——— ——— $41 : 13 : 07$
With the Angle $a z N$ = ——— ——— $134 : 39 : 56$
By which you'll find the Angle $z N a = 29° : 59' : 56''$, which reduced to Time is $1^h : 59' : 59''\frac{9}{10}$, which take from 12^h, and there will remain —— —— —— $10^h : 00' : 00''\frac{1}{10}$
And that's the Time of the second Observation.

Again, to the Angle $z N a = 29° : 59' : 56''$ add twice the Angle $q N a$, and the Sum will be — — — $59 : 59 : 48$
Which being reduced to Time is $3^h : 59' : 59''\frac{4}{5}$, and this taken from 12^h leaves the Time of the first Observation —— $8^h : 00' : 00''\frac{1}{5}$

Having now gone through some of the most useful Problems relating to the Course of the Sun (which has no Latitude), those that follow are intended to shew how to proceed with the Moon, the Planets and fixed Stars, that have Latitude.

PROB. 1.

Given the Latitude and Longitude of a Planet or fixed Star, to find the Right Ascension and Declination.

EXAMPLE.

The Bright Star of Aries being in Taurus $4° : 4' : 20''$, and having $9° : 57'$ of North Latitude; I demand its Right Ascension and Declination.

1. Draw the Primitive Circle $H z h N$, which you may call the Solstitial Colure, and quarter it with the Horizon $H h$ and Prime Vertical $z N$.

2. Set the Latitude of the Place from h to P, and from z to Æ, and draw S P for the Equinoctial Colure, and Æ ϖ for the Equinoctial.

3. Set off $23° : 29'$ from P to C, and from Æ to ♋, and draw the Diameter ♋, ♑, for the Ecliptick, and C $\varkappa$ for its Axis.

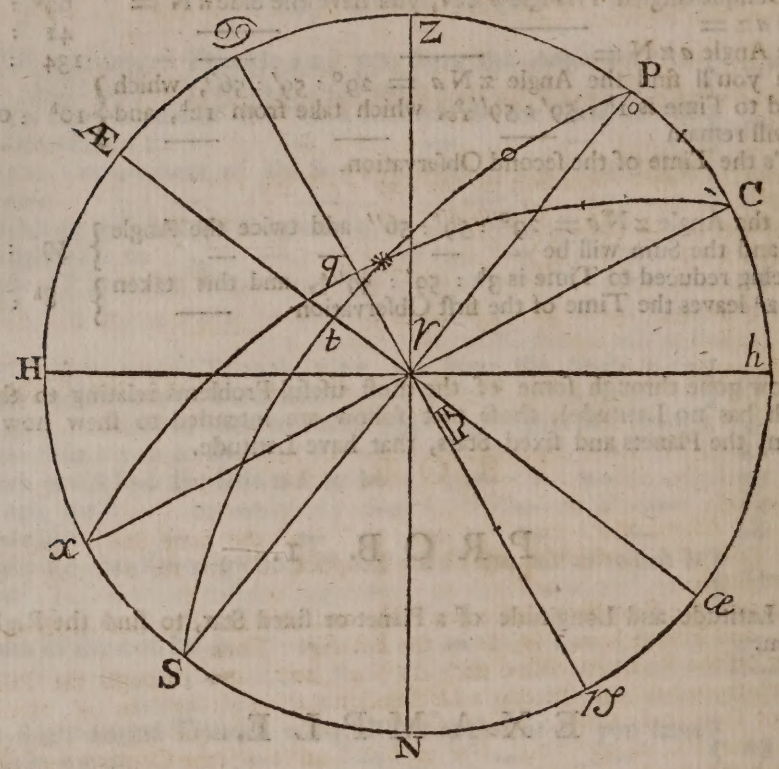
4. Take the Half-Tangent of $34° : 4' : 20''$ in your Compasses, and set that off from r to q, and draw the Circle of Longitude C $q \varkappa$, upon which set $9° : 57'$ from q to $*$, and thro' $*$ draw P $*$ S, and you will form the Oblique-angled Spherical Triangle P $*$ C.

Now, in this Figure you are to know, That

1. $q *$, is the Latitude of the Star, and $* C$ the Complement of the Latitude.
2. $t *$, is the Declination thereof, and $* P$ the Complement of the Declination.
3. $r q$, is the Longitude of the Star, and its Complement is $q ♋ = q C ♋$.
4. $r t = t P r$, is the Star's Right Ascension.
5. $P C = Æ ♋ = Æ r ♋$, is the Obliquity of the Ecliptick.

The Scheme being drawn you may find the Quantity of the Arch $t *$, and that will be the Declination; and also of the Arch $r t$, and that will be the Right Ascension.

By



By Calculation.

In the Oblique-angled Triangle P C ∗, you have

Given { The Complement of the Latitude of the Star ∗C = — 83° : 3' : 00"
The Complement of its Longitude q C ♋ = — 55 : 55 : 4
And the Distance of the Pole of the Equinoctial from the Pole of the Ecliptic — — = 23 : 29 : 00 }

1. To find the Complement of the Star's Declination P∗ ; this you may do by a Rule given at *Case* 3d, of Oblique-angled Spherical Triangles. Thus,

As Radius	—	10.
Is to the S. *c.* of q C ♋ = 55° : 55' : 4"	—	9·7483721
So is the Tangent of P C = 23° : 29'	—	9·6379563
To the Tangent of a fourth Arch 13° : 40' : 48"	—	9·3863284
The greateſt Side C∗ — 80 : 03 : 00		
The Differ. is the Reſidual Arch 66 : 22 : 12		

2. As

2. As the S. *c.* of the fourth Arch 13° : 40′ : 48″, *co. ar*	—	0.0125045
Is to the S. *c.* of the Reſidual Arch 66° : 22′ 12″	—	9.6029588
So is the S. *c.* of 23° : 29′	—	9.9624527
To the Sine of $t* = 22° : 13′ : 57″$, the Declination requir'd	—	9.5779160

AGAIN, to find the Right Aſcenſion of the Star *r t*. Say,

As the Sine of $P* = 67° : 46′ : 3″$, *co. ar.*	—	0.0335504
Is to the Sine of $q\,C\,\mathfrak{S} = 55° : 55′ : 4″$	—	9.9181532
So is the Sine of $*\,C = 80 : 03$	—	9.9934181
To the Sine of $t\,P\,Æ = 61 : 47 : 56 = Æ\,t$	—	9.9451217

And the Complement of 61 : 47 : 56 is 28° : 12′ : 4″ = *r t*, the Right Aſcenſion requir'd.

PROB. 2.

GIVEN the Right Aſcenſion *r t* = 28° : 12′ : : 4″ (of the ſame Star), and Declination thereof = 22° : 13′ : 57″, and Obliquity of the Ecliptick *Æ r* ♋ = 23° : 29′, to find its Latitude and Longitude.

1. HAVING drawn the Primitive Circle H *z b* N, and alſo the Equinoctial *Æ æ*, Equinoctial Colure P *r* S, and Ecliptick ♋ *r* ♑, as in the laſt *Fig.* Then,

2. FROM *r*, ſet off the Right Aſcenſion 28° : 12′ : 4″ to *t*, and through the Point *t* draw P *t* S, upon which from *t* ſet off the Star's Declination 22° : 13′ : 57″ to *, through which draw the Circle of Longitude C * *x*, which will make the Oblique-angled Triangle P C *, where you have given P C = 23° : 29′, P * = 67° : 46′ : 3″ (the Complement of the Star's Declination), and the Angle * P C = *t æ* the Supplement of *Æ t* = 118° : 12′ : 4″, to find C *, which is the Complement of the Latitude of the Star; and Angle P C * = ♋ *q*, whoſe Complement is *r q*, the Longitude of the Star from the Beginning or firſt Point of *r*. This being but the Reverſe of *Prob.* 1. the Reader may excuſe ſaying any more about it.

PROB. 3.

GIVEN the Meridian Altitude of an unknown Star or Planet, and its Diſtance from a known Star, to find the unknown Star's Right Aſcenſion, Declination, Longitude and Latitude.

EXAMPLE.

Suppoſe the Meridian Altitude of Regulus, or the Lyon's Heart, was found to be 51° : 58′ : 50″, and its Diſtance from Arcturus 59° : 46′ : 50″, in Antecedence; I demand the Right Aſcenſion, Declination, Longitude and Latitude of the Lyon's Heart, the Right Aſcenſion of Arcturus being then 210° : 22′ : 30″, and Declination 20° : 49′.

1. DRAW the Meridian Z O N H, the Horizon H O, and at Right Angles to it the Prime Vertical Z N.

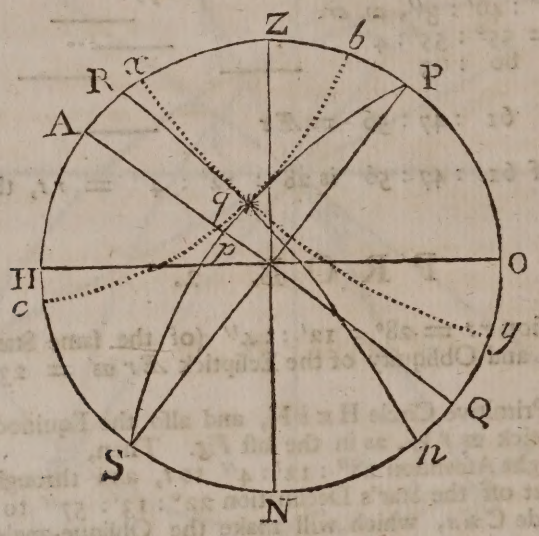
2. SET off the Latitude of the Place of Obſervation 51° : 28′ : 30″ from O to P, and the ſame from Z to A, and draw the Axis P S, and Equator A Q.

3. SET

3. SET off 51° : 58′ : 50″, the Meridian Altitude of Regulus, from H to R; and about the Point R, or Place of Regulus, as a Pole, deſcribe the ſmall Circle *b c*, at the Diſtance of 59° : 46′ : 50″ equal to the Diſtance between the two Stars.

4. ABOUT the Pole P, deſcribe the ſmall Circle *x y*, at the Diſtance of Arcturus from the Pole equal to 69° : 11′, and this will interſect the Circle *b c*, in the Point *q*, and that is the Place of Arcturus.

5. THROUGH *q*, and the Poles P S, draw the Circle of Right Aſcenſion (or Hour-Circle) P *q* S, and through R, *q*, *n*, the great Circle R *q n*, and the Thing is done.



1. To find the Declination of Regulus.

From the Meridian Altitude H R =	51° : 58′ : 50″
Take the Complement of the Latitude of the Place	38 : 31 : 30
Remains the Declination of Regulus North	13 : 27 : 20
The Complement of which is R P = the Star's Diſtance from the North-Pole	76 : 32 : 40

2. To find the Difference of the Right Aſcenſions of theſe two Stars, *viz.* the Angle R P *q*, the Meaſure of which A *p*, you have in the Oblique-angled Triangle R P *q*, the Side.

R P = 76° : 32′ : 40″
R *q* = 59 : 46 : 50
P *q* = 69 : 11 : 00
} to find R P *q*, *per Axiom* 5. inverted.

The Natural Verſed Sine of R *q* = 59° : 46′ : 50″ is	4966867
Natural Verſed Sine of R *p* — P *q* = 7° : 21′ : 40″ ſubſt.	82417
Remains	4884450
The Logar. of which, more twice Radius is	29.6888156
Logarithmic Sine of R P = 76° : 32′ : 40″	9.9879122
Logar. Sine of P *q* = 69 : 11 : 00	9.9706826
Sum ſubſtract.	19.9585948
Remains the Log. V. S. of R P *q* = A *p* = 62° : 26′ : 19″	9.7302208
R. A. of Arcturus	210 : 22 : 30
The Difference is	147 : 56 : 11 for the R. A. of the Lyon's Heart.

The

The Declination of Regulus being	—	—		13° : 27′ : 20″
And its R. A.	—	—	—	147 : 56 : 11

You may by theſe find the Longitude and Latitude thereof, as follows:

The Stereographic Solution.

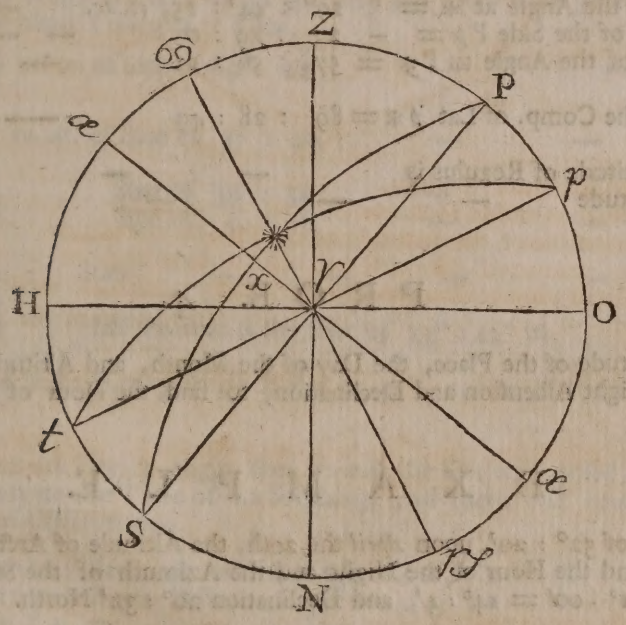
1. DRAW the Primitive Circle, or Solſtitial Colure H Z O N, and quarter it with the Prime Vertical Z N, and Horizon H O.

2. SET the Latitude of the Place (ſuppoſe 51° : 32′) from O to P, then draw the Axis P S, and at Right Angles thereto ♈, ♎, for the Equinoctial.

3. FROM 180°, take 147° : 56′ : 11″, and there will remain 32° : 3′ : 49″. Take the Half-Tangent thereof in your Compaſſes, and ſet that off upon the Equinoctial from ♈ to *x*, and thro' *x* draw P *x* S.

4. THE Declination of Regulus being 13° : 27′ : 20″ North, ſet that upon P *x* S, from *x* to ∗.

5. TAKE the Obliquity of the Ecliptick 23° : 29′ in your Compaſſes, (out of the Line of Chords) and lay that on the Primitive Circle from P to *p*, and from S to *t*, and through ∗, and *p*, *t*, draw the great Circle *p* ∗ *t*, and 'tis done; and the Angle ♋ *p* ∗ is the Longitude of Regulus from the Beginning of ♋, and *p* ∗ is the Complement of the Latitude.



By Calculation.

IN the Oblique-angled Triangle ∗ P *p*, you have given,

1. The Complement of the Star's Declination P ∗ =			—	76° : 32′ : 40″
2. The Obliquity of the Ecliptick P *p*	—	—	—	23 : 29 : 00
3. Included Angle ∗ P *p* =	—	—	—	122 : 03 : 49

To find the Angles P ∗ *p*, and P *p* ∗ ; which you may attain by a Rule given at *Caſe* the 4th, of Oblique-angled Spherical Triangles. Thus,

1. As the S. *c.* of half the Sum of the Sides	50°	: 00′	: 50″, *co. ar.*	—	0.1920580
Is to the S. *c.* of half their Difference	26	: 31	: 50	— —	9.9516756
So is the T. *c.* of hlaf the included Angle	61	: 1	: 54	—	9.7431857
To the Tangent of half P*p* ∗ + P∗*p*	37	: 37	: 25	— —	9.8869193

2. As the Sine of half the Sum of the Sides	50°	: 00′	: 50″, *co. ar.*	—	0.1156578
Is to the Sine of half their Difference	26	: 31	: 50	—	9.6499916
So is the T. *c.* of half ∗P*p* = —	61	: 1	: 25	—	9.7431857
To the Tangent of half the Difference of the other two Angles ——	17	: 53	: 10	——	9.5088351

Half the Sum of the other two Angles was found to be				—	37° : 37′ : 25″
Half their Difference — — —				—	17 : 53 : 10
The Sum of these is the Angle P*p* ∗ =				——	55 : 30 : 35
And their Difference is the Angle P∗*p* =				——	19 : 44 : 15

3. To find the Complement of the Star's Latitude *p*∗. Say,					
As the Sine of the Angle at ∗ =	19°	: 44′	: 15, *co. ar.*	—	0.4714543
Is to the Sine of the Side P*p* =	23	: 29	: 0	— —	9.6004090
So is the Sine of the Angle ♋ P∗ =	57	: 56	: 11	——	9.9281188
To the Sine of the Comp. of Lat. *p*∗ =	89	: 28	: 49	——	9.9999821
Hence the Latitude of Regulus is				— —	0° : 31′ : 11″
And its Longitude — — —					♌ 25 : 30 : 35

PROB. 4.

GIVEN the Latitude of the Place, the Day of the Month, and Altitude of a known Star, together with its Right Ascension and Declination, to find the Hour of the Night and Azimuth of the Star.

EXAMPLE.

IN the Latitude of 52° : 20′, upon *April* the 20th, the Altitude of Arcturus was found to be 50° : 00′; I demand the Hour of the Night and the Azimuth of the Star, its Right Ascension being then 211° . 00′ = 14ʰ : 4′, and Declination 20° : 32′ North.

The Stereographic Solution.

1. HAVING drawn the Primitive Circle A Z *h* N, and quarter'd it with the Horizon H *h*, and Prime Vertical Z N, take 52° : 20′ in your Compasses out of the Line of Chords, whose Radius is equal to *r* H, and set that from *h* to P, and from Z to *æ*, and then draw P S for the Axis, and *æ*, *æ*, for the Equinoctial.

2. DRAW

2. DRAW the Parallel of Altitude $q*t$, at the Diſtance of 50° : 0′ = $v*$ from the Horizon H h, and the Parallel of Declination $x*y$ 20° : 32′ from the Equinoctial (on the North Side thereof,) and theſe two Parallels will interſect in the Point $*$, which is its Place at that Time.

3. THROUGH $*$, and the Poles P and S, draw the Hour-Circle P$*$S, and through the ſame Point and Zenith and Nadir, draw the Azimuth Circle Z$*$N, and 'tis done.

By Calculation.

1. FIND the Time when Arcturus will be upon the South Part of the Meridian, by ſubſtracting the Right Aſcenſion of the Sun from his Right Aſcenſion. Thus,

The given R. A. of Arcturus was	— —	14^h : 4′ : 00″
The R. A. of ☉ that Day was	— —	2 : 33 : 00
Then the Time of the Southing of Arcturus is	—	11 : 31 : 00 P. M.

2. IN the Oblique-angled Triangle P$*$Z, you have

The Complement of the Latitude of the Place PZ =	—	37° : 40′
The Complement of the Star's Altitude Z$*$ =	—	40 : 00
And Complement of its Declination P$*$ =	—	69 : 28

To find the Horary Angle ZP$*$; which you may do, by a Rule given at the 11th *Caſe* of Oblique-angled Spherical Triangles. Thus,

P$*$ = 69° : 28″ } Pz = 37 : 40 }	*co. ar.* of Sine of 69° : 28′	— —	0.0285069
Differ. 31 : 48	*co. ar.* of Sine of 37 : 40	— —	0.2139114
½ Dif. is 15 : 54 } ½ z$*$ is 20 : 00 }	Sine of 35 : 54	— —	9.7681735
	Sine of 4 : 06	— —	8.8542905
Sum 35 : 54 } Differ. 4 : 06 }	Sum — —	— —	18.8648823
	Half the Sum is the Sine of 15° : 42′ =	—	9.4324411

Which being doubled is the Horary Angle ZP$*$ = 31° : 24′ = 2^h : 5′ : 36″.

HAVING thus found the Horary Angle, then becauſe the Star is ſuppoſed to be in the Eaſtern Hemiſphere, ſubſt. it from the Time of its Southing, and there will remain 9^h : 25′ : 24″ P. M. for the Time of Obſervation.

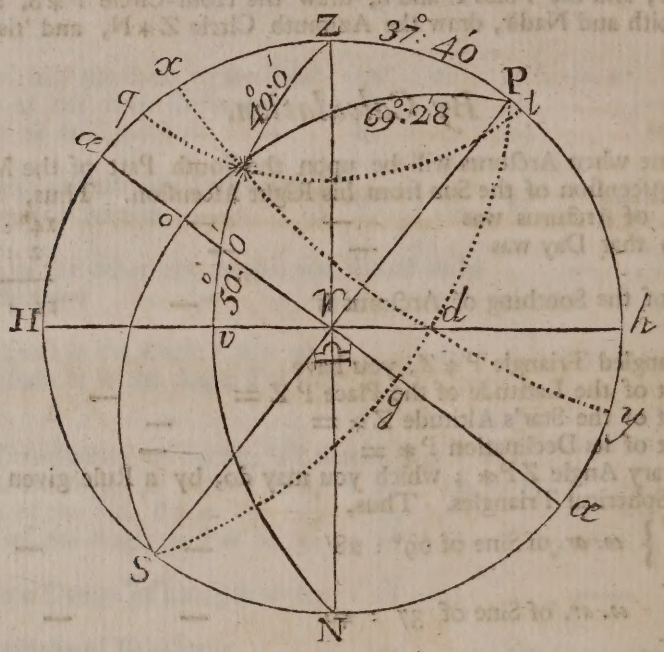
To find the Azimuth of Arcturus, or the Angle xZ$*$.

You have now the Horary Angle ZP$*$ =		31° : 24′ }
And its oppoſite Side Z$*$ =	—	40 : 00 }
Together with the Side P$*$ =	—	69 : 28 }

THEN, As the Sine of Z$*$ = 40° : 00′, *co. ar.*	—	—	0.1919325
Is to the Sine of ZP$*$ = 31 : 24	—	—	9.7168458
So is the Sine of P$*$ = 69 : 28	—	—	9.9714931
To the Sine of the Azimuth of Arcturus xZ$*$ = 49° : 22′ =		—	9.8802714

Or S. E. + 4° : 22′ Eaſterly.

N. B. That if the Star had been in the Weſtern Hemiſphere at the Time of Obſervation, then you muſt have added the Horary Angle to the Time of the Star's Southing, and you would have had $13^h : 36' : 36''$, for the Time of Obſervation.



PROB. 5.

Given the Latitude of the Place, the Day of the Month, and Right Aſcenſion and Declination of a Star, to find its Amplitude, the Time of its Riſing and Setting, and Continuance above, or below the Horizon.

EXAMPLE.

Let it be requir'd to find the Amplitude, Time of Riſing, Setting, *&c.* of Arcturus, at *London*; whoſe Latitude is 51° : 32′ North, upon *April* 20th, 1743.

The Stereographic Projection.

1. Draw the Primitive Circle H Z *h* N, the Horizon H *h*, and Prime Vertical Z N; then take the Latitude of the Place 51° : 32′ in your Compaſſes out of the Line of Chords, and ſet that from *h* to P, and from Z to *æ*, and draw *æ æ* for the Equinoctial, and P S for the Axis of the World.

2. Draw the Parallel of the Star's Declination *x* ∗ *y*, at the Diſtance of 20° : 32′ from the Equinoctial; and where it cuts the Horizon, as at ∗, there the Star is at his Riſing or Setting.

3. Through P ∗ and S, draw the Horary Circle P *g* S, and 'tis done.

By

By Calculation.

In the Right-angled Triangle $r*g$, you have the Angle $*rg = 38° : 28'$, being the Complement of the Latitude; and alſo the Declination of the Star $*g = 20° : 32'$.

To find its Amplitude.

As the Sine of $gr* = 38° : 28'$	9.7938317
Is to the Sine of the Declination $*g = 20° : 32'$	9.5450005
So is Radius	10.
To the Sine of the Amplitude requir'd $34° : 19'$	9.7511688

This is North, becauſe the Declination is ſo; for the Declination and Amplitude of the Sun, or any Star, are always of the ſame Denomination.

To find the Aſcenſional Difference rg.

Say, As the Tangent of the Angle $*rg = 38° : 28'$	9.9000865
Is to the Tangent of $*g = 20° : 32'$	9.5735074
So is Radius	10.
To the Sine of the Aſcenſional Difference $rg = 28° : 8'$	9.6734209

To

To which add	——	$ar =$	$90^\circ : 0'$
The Sum is	——		$118 : 8$ in Time is $7^h : 52' : 32''$

And this is half the Time of the Star's Continuance above the Horizon. The Time of the Southing (as at the laſt *Prob.*) was at —— $11^h : 31' : 00''$ P. M.
To which add —— —— $7 : 52 : 32$

The Sum is the Time of Setting —— $19 : 23 : 32$

That is 23 Minutes and 32 Seconds after Seven in the Morning of the 21ſt Day of *April*.

Subſtract the ſame from the Time of Southing, and you'll have the Time of the Star's Riſing, *viz.* $3^h : 38' : 28''$; that is, 38 Minutes and 28 Seconds after Three in the Morning of the given Day.

CHAP. XVI.

To draw the Ecliptic upon the Plane of the Meridian for any given Time or Place.

THIS is of great Uſe to thoſe who would calculate an Eclipſe of the Sun, or delineate a Type of an Eclipſe of the Moon; becauſe in either to know the Altitude of the *Nonageſſima* Degree, or higheſt Point of the Ecliptick, the Meridian-Angle, *&c.* (as farther on) is abſolutely neceſſary; and is moſt agreeable to the cæleſtial Phænomena in the ſecond.

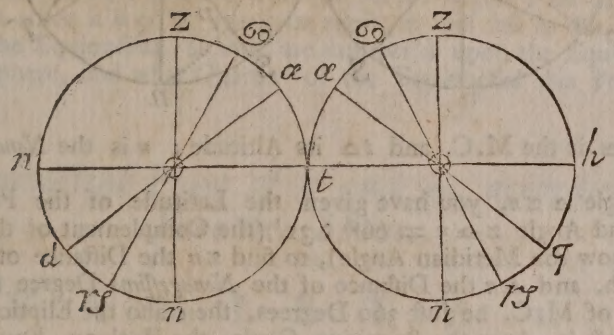
WHEN I have been about ſuch Work, and had found the apparent Time of the true Ecliptic, Conjunction or Oppoſition of the Luminaries, the Place of the Sun and his Right Aſcenſion, and by theſe the R. A. of M. C. or *Medium Cœli*; I have been at a Loſs to know how to project that Part of the Ecliptic, which I was to uſe in its due Poſition, without looking upon a Globe; which not being always at hand, put me upon thinking of ſome way of doing it, that might be the moſt eaſy and ready; and that at an Eclipſe of the Moon, by knowing the Longitude and Latitude of any other Planet or fixed Star, to find if it is above or below the Horizon; and if above, to find whereabouts in the Heavens to look for it, as if the Eye was placed in the Center of a Sphere of Glaſs, having the Planets and fixed Stars painted upon it.

IN order to do this, the firſt Way that offer'd was to divide the Sphere into two Hemiſpheres, like thoſe uſed in Geography, upon which the World is drawn; with this Difference, that whereas thoſe join or touch one another where the Meridian interſects the Equator, from which Point the Longitude of Places are reckoned; theſe join or touch at the Interſection of the Meridian and Horizon, from whence you may count the Azimuth of the Sun, Moon or Stars, and muſt be held between you and the South Part of the Meridian as near as you can, when you compare what you have put upon the Projection with their Correſpondents in the Heavens.

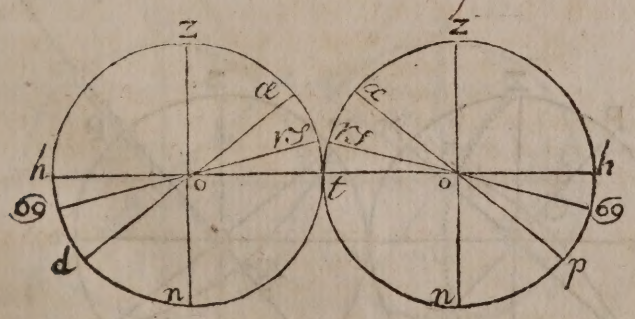
THE

THE Way is thus: Having found the apparent Time of the Ecliptic Conjunction or Oppoſition of the Sun and Moon, reduce it into Degrees of the Equinoctial; to which add the Right Aſcenſion of the Sun, and the Sum is the R. A. of M. C. or Point of the Ecliptic then upon the Meridian; which being known, and the Equinoctial drawn in each Hemiſphere, according to the Latitude of the Place, the next Thing to be done is to draw the Ecliptick, or that half of it that is moſt proper for your Purpoſe, and this will admit of *Four Varieties*; each Figure (as aboveſaid) is compoſed of two Hemiſpheres, the better to ſee whereabouts all the Signs of the Zodiac are, which are above or below the Horizon, and which are Riſing, Culminating and Setting, *&c.* That on the Left-hand, I call the Eaſtern Hemiſphere, and that on the Right-Hand, the Weſtern; the Primitive Circle (which here repreſents the Meridian) being as it were ſplit into two Meridians (tho' both are underſtood to be the ſame Meridian) each containing one Hemiſphere.

1. *Note*, THAT if the R.A. of the M. C. be juſt 90 Degrees, then the Ecliptick becomes a Right Circle; to draw which, you muſt take 23° : 29′ in your Compaſſes from a Line of Chords, and lay off that Extent upon the Meridian from the Equinoctial at ♈ to ♋ in both Hemiſpheres (if you pleaſe), and then the Point ♋ will be the M. C. and *Nonageſſima* Degree, and its Altitude will be *t* ♋: From ♋, and thro' the Center at O, draw ♋ ♑, for the Ecliptick.



2. IF the R. A. of M. C. be 270 Degrees, then alſo the Ecliptick will be a Right Circle; to draw which, you muſt by your Line of Chords ſet 23° : 29′ upon the Meridian from ♈, downwards to ♑, and thence thro' the Center at O, draw ♑ ♋ for the Ecliptick, the Point ♑ is both the M. C. and *Nonageſſima* Degree, and their Altitude is *t* ♑. *See the next Fig.*

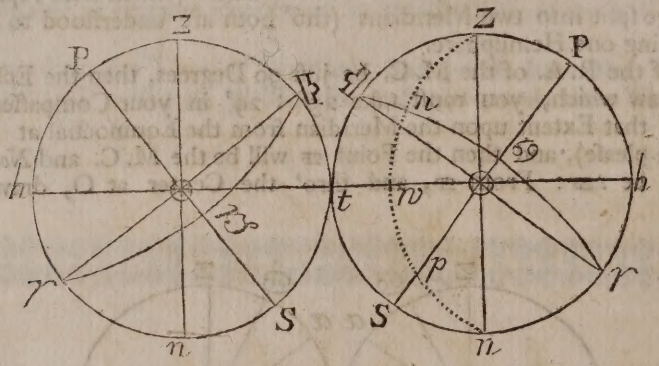


3. IF

3 If the R. A. of M. C. be juſt 180 Degrees, then the Ecliptick will be an Oblique Circle; and, to draw it, you muſt,

(1.) Set the Complement of the Latitude of the Place from t to $\triangle$, and from h to r; and through theſe Points draw $r\triangle$ for the Equinoctial, and croſs it at Right Angles with S P, for the Axis of the World.

(2.) Take the Secant of 66° : 31′ in your Compaſſes, and ſetting one Foot in $\triangle$ turn the other about, until it fall upon the Axis S P produced above P, in the Eaſtern Hemiſphere, and below S in the Weſtern, and where it reſts is the Center of half the Ecliptick, upon which deſcribe it.

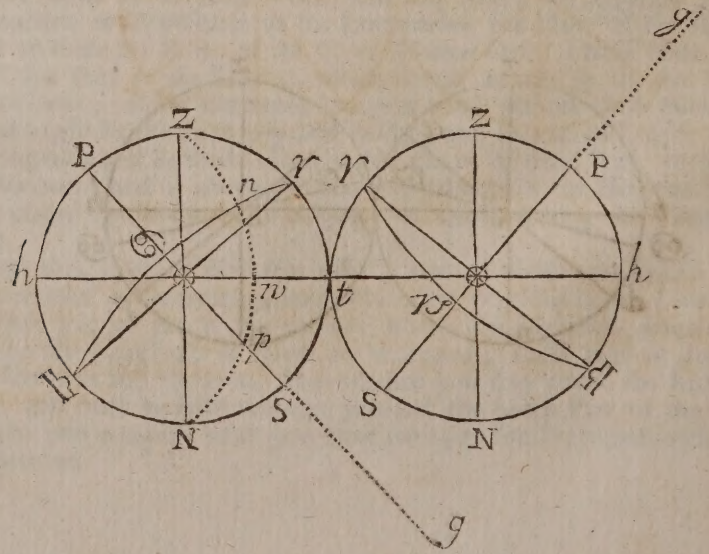


Here the Point $\triangle$ is the M. C. and $t\triangle$ its Altitude; n is the *Nonageſſima* Degree, and $w\,n$ its Altitude.

(3.) In the Triangle $\triangle z\,n$, you have given the Latitude of the Place $z\triangle$, ſuppoſe of *London* 51° : 32′, and Angle $z\triangle n = 66° : 31'$ (the Complement of the Sun's greateſt Declination, which is now the Meridian Angle), to find $z\,n$ the Diſtance of the *Nonageſſima* Degree from the Zenith, and $\triangle n$ the Diſtance of the *Nonageſſima* Degree from M. C.

(4.) If the R. A. of M. C. be juſt 360 Degrees, then alſo the Eliptick will be an Oblique Circle; and to project it, draw the Primitive Circle, the Horizon, Equinoctial and Axis, as before.

Then take the Secant of 66° : 31′ in your Compaſſes, and ſet one Foot in r, and turn the other about until it falls on the Axis PS, produced below S in the Eaſtern Hemiſphere, and above P in the Weſtern; ſo will you have the Centers g and g, upon each with the ſame Extent of your Compaſſes draw the Ecliptick in each Hemiſphere, as under.



In

In this Figure zn is the Diſtance of the *Nonageſſima* Degree from the Zenith, and nw its Altitude; nr its Diſtance from the M. C. zrn the Meridian Angle $=$ 66° : 31′, and zr the Latitude of the Place.

Now, to go on, I have made choice of four different R. A's. of the M. C. and thoſe, with the four following Figures, will be ſufficient to inform the Learner, how to proceed, when the R. A. of M. C. is any other Number of Degrees, *&c.*

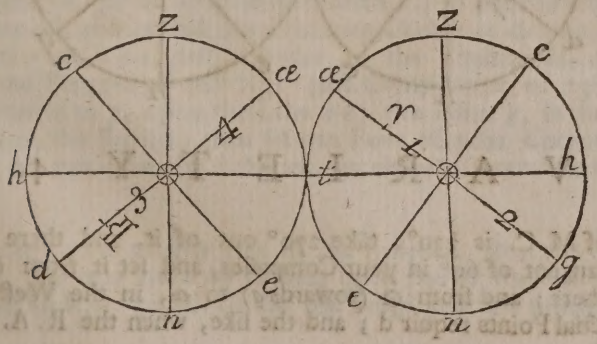
Varieties $\left\{\begin{matrix}1\\2\\3\\4\end{matrix}\right\}$ when the R. A of M. C. is $\left\{\begin{matrix}30\\150\\210\\330\end{matrix}\right\}$ Degrees.

Then, when you have drawn the Primitive Circles for each Variety, and quarter'd them with the Horizon hth, and Prime Vertical zn; from the South Point of the Horizon (or Point of Contact at t,) ſet the Complement of the Latitude of the Place to $æ$, and thence through the Center at $\odot$, draw the Diameters $æ\odot q$, and $æ\odot d$, for the Equinoctial; through two Points of which the Ecliptick muſt paſs, one of them is the Equinoctial Poinr of r, which will fall upon the Meridian at $æ$, when the R. A. of M. C. is 360°, or 24 Hours; but, at any other time, except when it is 90° : 270°, or 180°, it will fall in the firſt, ſecond, third or fourth Quarter of the Equinoctial, as they are number'd upon the Equinoctial; but, to know in which Hemiſphere, and what Quarter of the Equinoctial this Point will fall, obſerve what follows.

To find the Equinoctial Point of ♈ and ♎, according to the four Varieties above.

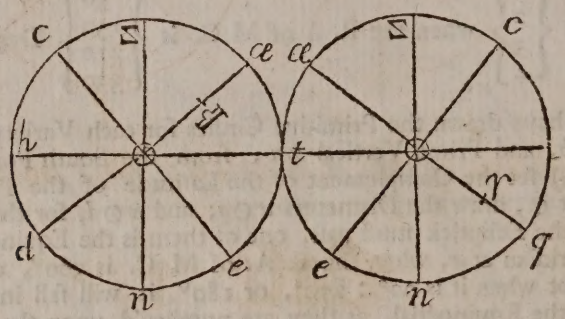
VARIETY 1.

When the R. A. of M. C. is 30°, take the Half-Tangent of 60° in your Compaſſes, and ſet it from $\odot$ (towards $æ$) to r, in the firſt Quarter of the Equinoctial in the Weſtern Hemiſphere; and from $\odot$ (towards d) to ♎, in the Eaſtern Hemiſphere; and theſe are Equinoctial Points requir'd. Do the like, when the R. A. of M. C. is any thing under 90°.



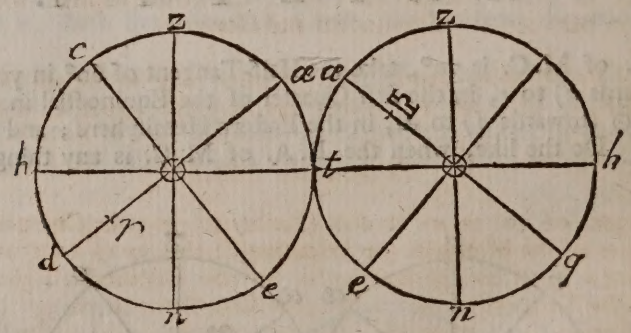
VARIETY 2.

When the R.A. of M.C. is 150°, take 90° out of 150°, and there will remain 60°. Take the Half-Tangent of 60° in your Compaſſes, and ſet that from ⊙ (toward *q*) to *r*, in the Weſtern Hemiſphere; and from ⊙ (towards *æ*) to ♎, in the Eaſtern Hemiſphere; and the like when the R. A. of M.C. is any thing above 90°, and under 180°.



VARIETY 3.

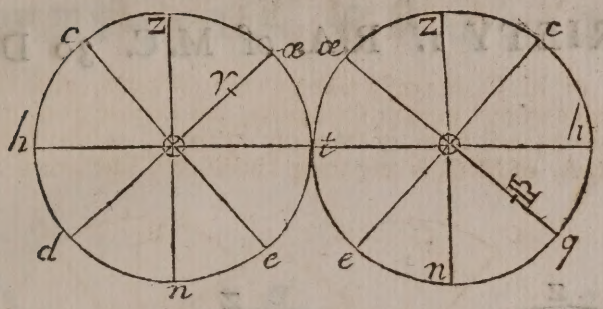
When the R.A. of M.C. is 210°, take 180° out of it, and there will remain 30° = *dr*, whoſe Complement is ⊙*r* = 60°; the Half-Tangent of which ſet from ⊙ (towards *d*) to *r*, in the Eaſtern Hemiſphere; and from ⊙ (towards *æ*) to ♎, in the Weſtern; and the like when R. A. of M.C. is above 180°, and under 270°.



VARIETY 4.

When the R. A. of M.C. is 330°, take 270° out of it, and there will remain 60° = ⊙*r*; take the Half-Tangent of 60° in your Compaſſes, and ſet it from ⊙ (towards *æ*) to *r*, in the Eaſtern Hemiſphere; and from ⊙ (towards *q*) to ♎, in the Weſtern Hemiſphere; ſo have you the Equinoctial Points requir'd; and the like, when the R. A. of M.C. is above 270°, and under 360°.

To



To find the Position of the Ecliptick: This admits of two Cases.

Case 1.

When the R.A. of M.C. is under 180°, the Ecliptick will intersect the South Part of the Meridian between the Equinoctial and Zenith.

Case 2.

When the R.A. of M.C. is more than 180°, then the Ecliptick will intersect the South Part of the Meridian between the Equinoctial and Horizon.

The Equinoctial Points being found, the Ecliptick may be drawn, as directed at the third *Case* of the second *Prob.* of *Chap.* 11. which you may take again, as under.

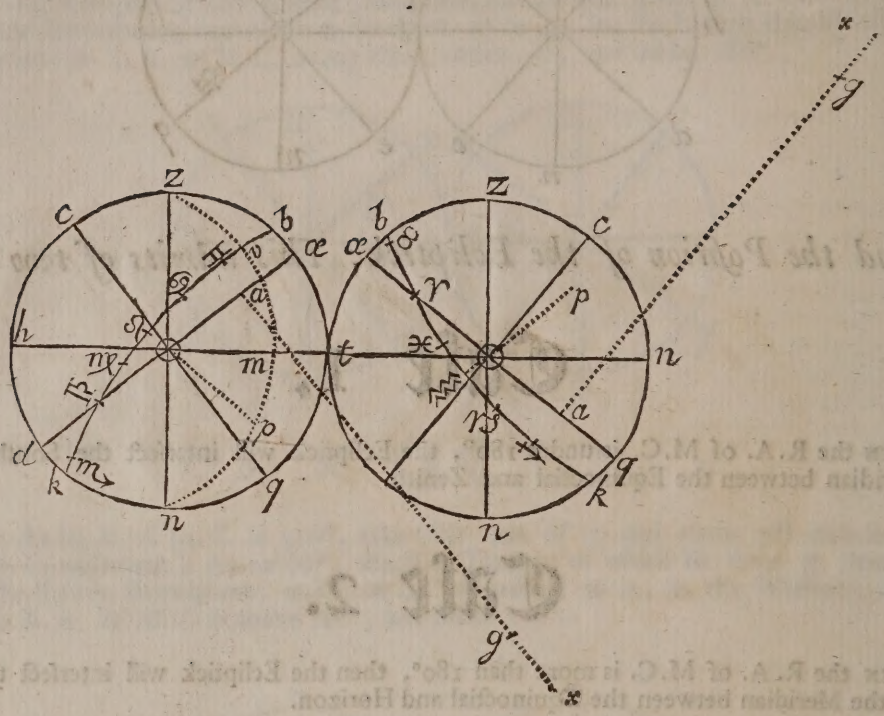
To draw the Ecliptick.

Take the Tangent of 30° = *ær* = *d* ♎ (*Variety* 1.) in your Compasses, and set that Extent from the Center of the Meridian (or Primitive Circle) at ⊙ to *a*, in each Hemisphere; then upon the Point *a* erect *a x*, perpendicular to the Equinoctial, and making *r a* = ♎ *a*, Radius. Lay off the Tangent of 66° : 31′ (the Complement of 23° : 29′, the Obliquity of the Ecliptick) from *a* to *g*, upon the Line *a x*; this Point *g*, is the Center of the Ecliptick, and *g r*, or *g* ♎, the Radius; then set one Foot of your Compasses in *g*, and extend the other to *r*, or ♎, and draw the Ecliptick, in either or both the Hemispheres, as Occasion may require.

Now, whether the Center of that Part of the Ecliptick, that you are to use, fall above or below the Horizon, you may know by looking upon one of the Figures following, *viz.* that which agrees nearest to R.A. of M.C. for the Time you are working for.

Having drawn the Parts of the Ecliptick in each Hemisphere, you may find the Poles at *p*, and by them divide each Part into Signs; as in the next *Figure*, &c.

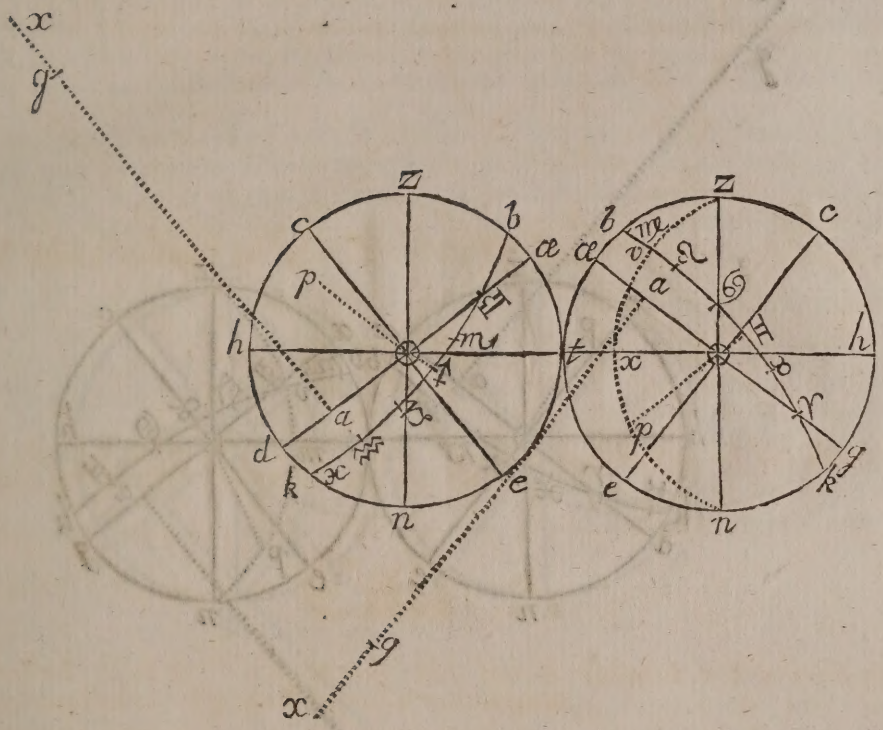
VARIETY 1. R.A. of M.C. 30 Degrees.



When the R. A. of M. C. is 30 Degrees, or any Thing under 90 Degree, this Figure may be ſufficient to work with, if you let $r æ$ repreſent the R. A of M. C. then will b repreſent the Place of M. C. in the Ecliptick, and $æ b$ its Declination North ; $r b æ$ is the Meridian Angle $= z b v$; v repreſents the *Nonageſſima* Degree, and $v m$ its Altitude, $v b$ repreſents the Diſtance of the *Nonageſſima* Degree from the M. C. which, in this *Variety*, muſt be added to the Place of M. C.

VARIETY

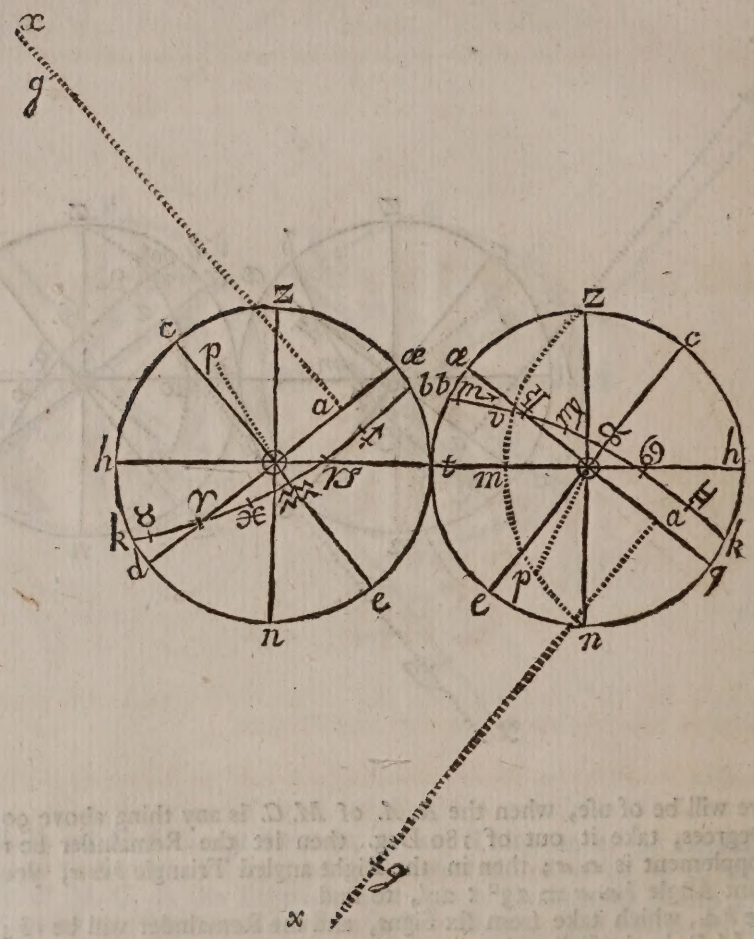
VARIETY 2. R. A. of M. C. equal 150.



THIS Figure will be of uſe, when the *R. A.* of *M. C.* is any thing above 90 Degrees, and under 180 Degrees, take it out of 180 Deg. then let the Remainder be repreſented by $r æ$, whoſe Supplement is $\triangle æ$; then in the Right angled Triangle $b \triangle æ$, you'll have $\triangle æ$, and the conſtant Angle $b \triangle æ = 23° : 29'$, to find

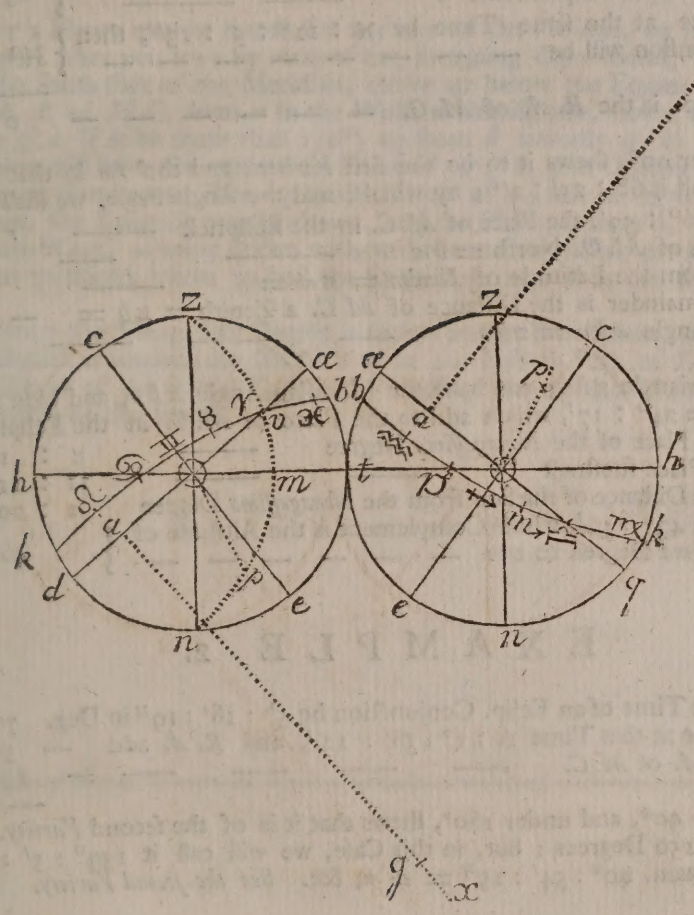
1. The Side $b \triangle$, which take from ſix Signs, and the Remainder will be rb; b is the Place of the *M. C.* in the Ecliptick.
2. The Declination of *M. C.* $= b æ$.
3. The Meridian Angle $\triangle b æ = z b v$. Now in the Triangle $z b v$, you will have $z b$, and $z b v$, to find
4. The Side $b v$, which is the Diſtance of the *Nonageſſima* Degree from the *M. C.* and muſt be ſubſtracted from the Place of *M. C.* and you'll have the Place of the *Nonageſſima* Degree at v.
5. The Side $z v$, whoſe Complement is $v x$, is the Altitude of the *Nonageſſima* Degree.

VARIETY 3. R.A. of M.C. 210.



THIS Figure may ſerve when the *R. A.* of *M. C.* is above 180 Deg. and under 270 Deg. take 180° out of it, and the Remainder may be repreſented by ♎*æ*; then in the Triangle ♎*bæ*, you'll have ♎*æ*, and the Angle *b*♎*æ* = 23° : 29′, to find the Declination of *M. C.* South; which add to *æz*, the Latitude = 51° : 32′, and the Sum will be the Diſtance of *M. C.* from the Zenith = *zb*. The Meridian Angle is ♎*bæ*, and the Side *b*♎, added to fix Signs will be the Place of *M. C.* in the Ecliptick, at *b*; then in the Triangle *zbv*, you'll have *zbv* and *zb*, to find *bv* and *zv*, whoſe Complement is *vm* = the Altitude of the *Nonageſima* Degree.

VARIETY 4. R. A. of M. C. 330 Deg.



THIS Figure may ſerve when the *R. A.* of *M. C.* is more than 270°, and under 360°: Then what it wants of 360°, may be repreſented by *r æ*. So in the Triangle *r æ b*, you'll have *r æ*, and the Angle *æ r b*, to find *r b*; which take out of twelve Signs, and what remains will be the Place of *M. C.* in the Ecliptick at *b*. The Declination of *M. C.* is *æ b*, and Diſtance from the Zenith *z b*; the Meridian Angle is *z b v*: By theſe you may find the Diſtance of the *Nonageſſima* Degree at *v*, from the *M. C.* at *b*; and *v m* the Altitude of the *Nonageſſima* Degree, being the Complement of *z v*. Thus,

If the *R. A.* of *M. C.* was 330°, then $360° - 330° = 30° = r æ$, &c.

ALTHOUGH what has been ſaid, under the Figures of each *Variety*, may be ſufficient for ſome; yet, perhaps, others may not underſtand my Meaning ſo well, without being farther informed: To this End, I have aſſumed Right Aſcenſions of *M. C.* different from thoſe given at each *Variety*, to ſhew that the ſame Figures of any of the *Varieties* may ſerve under ſuch and ſuch Limitations, without being obliged to draw a new one for every Caſe that might happen.

EXAMPLE

EXAMPLE 1.

LET the apparent Time of an Ecliptick Conjunction by $1^h : 35' : 27''$, this in Degrees of the Equinoctial is ——— ——— } $23° : 51' : 46''$

Let the Sun's Place at the ſame Time be ♈ : 11° : 5′ : 13″, then his Right Aſcenſion will be — — — — — — — — } $342 : 33 : 16$

The Sum want. 360. is the *R. A.* of *M. C.* — — — — — — $6 : 25 : 2$

THIS being under 90°, ſhews it to be the firſt *Variety*; and tho' *r æ* in that Figure is 30°, yet now we may call it $6° : 25' : 2''$; by which, and the Angle $b \triangleq æ$, we ſhall find

1. The Side $r b = 6° : 50'$, the Place of *M. C.* in the Ecliptick ——— $r : 6° : 59'$
2. The Declination of *M. C.* North $= b æ$ ——— ——— $2 : 47$
   Take this from the Latitude of *London* $= z æ =$ ——— $51 : 32$
   And the Remainder is the Diſtance of *M. C.* a Zenith $= z b =$ — $48 : 45$
3. The Meridian Angle $æ b r = z b v$ ——— ——— ——— $66 : 41$

NOW, in the Triangle $z b v$, you have the Meridian Angle $z b v$, and Side $z b$; by theſe you will find $b v = 24° : 17'$, which add to the Place of *M. C.* in the Ecliptick at *b*, and the Sun will be the Place of the *Nonageſſima* Degree ——— $8 : 1° : 16'$

The Sun's Place ſubſtract ——— ——— $11 : 11 : 5 : 13$

Remains the Diſtance of the Sun from the *Nonageſſima* Degree $1 : 20 : 10 : 47$

4. The Side $z v = 43° : 40'$, whoſe Complement is the Altitude of the *Nonageſſima* Degree $= v m$ — — — — — — } $46 : 20$

EXAMPLE 2.

LET the apparent Time of an Eclip. Conjunction be $5^h : 18' : 19''$ in Deg. $79° : 34' : 45''$

The Sun's true Place at that Time ♊ : 1° : 38′ : 12″, and *R. A.* add — $59 : 31 : 00$

The Sum is the *R. A.* of *M. C.* ——— ——— ——— ——— — $139 : 5 : 45$

This being above 90°, and under 180°, ſhews that it is of the ſecond *Variety*. Where, in that Figure, *r æ* is 150 Degrees; but, in this Caſe, we will call it $139° : 5' : 45''$: Take this from 180°, remain. $40° : 54' : 15'' = \triangleq æ$, &c. *See the ſecond Variety.*

THESE two Examples, together with what is ſaid under the Figures of the Third and Fourth *Varieties*, will be enough to know how to find the Requiſites in any Caſe.

HAVING thus found the Diſtance of the Sun from the *Nonageſſima* Degree, and the Altitude of the *Nonageſſimal* Degree, together with the Horizontal Parallax of the Moon from the Sun, the next Buſineſs will be to find the Parallax of the Moon, both in Longitude and Latitude; and having Examples of this by *Street*'s Logiſtical Logarithms, in other Places of this Book, you may do it by *Shakerley*'s. Thus,

To find the Parallax of Longitude.

ADD together the Logiſtical Logarithm of the Horizontal Parallax of the Moon from the Sun, the Sine of the Altitude of the *Nonageſſima* Degree, and the Sine of the Diſtance of the Sun from the *Nonageſſima* Degree; the Sum, ſubſtracting double the Radius, is the Logiſtical Logarithm of the Parallax in Longitude.

To find the Parallax of Latitude.

ADD the Logiſtical Logarithm of the Horizontal Parallax of the Moon, from the Sun to the Co-ſine of the Altitude of the *Nonageſſima* Degree, the Sum is the Logiſtical Logarithm of the Parallax of Latitude.

Note, THAT a Figure may be drawn for any one of the *Varieties*, by Hand upon a Slate, or Piece of Paper, after you have by one of the foregoing *Caſes* found, whether the Ecliptick will cut the South Part of the Meridian, above or below the Equinoctial; if you begin to count the *R. A.* of *M. C.* from ♈ in the Weſtern Hemiſphere, down towards *q*; and ſo upon Occaſion (*i. e.* if it be more than 110°) up from *d* towards ♈, in the Eaſtern Hemiſphere; then you will have the equinoctial Point of ♈, in that Quarter of the Equinoctial where your Count terminates: The equinoctial Point of ♎, will be in the oppoſite Quarter. This being done, the Ecliptick may be drawn, and the Triangle form'd, *&c.* Now, though this be but Gueſs-Work, as being drawn without Scale or Compaſſes, yet a Figure thus made will be ſufficient to inform where to find the Requiſites; as well as one drawn more exact.

Note alſo, THAT the *Nonageſſima* Degree is always in that Hemiſphere, in which the convex Side of the Ecliptick is towards the Zenith; as in the Eaſtern Part of *Variety* the firſt, the Weſtern Hemiſphere of the ſecond *Variety*, the Weſtern Hemiſphere of the third *Variety*, and Eaſtern Hemiſphere of the fourth.

Aſtronomical Tables

OF THE

Motion of the SUN and MOON,

According to the laſt Improvement of

The Newtonian Theory.

By which may be found their true PLACES for any TIME, and alſo their ECLIPSES.

A TABLE of the Equation of Time: Found by the Sun's Place.

Add to the equal, or subſtract from the apparent Time.

☉ in	♈ : ♎	♉ : ♏	♊ : ♐	☉ in
	′ : ″	′ : ″	′ : ″	
0	0 : 0	8 : 24	8 : 46	30
1	0 : 20	8 : 35	8 : 36	29
2	1 : 40	8 : 45	8 : 25	28
3	1 : 0	8 : 54	8 : 14	27
4	1 : 19	9 : 3	8 : 1	26
5	1 : 39	9 : 11	7 : 49	25
6	1 : 59	9 : 18	7 : 35	24
7	2 : 18	9 : 25	7 : 21	23
8	2 : 37	9 : 31	7 : 6	22
9	2 : 57	9 : 36	6 : 51	21
10	3 : 16	9 : 41	6 : 35	20
11	3 : 34	9 : 45	6 : 19	19
12	3 : 53	9 : 49	6 : 2	18
13	4 : 11	9 : 51	5 : 45	17
14	4 : 29	9 : 53	5 : 27	16
15	4 : 47	9 : 55	5 : 9	15
16	5 : 4	9 : 55	4 : 50	14
17	5 : 21	9 : 55	4 : 31	13
18	5 : 38	9 : 54	4 : 12	12
19	5 : 54	9 : 52	3 : 52	11
20	6 : 10	9 : 50	3 : 32	10
21	6 : 26	9 : 47	3 : 12	9
22	6 : 41	9 : 43	2 : 51	8
23	6 : 55	9 : 38	2 : 30	7
24	7 : 10	9 : 33	2 : 9	6
25	7 : 23	9 : 27	1 : 48	5
26	7 : 36	9 : 20	1 : 27	4
27	7 : 49	9 : 13	1 : 5	3
28	8 : 1	9 : 5	0 : 43	2
29	8 : 13	8 : 56	0 : 22	1
30	8 : 24	8 : 46	0 : 0	0
	′ : ″	′ : ″	′ : ″	
☉ in	♓ : ♍	♒ : ♎	♑ : ♑	☉ in

Subſtract from the equal Time, or add to the apparent.

The

The Second PART of the Equation of Time, Found by the Sun's Mean Anomaly.

Mean Anom. ☉	Add to the equal Time, or subſtract from the apparent.						Mean Anom. ☉
	♈ . 0	♉ . 1	♊ . 2	♋ . 3	♌ . 4	♍ . 5	
	′ : ″	′ : ″	′ : ″	′ : ″	′ : ″	′ : ″	
0	0 : 0	3 : 46	6 : 34	7 : 40	6 : 42	3 : 54	30
1	0 : 8	3 : 53	6 : 38	7 : 40	6 : 38	3 : 47	29
2	0 : 16	3 : 59	6 : 42	7 : 40	6 : 34	3 : 40	28
3	0 : 23	4 : 6	6 : 46	7 : 40	6 : 30	3 : 33	27
4	0 : 31	4 : 13	6 : 49	7 : 39	6 : 26	3 : 25	26
5	0 : 39	4 : 19	6 : 53	7 : 39	6 : 21	3 : 18	25
6	0 : 47	4 : 26	6 : 57	7 : 38	6 : 17	3 : 11	24
7	0 : 55	4 : 32	7 : 0	7 : 38	6 : 12	3 : 3	23
8	1 : 3	4 : 38	7 : 3	7 : 37	6 : 7	2 : 56	22
9	1 : 10	4 : 45	7 : 6	7 : 36	6 : 2	2 : 48	21
10	1 : 18	4 : 51	7 : 9	7 : 34	5 : 57	2 : 40	20
11	1 : 26	4 : 57	7 : 12	7 : 33	5 : 52	2 : 33	19
12	1 : 34	5 : 3	7 : 14	7 : 32	5 : 43	2 : 25	18
13	1 : 41	5 : 9	7 : 17	7 : 30	5 : 41	2 : 17	17
14	1 : 49	5 : 15	7 : 19	7 : 28	5 : 36	2 : 9	16
15	1 : 57	5 : 20	7 : 22	7 : 26	5 : 30	2 : 1	15
16	2 : 4	5 : 26	7 : 24	7 : 24	5 : 24	1 : 53	14
17	2 : 12	5 : 32	7 : 26	7 : 22	5 : 18	1 : 45	13
18	2 : 19	5 : 37	7 : 28	7 : 20	5 : 12	1 : 37	12
19	2 : 27	5 : 42	7 : 30	7 : 18	5 : 6	1 : 25	11
20	2 : 34	5 : 48	7 : 31	7 : 15	5 : 0	1 : 21	10
21	2 : 42	5 : 53	7 : 33	7 : 12	4 : 54	1 : 13	9
22	2 : 49	5 : 58	7 : 34	7 : 10	4 : 48	1 : 5	8
23	2 : 56	6 : 3	7 : 35	7 : 7	4 : 41	0 : 57	7
24	3 : 3	6 : 7	7 : 36	7 : 4	4 : 35	0 : 49	6
25	3 : 11	6 : 12	7 : 37	7 : 0	4 : 28	0 : 41	5
26	3 : 18	6 : 17	7 : 38	6 : 57	4 : 22	0 : 33	4
27	3 : 25	6 : 21	7 : 39	6 : 54	4 : 15	0 : 24	3
28	3 : 32	6 : 26	7 : 40	6 : 50	4 : 8	0 : 16	2
29	3 : 39	6 : 30	7 : 40	6 : 46	4 : 1	0 : 8	1
30	3 : 46	6 : 34	7 : 40	6 : 42	3 : 54	0 : 0	0
Mean Anom. ☉	♐ . 11	♑ . 10	♒ . 9	♓ . 8	♎ . 7	♏ . 6	Mean Anom. ☉
	Subſtract from the equal Time, or add to the apparent.						

A TABLE of the Sun's Mean Motion in Years.

For Years compleat.

Years.	Long. ⊙. ∫ : ° : ′ : ″	Anom. ⊙. ∫ : ° : ′ : ″
1	11 29 45 40	11 29 44 37
2	11 29 31 20	11 29 29 14
3	11 29 17 0	11 29 13 51
B 4	0 0 1 49	11 29 57 37
5	11 29 47 29	11 29 42 14
6	11 29 33 9	11 29 26 51
7	11 29 18 49	11 29 11 28
B 8	0 0 3 37	11 29 55 13
9	11 29 49 18	11 29 39 51
10	11 29 34 58	11 29 24 28
11	11 29 20 38	11 29 9 5
B 12	0 0 5 26	11 29 52 50
13	11 29 51 6	11 29 37 27
14	11 29 36 47	11 29 22 5
15	11 29 22 27	11 29 6 42
B 16	0 0 7 15	11 29 50 27
17	11 29 52 55	11 29 35 4
18	11 29 38 35	11 29 19 41
19	11 29 24 15	11 29 4 18
B 20	0 0 9 4	11 29 48 4
40	0 0 18 8	11 29 36 8
60	0 0 27 12	11 29 24 12
80	0 0 36 16	11 29 12 16
100	0 0 45 20	11 29 0 20
200	0 1 30 40	11 28 0 40
300	0 2 16 0	11 27 1 0
400	0 3 1 20	11 26 1 20
500	0 3 46 40	11 25 1 40
600	0 4 32 0	11 24 2 0
700	0 5 17 20	11 23 2 20
800	0 6 2 40	11 22 2 40
900	0 6 48 0	11 21 3 0
1000	0 7 33 20	11 20 3 20
2000	0 15 6 40	11 10 6 40
3000	0 22 40 0	11 0 10 0
4000	1 0 13 20	10 20 13 20
5000	1 7 46 40	10 10 16 40

For Years of CHRIST current.

Years.	Long. ⊙. ∫ : ° : ′ : ″	Anom. ⊙. ∫ : ° : ′ : ″
1	9 7 53 10	7 0 0 40
101	9 8 58 30	6 29 1 0
201	9 9 23 50	6 28 1 20
301	9 10 9 10	6 27 1 40
401	9 10 54 30	6 26 2 0
501	9 11 39 50	6 25 2 20
601	9 12 25 10	6 24 2 40
701	9 13 10 30	6 23 3 0
801	9 13 15 50	6 22 3 20
901	9 14 41 10	6 21 3 40
1001	9 15 26 30	6 20 4 0
1101	9 16 11 50	6 19 4 20
1201	9 16 57 10	6 18 4 40
1301	9 17 42 30	6 17 5 0
1401	9 18 27 50	6 16 5 20
1501	9 19 13 10	6 15 5 40
1601	9 19 58 30	6 14 6 0
1621	9 20 7 34	6 13 54 24
1641	9 20 16 38	6 13 42 28
1661	9 20 25 42	6 13 30 12
1681	9 20 34 46	6 13 18 16
1701	9 20 43 50	6 13 6 20
1721	9 20 52 54	6 12 54 24
1722	9 20 38 34	6 12 39 01
1723	9 20 24 14	6 12 23 38
1724	9 20 9 54	6 12 8 15
1725	9 20 54 43	6 12 52 01
1726	9 20 40 23	6 12 36 38
1727	9 20 26 3	6 12 21 15
1728	9 20 11 43	6 12 5 52
1729	9 20 56 32	6 12 49 37
1730	9 20 42 12	6 12 34 14
1731	9 20 27 52	6 12 18 51
1732	9 20 13 32	6 12 3 28
1733	9 20 58 21	6 12 47 13
1734	9 20 44 01	6 12 31 50
1735	9 20 29 41	6 12 16 27

The Sun's Mean Motion in Years.

Years.	Long. ☉.				Anom. ☉.			
	ſ	°	′	″	ſ	°	′	″
1736	9	20	15	21	6	12	1	12
1737	9	21	0	10	6	12	44	57
1738	9	20	45	50	6	12	29	34
1739	9	20	31	30	6	12	14	11
1740	9	20	17	10	6	11	58	48
1741	9	21	1	58	6	12	42	28
1742	9	20	47	38	6	12	27	5
1743	9	20	33	18	6	12	11	42
1744	9	20	18	58	6	11	56	19
1745	9	21	3	48	6	12	40	7
1746	9	20	49	27	6	12	24	41
1747	9	20	35	7	6	12	9	18
1748	9	20	20	47	6	11	53	55
1749	9	21	5	36	6	12	37	44
1750	9	20	51	16	6	12	22	17
1751	9	20	36	56	6	12	6	54
1752	9	20	22	36	6	11	51	31
1753	9	21	7	25	6	12	35	16
1754	9	20	53	5	6	12	19	53
1755	9	20	38	45	6	12	4	30
1756	9	20	24	25	6	11	49	7
1757	9	21	9	14	6	12	32	52
1758	9	20	54	54	6	12	17	29
1759	9	20	40	34	6	12	2	6
1760	9	20	26	14	6	11	46	43
1761	9	21	11	2	6	12	30	32
1762	9	20	56	44	6	12	15	9
1763	9	20	42	24	6	11	59	46
1764	9	20	28	4	6	11	44	23
1765	9	21	12	52	6	12	28	9
1766	9	20	58	32	6	12	12	46
1767	9	20	44	12	6	11	57	23
1768	9	20	29	52	6	11	42	0
1769	9	21	14	42	6	12	25	45
1770	9	21	0	22	6	12	10	23
1771	9	20	46	2	6	11	55	0
1772	9	20	31	42	6	11	39	37
1773	9	21	16	31	6	12	23	22
1774	9	21	2	8	6	12	7	59
1775	9	20	47	49	6	11	52	37
5776	9	20	33	29	6	11	37	14
1777	9	21	18	17	6	12	20	59
1778	9	21	3	57	6	12	5	36
1779	9	20	49	37	6	11	50	13
1780	9	20	35	17	6	11	34	50
1781	9	21	20	6	6	12	18	36
1782	9	21	5	46	6	12	3	13
1783	9	20	51	26	6	11	47	50
1784	9	20	37	6	6	11	32	7
1785	9	21	21	55	6	12	16	13
1786	9	21	7	35	6	12	0	50
1787	9	20	53	15	6	11	45	27
1788	9	20	38	55	6	11	30	4
1789	9	21	23	43	6	11	13	49
1790	9	21	9	24	6	11	58	27
1791	9	20	55	4	6	11	43	4
1792	9	20	40	44	6	11	27	41
1793	9	21	25	32	6	12	11	26
1794	9	21	11	12	6	11	56	3
1795	9	20	56	53	6	11	40	41
1796	9	20	42	33	6	11	25	18
1797	9	21	27	21	6	12	9	3
1798	9	21	13	1	6	11	53	4
1799	9	20	58	41	6	11	38	17
1800	9	20	44	21	6	11	22	54
1801	9	21	29	10	6	12	6	40
1802	9	21	14	50	6	11	51	17
1803	9	21	0	30	6	11	35	54
1804	9	20	46	10	6	11	20	31
1805	9	21	30	59	6	12	4	17
1806	9	21	16	39	6	11	48	54
1807	9	21	2	19	6	11	33	31
1808	9	20	47	59	6	11	18	8
1809	9	21	32	47	6	12	1	53
1810	9	21	18	28	6	11	46	31
1811	9	21	4	8	6	11	31	8

The Sun's Mean Motion in Years.

Years.	Long. ☉.				Anom. ☉.			
	s	°	′	″	*s*	°	′	″
1812	9	20	49	48	6	11	15	45
1813	9	21	34	36	6	11	59	30
1814	9	21	20	16	6	11	44	7
1815	9	21	5	57	6	11	28	45
1816	9	20	51	37	6	11	13	22
1817	9	21	36	25	6	11	57	7
1818	9	21	20	5	6	11	41	44
1819	9	21	7	45	6	11	26	21
1820	9	20	53	25	6	11	10	58
1821	9	21	38	14	6	11	54	35
1822	9	21	23	54	6	11	39	21
1823	9	21	9	34	6	11	23	58
1824	9	20	55	14	6	11	8	35
1825	9	21	40	3	6	11	52	21
1826	9	21	25	43	6	11	36	58
1827	9	21	11	23	6	11	21	35
1828	9	20	57	3	6	11	6	12
1829	9	21	41	51	6	11	49	57
1830	9	21	28	32	6	11	34	35
1831	9	21	13	12	6	11	19	12
1832	9	20	58	52	6	11	3	49
1833	9	21	43	40	6	11	47	34
1834	9	21	29	20	6	11	32	11
1835	9	21	15	1	6	11	16	49
1836	9	21	0	41	6	11	1	26
1837	9	21	45	29	6	11	45	11
1838	9	21	31	9	6	11	29	48
1839	9	21	16	49	6	11	14	25
1840	9	21	2	29	6	10	59	2
1841	9	21	47	18	6	11	42	48
1842	9	21	32	58	6	11	27	25
1843	9	21	18	38	6	11	12	2
1844	9	21	4	18	6	10	56	39
1845	9	21	49	7	6	11	40	25
1846	9	21	35	47	6	11	24	2
1847	9	21	21	27	6	11	9	39
1848	9	21	6	7	6	10	54	16
1849	9	21	50	55	6	11	38	1
1850	9	21	36	36	6	11	22	30
1851	9	21	22	16	6	11	7	16
1852	9	21	7	56	6	10	51	53
1853	9	21	52	44	6	11	35	38
1854	9	21	38	24	6	11	20	15

The

The Sun's Mean Motion in Months and Days.

Common Year.	January. Long. ⊙.				January. Anom. ⊙.				February. Long. ⊙.				February. Anom. ⊙.				Bissex.
	∫	°	′	″	∫	°	′	″	∫	°	′	″	∫	°	′	″	
1	0	0	59	8	0	0	59	8	1	1	32	26	1	2	31	30	0
2	0	1	58	17	0	1	58	17	1	2	31	35	1	2	31	30	1
3	0	2	57	25	0	2	57	25	1	3	30	43	1	3	30	38	2
4	0	3	56	33	0	3	56	33	1	4	29	51	1	4	29	46	3
5	0	4	55	42	0	4	55	42	1	5	29	0	1	5	28	56	4
6	0	5	54	50	0	5	54	50	1	6	28	8	1	6	28	3	5
7	0	6	53	58	0	6	53	57	1	7	27	16	1	7	27	11	6
8	0	7	53	7	0	7	53	6	1	8	26	25	1	8	26	10	7
9	0	8	52	15	0	8	52	14	1	9	25	33	1	9	25	27	8
10	0	9	51	23	0	9	51	22	1	10	24	41	1	10	24	35	9
11	0	10	50	31	0	10	50	30	1	11	23	49	1	11	23	43	10
12	0	11	49	40	0	11	49	39	1	12	22	58	1	12	22	52	11
13	0	12	48	48	0	12	48	47	1	13	22	6	1	13	22	0	12
14	0	13	47	57	0	13	47	55	1	14	21	15	1	14	21	9	13
15	0	14	47	5	0	14	47	3	1	15	20	23	1	15	20	17	14
16	0	15	46	13	0	15	46	10	1	16	19	31	1	16	19	24	15
17	0	16	45	22	0	16	45	9	1	17	18	40	1	17	18	32	16
18	0	17	44	30	0	17	44	27	1	18	17	48	1	18	17	40	17
19	0	18	43	38	0	18	43	35	1	19	16	56	1	19	16	48	18
20	0	19	42	47	0	19	42	44	1	20	16	5	1	20	15	57	19
21	0	20	41	55	0	20	41	52	1	21	15	13	1	21	15	5	20
22	0	21	41	3	0	21	41	0	1	22	14	21	1	22	14	12	21
23	0	22	40	12	0	22	40	8	1	23	13	30	1	23	13	21	22
24	0	23	39	20	0	23	39	16	1	24	12	38	1	24	12	29	23
25	0	24	38	28	0	24	38	24	1	25	11	46	1	25	11	37	24
26	0	25	37	37	0	25	37	33	1	26	10	55	1	26	10	46	25
27	0	26	36	45	0	26	36	41	1	27	10	3	1	27	9	54	26
28	0	27	35	53	0	27	35	48	1	28	9	11	1	28	9	1	27
29	0	28	35	2	0	28	34	56	1	29	8	20	1	29	8	10	28
30	0	29	34	10	0	29	34	5									29
31	1	0	33	18	1	0	33	14									30

The

The Sun's Mean Motion in Months and Days.

Common Year.	March. Long. ⊙.				March. Anom. ⊙.				April. Long. ⊙.				April. Anom. ⊙.				Bissex.
	ʃ	°	′	″	ʃ	°	′	″	ʃ	°	′	″	ʃ	°	′	″	
1	1	29	8	20	1	29	8	10	2	29	41	38	2	29	41	23	0
2	2	0	7	28	2	0	7	18	3	0	40	46	3	0	40	32	1
3	2	1	6	36	2	1	6	26	3	1	39	53	3	1	39	40	2
4	2	2	5	44	2	2	5	33	3	2	39	3	3	2	38	48	3
5	2	3	4	53	2	3	4	42	3	3	38	12	3	3	37	56	4
6	2	4	4	1	2	4	3	50	3	4	37	20	3	4	37	4	5
7	2	5	3	9	2	5	2	58	3	5	36	28	3	5	36	12	6
8	2	6	2	18	2	6	2	7	3	6	35	37	3	6	35	20	7
9	2	7	1	26	2	7	1	15	3	7	34	45	3	7	34	28	8
10	2	8	0	34	2	8	0	22	3	8	33	53	3	8	33	36	9
11	2	8	59	42	2	8	59	30	3	9	33	1	3	9	32	44	10
12	2	9	58	51	2	9	58	39	3	10	32	10	3	10	31	53	11
13	2	10	57	59	2	10	57	47	3	11	31	18	3	11	31	1	12
14	2	11	57	8	2	11	56	56	3	12	30	27	3	12	30	10	13
15	2	12	56	16	2	12	56	4	3	13	29	35	3	13	29	18	14
16	2	13	55	24	2	13	55	11	3	14	28	43	3	14	28	26	15
17	2	14	54	33	2	14	54	20	3	15	27	52	3	15	27	34	16
18	2	15	53	41	2	15	53	28	3	16	27	0	3	16	26	42	17
19	2	16	52	49	2	16	52	36	3	17	26	8	3	17	25	50	18
20	2	17	51	48	2	17	51	45	3	18	25	17	3	18	24	58	19
21	2	18	51	6	2	18	50	53	3	19	24	25	3	19	24	7	20
22	2	19	50	14	2	19	50	0	3	20	23	33	3	20	23	15	21
23	2	20	49	23	2	20	49	9	3	21	22	42	3	21	22	23	22
24	2	21	48	31	2	21	48	17	3	22	21	50	3	22	21	31	23
25	2	22	47	39	2	22	47	25	3	23	20	58	3	23	20	39	24
26	2	23	46	48	2	23	46	34	3	24	20	7	3	24	19	48	25
27	2	24	45	56	2	24	45	42	3	25	19	15	3	25	18	56	26
28	2	25	45	4	2	25	44	49	3	26	18	23	3	26	18	4	27
29	2	26	44	13	2	26	43	58	3	27	17	32	3	27	17	12	28
30	2	27	43	21	2	27	43	6	3	28	16	40	3	28	16	30	29
31	2	28	42	29	2	28	42	15									
									3	29	15	48	3	29	15	28	30

The Sun's Mean Motion in Months and Days.

Common Year.	May. Long. ☉.				May. Anom. ☉.				June. Long. ☉.				June. Anom. ☉.				Bissextile.
	f	☌	′	″	*f*	☌	′	″	*f*	☌	′	″	*f*	☌	′	″	
1	3	29	15	48	3	29	15	28	4	29	49	6	4	29	48	41	0
2	4	0	14	56	3	0	14	36	5	0	48	14	5	0	47	50	1
3	4	1	14	4	4	1	13	44	5	1	47	23	5	1	46	58	2
4	4	2	13	12	4	2	12	51	5	2	46	31	5	2	46	6	3
5	4	3	12	21	4	3	12	0	5	3	45	40	5	3	45	14	4
6	4	4	11	29	4	4	11	8	5	4	44	48	5	4	44	22	5
7	4	5	10	37	4	5	10	6	5	5	43	56	5	5	43	31	6
8	4	6	9	46	4	6	9	25	5	6	43	5	5	6	42	39	7
9	4	7	8	54	4	7	8	33	5	7	42	13	5	7	41	47	8
10	4	8	8	2	4	8	7	40	5	8	41	21	5	8	40	55	9
11	4	9	7	10	4	9	6	48	5	9	40	29	5	9	40	3	10
12	4	10	6	19	4	10	5	57	5	10	39	38	5	10	39	11	11
13	4	17	5	27	4	11	5	5	5	11	38	46	5	11	38	19	12
14	4	12	4	36	4	12	4	14	5	12	37	55	5	12	37	27	13
15	4	13	3	44	4	13	3	22	5	13	37	3	5	13	36	35	14
16	4	14	2	52	4	14	2	29	5	14	36	11	5	14	35	43	15
17	4	15	2	1	4	15	1	38	5	15	35	20	5	15	34	52	16
18	4	16	1	9	4	16	0	46	5	16	34	28	5	16	34	0	17
19	4	17	0	17	4	16	59	54	5	17	33	36	5	17	33	8	18
20	4	17	59	26	4	17	59	3	5	18	32	45	5	18	32	17	19
21	4	18	58	34	4	18	58	11	5	19	31	53	5	19	31	25	20
22	4	19	57	42	4	19	57	18	5	20	31	1	5	20	30	32	21
23	4	20	56	51	4	20	56	27	5	21	30	10	5	21	29	30	22
24	4	21	55	59	4	21	55	35	5	22	29	18	5	22	28	49	23
25	4	22	55	7	4	22	54	43	5	23	28	26	5	23	27	57	24
26	4	23	54	16	4	23	53	52	5	24	27	35	5	24	27	6	25
27	4	24	53	24	4	24	53	0	5	25	26	43	5	25	26	14	26
28	4	25	52	32	4	25	52	7	5	26	25	51	5	26	25	22	27
29	4	26	51	41	4	26	51	16	5	27	25	0	5	27	24	30	28
30	4	27	50	49	4	27	50	24	5	28	24	8	5	28	23	38	29
31	4	28	49	57	4	28	49	32	5	29	23	16	5	29	22	46	30

The

The Sun's Mean Motion in Months and Days.

Common Year.	July. Long. ⊙.				July. Anom. ⊙.				August. Long. ⊙.				August. Anom. ⊙.				Biſſextile.
	ſ	°	′	″	ſ	°	′	″	ſ	°	′	″	ſ	°	′	″	
1	5	29	23	16	5	29	22	46	6	29	56	33	6	29	55	57	0
2	6	0	22	14	6	0	21	53	7	0	55	41	7	0	55	6	1
3	6	1	21	32	6	1	21	1	7	1	54	50	7	1	54	14	2
4	6	2	20	40	6	2	20	9	7	2	53	58	7	2	53	32	3
5	6	3	19	49	6	3	19	17	7	3	53	7	7	3	52	3	4
6	6	4	18	57	6	4	18	25	7	4	52	15	7	4	51	37	5
7	6	5	18	5	6	5	17	33	7	5	51	23	7	5	50	45	6
8	6	6	17	14	6	6	16	42	7	6	50	32	7	6	49	53	7
9	6	7	16	22	6	7	15	50	7	7	49	40	7	7	49	2	8
10	6	8	15	30	6	8	14	58	7	8	48	48	7	8	48	10	9
11	6	9	14	38	6	9	14	6	7	9	47	56	7	9	47	18	10
12	6	10	13	47	6	10	13	14	7	10	47	5	7	10	46	26	11
13	6	11	12	55	6	11	12	22	7	11	46	13	7	11	45	34	12
14	6	12	12	4	6	12	11	31	7	12	45	22	7	12	44	43	13
15	6	13	11	12	6	13	10	39	7	13	44	30	7	13	43	51	14
16	6	14	10	20	6	14	9	47	7	14	43	38	7	14	52	58	15
17	6	15	9	29	6	15	8	55	7	15	42	47	7	15	42	7	16
18	6	16	8	37	6	16	8	3	7	16	41	55	7	16	41	15	17
19	6	17	7	45	6	17	7	11	7	17	41	3	7	17	40	23	18
20	6	18	6	54	6	18	6	20	7	18	40	12	7	18	39	32	19
21	6	19	6	2	6	19	5	28	7	19	39	20	7	19	38	40	20
22	6	20	5	10	6	20	4	36	7	20	38	28	7	20	37	48	21
23	6	21	4	19	6	21	3	34	1	21	37	37	7	21	36	56	22
24	6	22	3	27	6	22	2	52	7	22	36	45	7	22	36	4	23
25	6	23	2	35	6	23	2	0	7	23	35	53	7	23	35	12	24
26	6	24	1	44	6	24	1	9	7	24	35	2	7	24	34	21	25
27	6	25	0	52	6	25	0	17	7	25	34	10	7	25	33	29	26
28	6	26	0	0	6	25	59	25	7	26	33	18	7	26	32	37	27
29	6	26	59	9	6	26	58	33	7	27	32	27	7	27	31	45	28
30	6	27	58	17	6	27	57	41	7	28	31	35	7	28	30	33	29
31	6	28	57	25	6	28	56	49	7	29	30	43	7	29	30	1	30

The Sun's Mean Motion in Months and Days.

Common Years.	September.								October.								Bissextile.
	Long. ☉.				Anom. ☉.				Long. ☉.				Anom. ☉.				
	ſ	°	′	″	ſ	°	′	″	ſ	°	′	″	ſ	°	′	″	
1	8	0	29	52	8	0	29	10	9	0	4	2	9	0	3	17	0
2	8	1	29	0	8	1	28	18	9	1	3	11	9	1	2	15	1
3	8	2	28	9	8	2	27	26	9	2	2	19	9	2	1	33	2
4	8	3	27	17	8	3	26	35	9	3	1	27	9	3	0	41	3
5	8	4	26	26	8	4	25	44	9	4	0	36	9	3	59	50	4
6	8	5	25	34	8	5	24	52	9	4	59	44	9	4	58	58	5
7	8	6	24	42	8	6	24	0	9	5	58	52	9	5	58	6	6
8	8	7	23	51	8	7	23	8	9	6	58	1	9	6	57	14	7
9	8	8	22	59	8	8	22	16	9	7	57	9	9	7	56	22	8
10	8	9	22	7	8	9	21	24	9	8	56	17	9	8	55	28	9
11	8	10	21	15	8	10	20	32	9	9	55	25	9	9	54	36	10
12	8	11	20	24	8	11	19	41	9	10	54	34	9	10	53	45	11
13	8	12	19	32	8	12	18	50	9	11	53	42	9	11	52	53	12
14	8	13	18	40	8	13	17	57	9	12	52	49	9	12	52	2	13
15	8	14	17	48	8	14	17	5	9	13	51	58	9	13	51	11	14
16	8	15	16	57	8	15	16	30	9	14	51	7	9	14	50	19	15
17	8	16	16	6	8	16	15	22	9	15	50	16	9	15	49	28	16
18	8	17	15	14	8	17	14	30	9	16	49	24	9	16	48	36	17
19	8	18	14	22	8	18	13	38	9	17	48	32	9	17	47	44	18
20	8	19	13	31	8	19	12	47	9	18	47	41	9	18	46	52	19
21	8	20	12	39	8	20	11	55	9	19	46	49	9	19	46	1	20
22	8	21	11	47	8	21	11	3	9	20	45	57	9	20	45	9	21
23	8	22	10	56	8	22	10	11	9	21	45	6	9	21	44	17	22
24	8	23	10	4	8	23	9	19	9	22	44	14	9	22	43	25	23
25	8	24	9	12	8	24	8	27	9	23	43	22	9	23	42	33	24
26	8	25	8	21	8	25	7	35	9	24	42	31	9	24	41	41	25
27	8	26	7	29	8	26	6	44	9	25	41	39	9	25	40	49	26
28	8	27	6	37	8	27	5	52	9	26	40	47	9	26	39	57	27
29	8	28	5	46	8	28	5	1	9	27	39	56	9	27	39	5	28
30	8	29	4	54	8	29	4	9	9	28	39	4	9	28	38	13	29
31	9	0	4	2	9	0	3	17	9	29	38	12	9	29	37	20	30

The Sun's Mean Motion in Months and Days.

Common Year.	November.								December.								Bissex.
	Long. ☉.				Anom. ☉.				Long. ☉.				Anom. ☉.				
	∫	°	′	″	∫	°	′	″	∫	°	′	″	∫	°	′	″	
1	10	0	37	20	10	0	36	28	11	0	11	30	11	0	10	32	0
2	10	1	36	29	10	1	35	36	11	1	10	39	11	1	9	40	1
3	10	2	35	37	10	2	34	44	11	2	9	47	11	2	8	48	2
4	10	3	34	45	10	3	34	52	11	3	8	55	11	3	7	56	3
5	10	4	33	54	10	4	33	0	11	4	8	4	11	4	7	4	4
6	10	5	33	2	10	5	32	8	11	5	7	12	11	5	6	12	5
7	10	6	32	10	10	6	31	16	11	6	6	20	11	6	5	20	6
8	10	7	31	19	10	7	30	25	11	7	5	29	11	7	4	28	7
9	10	8	30	27	10	8	29	33	11	8	4	37	11	8	3	36	8
10	10	9	29	35	10	9	28	40	11	9	3	45	11	9	2	44	9
11	10	10	28	43	10	10	27	48	11	10	2	53	11	10	1	52	10
12	10	11	27	52	10	11	26	57	11	11	2	2	11	11	1	0	11
13	10	12	27	0	10	12	26	5	11	12	1	10	11	12	0	8	12
14	10	13	26	9	10	13	25	14	11	13	0	19	11	12	59	17	13
15	10	14	25	17	10	14	24	22	11	13	59	27	11	13	58	25	14
16	10	15	24	25	10	15	23	29	11	14	58	35	11	14	57	33	15
17	10	16	23	34	10	16	22	38	11	15	57	44	11	15	56	42	16
18	10	17	22	42	10	17	21	46	11	16	56	52	11	16	55	50	17
19	10	18	21	50	10	18	20	54	11	17	56	0	11	17	54	58	18
20	10	19	20	59	10	19	20	3	11	18	55	9	11	18	54	6	19
21	10	20	20	7	10	20	19	11	11	19	54	17	11	19	53	14	20
22	10	21	19	15	10	21	18	18	11	20	53	25	11	20	52	22	21
23	10	22	18	24	10	22	17	27	11	21	52	34	11	21	51	30	22
24	10	23	17	32	10	23	16	35	11	22	51	42	11	22	50	39	23
25	10	24	16	40	10	24	15	43	11	23	50	50	11	23	49	47	24
26	10	25	15	49	10	25	14	52	11	24	49	59	11	24	48	55	25
27	10	26	14	57	10	26	14	0	11	25	49	7	11	25	48	3	26
28	10	27	14	5	10	27	13	8	11	26	48	15	11	26	47	11	27
29	10	28	13	14	10	28	12	16	11	27	47	24	11	27	46	19	28
30	10	29	12	22	10	29	11	24	11	28	46	32	11	28	45	28	29
31	11	0	11	30	11	0	10	32	11	29	45	40	11	29	44	37	30

The Sun's Mean Motion in Hours, Minutes, &c.

	Long. ☉.			Anom. ☉.		
H.	°	′	″	°	′	″
′	′	″	‴	′	″	‴
″	″	‴	⁗	″	‴	⁗
1	0	2	28	0	2	28
2	0	4	56	0	4	56
3	0	7	24	0	7	24
4	0	9	51	0	9	51
5	0	12	19	0	12	19
6	0	14	47	0	14	47
7	0	17	15	0	17	15
8	0	19	43	0	19	43
9	0	22	11	0	22	11
10	0	24	38	0	24	38
11	0	27	6	0	27	6
12	0	29	34	0	29	34
13	0	32	2	0	32	2
14	0	34	30	0	34	30
15	0	36	58	0	36	58
16	0	39	25	0	39	25
17	0	41	53	0	41	53
18	0	44	21	0	44	21
19	0	46	49	0	46	49
20	0	49	17	0	49	17
21	0	51	45	0	51	45
22	0	54	13	0	54	13
23	0	56	40	0	56	40
24	0	59	8	0	59	8
25	1	1	36	1	1	36
26	1	4	4	1	4	4
27	1	6	32	1	6	32
28	1	9	0	1	9	0
29	1	11	27	1	11	27
30	1	13	55	1	13	55
31	1	16	23	1	16	23
52	1	18	51	1	18	51
33	1	21	19	1	21	19
34	1	23	47	1	23	47
35	1	26	15	1	26	15
36	1	28	43	1	28	43
37	1	31	11	1	31	11
38	1	33	39	1	33	39
39	1	36	6	1	36	6
40	1	38	44	1	38	44
41	1	41	2	1	41	2
42	1	43	30	1	43	30
43	1	45	58	1	45	58
44	1	48	25	1	48	25
45	1	50	53	1	50	53
46	1	53	21	1	53	21
47	1	55	49	1	55	49
48	1	58	17	1	58	17
49	2	0	44	2	0	44
50	2	3	12	2	3	12
51	2	5	40	2	5	40
52	2	8	8	2	8	8
53	2	10	36	2	10	36
54	2	13	3	2	13	3
55	2	15	31	2	15	31
56	2	17	59	2	17	59
57	2	20	27	2	20	27
58	2	22	55	2	22	55
59	2	25	23	2	25	23
60	2	27	51	2	27	51

A TABLE of the Sun's Equation.

	SUBSTRACT															
Sig.	0. ♈			1. ♉			2. ♊			3. ♋			4. ♌			5. ♍			Sig.
Deg.	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	Deg.
0	0	0	0	0	57	7	1	39	41	1	56	19	1	41	48	0	59	15	30
1	0	1	59	0	58	51	1	40	42	1	56	20	1	40	48	0	57	26	29
2	0	3	59	1	0	33	1	41	41	1	56	19	1	39	46	0	55	37	28
3	0	5	58	1	2	15	1	42	39	1	56	16	1	38	41	0	53	48	27
4	0	7	57	1	3	55	1	43	35	1	56	12	1	37	35	0	51	57	26
5	0	9	56	1	5	35	1	44	29	1	56	5	1	36	27	0	50	5	25
6	0	11	55	1	7	14	1	45	21	1	55	56	1	35	17	0	48	13	24
7	0	13	53	1	8	51	1	46	0	1	55	45	1	34	5	0	46	19	23
8	0	15	51	1	10	27	1	47	0	1	55	31	1	32	52	0	44	25	22
9	0	17	49	1	12	1	1	47	46	1	55	15	1	31	37	0	42	30	21
10	0	19	47	1	13	35	1	48	31	1	54	58	1	30	20	0	40	34	20
11	0	21	45	1	15	7	1	49	13	1	54	38	1	29	1	0	38	37	19
12	0	23	42	1	16	38	1	49	53	1	54	16	1	27	41	0	36	40	18
13	0	25	38	1	18	7	1	50	32	1	53	52	1	26	19	0	34	41	17
14	0	27	37	1	19	36	1	51	9	1	53	26	1	24	55	0	32	42	16
15	0	29	30	1	21	3	1	51	44	1	52	58	1	23	30	0	30	43	15
16	0	31	25	1	22	28	1	52	17	1	52	27	1	22	3	0	28	43	14
17	0	33	20	1	23	51	1	52	48	1	51	55	1	20	35	0	26	42	13
18	0	35	14	1	25	14	1	53	16	1	51	20	1	19	5	0	24	41	12
19	0	37	8	1	26	35	1	53	43	1	50	44	1	17	33	0	22	39	11
20	0	39	1	1	27	55	1	54	7	1	50	5	1	16	0	0	20	37	10
21	0	40	53	1	29	13	1	54	30	1	49	24	1	14	26	0	18	34	9
22	0	42	44	1	30	29	1	54	50	1	48	42	1	12	50	0	16	31	8
23	0	44	35	1	31	43	1	55	9	1	47	57	1	11	13	0	14	28	7
24	0	46	25	1	32	56	1	55	25	1	47	11	1	9	34	0	12	24	6
25	0	48	14	1	34	8	1	55	39	1	46	22	1	7	54	0	10	21	5
26	0	50	2	1	35	18	1	55	51	1	45	31	1	6	13	0	8	17	4
27	0	51	50	1	36	26	1	56	1	1	44	38	1	4	31	0	6	13	3
28	0	53	36	1	37	33	1	56	9	1	43	43	1	2	47	0	4	9	2
29	0	55	22	1	38	38	1	56	15	1	42	47	1	1	2	0	2	4	1
30	0	57	7	1	39	41	1	56	19	1	41	48	0	59	15	0	0	0	0
Sig.	11. ♓			10. ♒			9. ♑			8. ♐			7. ♏			6. ♎			

ADD

The

The Moon's Mean Motion in Years.

	Long. ☽.				Apogee ☽.				Node ☽.			
	ſ :	° :	′ :	″	ſ :	° :	′ :	″	ſ :	° :	′ :	″
1	4	2	2	55	0	12	8	45	8	28	34	35
101	2	9	53	20	1	1	20	0	4	14	23	20
201	0	17	43	45	4	20	21	15	0	0	12	5
301	10	25	34	10	8	9	42	30	7	16	0	50
401	9	3	24	35	11	28	53	45	3	1	49	35
501	7	11	15	0	3	18	5	0	10	17	38	20
601	5	19	5	25	7	7	16	15	6	3	27	5
701	3	26	55	50	10	26	27	30	1	19	15	50
801	2	4	46	15	2	15	38	45	9	5	4	35
901	0	12	36	40	6	4	50	0	4	20	53	20
1001	10	20	27	5	9	24	1	15	0	6	42	5
1101	8	8	17	30	1	13	12	30	7	22	30	50
1201	7	16	7	55	5	2	23	45	3	8	19	35
1301	5	13	58	20	8	21	35	0	10	24	8	20
1401	3	21	48	45	0	10	46	15	6	9	57	5
1501	1	29	39	10	3	29	57	30	1	25	45	50
1601	0	7	29	35	7	19	8	45	9	11	34	35
1621	4	21	3	40	10	22	59	0	8	14	44	20
1641	9	4	37	45	1	26	49	15	7	17	54	5
1661	1	8	11	50	5	0	39	30	6	21	3	50
1681	6	1	45	55	8	4	29	45	5	24	13	35
1701	10	15	20	0	11	8	20	0	4	27	23	2
1721	2	28	54	5	2	12	10	15	4	0	33	5
1722	7	8	17	8	3	22	50	5	3	11	13	22
1723	11	17	40	11	5	3	29	56	2	21	53	39
1724	3	27	3	14	6	14	9	46	2	2	33	56
1725	8	19	36	52	7	24	56	18	1	13	11	2
1726	0	28	59	55	9	5	36	8	0	23	51	19
1727	5	8	22	58	10	16	15	59	0	4	31	36
1728	9	17	46	1	11	26	55	49	11	15	11	53
1729	2	10	19	39	1	7	42	20	10	25	48	59
1730	6	19	42	42	2	18	21	10	10	6	29	16
1731	10	29	5	46	3	29	2	1	9	17	9	33
1732	3	8	28	50	5	9	41	51	8	27	49	50
1733	8	1	2	29	6	20	28	23	8	8	26	56
2734	0	10	25	33	8	1	8	13	3	19	7	13
1735	4	19	48	37	9	11	48	4	6	29	47	30
1736	8	29	11	41	10	22	27	54	6	10	27	47
1737	1	21	45	20	0	3	14	26	5	21	4	53
1738	6	1	8	24	1	13	54	16	5	1	45	10

The

The Moon's Mean Motion in Years.

	Long. ☽.				Apogee ☽.				Node ☽.			
	∫ :	° :	′ :	″	∫ :	° :	′ :	″	∫ :	° :	′ :	″
1739	10	10	31	28	2	24	34	7	4	12	25	27
1740	2	19	54	32	4	5	13	56	3	23	5	44
1741	7	12	28	10	5	16	0	30	3	3	42	50
1742	11	21	51	13	6	26	40	20	3	14	23	7
1743	4	1	14	17	8	7	20	11	1	25	3	24
1744	8	30	37	20	9	18	0	1	1	5	43	41
1745	1	3	10	59	10	28	46	33	0	16	20	47
1746	5	12	34	2	0	9	26	23	11	27	1	4
1747	9	21	57	6	1	20	6	14	11	7	41	21
1748	2	1	20	9	3	0	46	4	10	18	21	38
1749	6	23	53	48	4	11	32	35	9	28	58	44
1750	11	3	16	51	5	22	12	26	9	9	39	1
1751	3	12	39	55	7	2	52	17	8	20	19	18
1752	7	22	2	58	8	13	32	7	8	0	59	35
1753	0	14	36	36	9	24	18	39	7	11	36	41
1754	4	23	59	40	11	4	58	28	6	22	16	58
1755	9	3	22	43	0	15	38	19	6	2	57	15
1756	1	12	45	47	1	26	18	9	5	13	37	32
1757	6	5	19	26	3	7	4	41	4	24	14	38
1758	10	14	42	29	4	17	44	31	4	4	54	55
1759	2	24	5	33	5	28	24	21	3	15	35	12
1760	7	3	28	36	7	9	4	12	2	26	15	28
1761	4	26	2	15	8	9	50	45	2	6	52	35
1762	4	5	25	18	10	0	30	34	1	17	32	52
1763	8	14	48	22	11	11	10	25	0	28	13	9
1764	0	24	11	25	0	21	50	15	0	8	53	26
1765	5	16	45	4	2	2	36	47	11	19	30	32
1766	9	26	8	7	3	13	16	37	11	0	10	49
1767	2	5	31	11	4	23	56	28	10	10	51	6
1768	6	14	54	14	6	4	36	18	9	21	31	23
1769	11	7	27	53	7	15	22	49	9	2	8	29
1770	3	16	50	56	8	26	2	40	8	12	48	46
1771	7	26	14	0	10	6	42	31	7	23	29	3
1772	0	5	37	3	11	17	22	21	7	4	9	20
2773	4	28	10	41	0	28	8	53	6	14	46	26
1774	9	7	33	45	2	8	48	42	5	25	26	43
1775	1	16	56	48	3	19	28	33	5	6	7	0
1776	5	26	19	52	5	0	8	23	4	16	47	17
1777	10	18	56	31	6	10	54	55	3	27	24	23

The

The Moon's Mean Motion in Years.

	Long. ☽.				Apogee ☽.				Node ☽.			
	f :	° :	′ :	″	*f* :	° :	′ :	″	*f* :	° :	′ :	″
1778	2	28	16	34	7	21	34	45	3	8	4	40
1779	7	7	39	38	9	2	14	35	2	18	44	57
1780	11	17	2	41	10	12	54	26	1	29	25	13
1781	4	9	36	20	11	23	41	0	1	10	2	20
1782	8	18	59	23	1	4	20	50	0	20	42	37
1783	0	28	22	27	2	15	0	41	0	1	22	54
1784	5	7	45	30	3	25	40	31	11	12	3	11
1785	10	0	19	9	5	6	27	3	10	22	40	17
1786	2	9	42	12	6	17	6	53	10	3	20	34
1787	6	19	5	16	7	27	46	44	9	14	0	51
1788	10	28	28	19	9	8	26	34	8	24	41	8
1789	3	21	1	58	10	19	13	5	8	5	18	14
1790	8	0	25	1	11	29	52	56	7	15	58	31
1791	0	9	48	5	1	10	32	47	6	26	38	48
1792	4	19	11	8	2	21	12	37	6	7	19	5
1793	0	11	44	46	4	1	59	9	5	17	56	11
1794	1	21	7	50	5	12	38	58	4	28	36	28
1795	6	0	30	53	6	23	18	49	4	9	16	45
1796	10	9	53	57	8	3	58	39	3	19	57	2
1797	3	2	27	36	9	14	45	11	3	0	34	8
1798	7	11	50	39	10	25	25	1	2	11	14	25
1799	11	21	13	43	0	6	4	51	1	21	54	42
1800	4	0	36	46	1	16	44	42	1	2	34	58
1801	8	23	10	25	2	27	31	15	0	13	12	5
1802	1	2	33	28	4	8	11	5	11	23	52	22
1803	5	11	56	32	5	18	50	56	11	4	32	39
1804	9	21	19	35	6	29	30	46	10	15	12	56
1805	2	13	53	14	8	10	17	18	9	25	15	2
1806	6	23	16	17	9	20	15	8	9	6	30	19
1807	11	2	39	21	11	1	36	59	8	17	10	36
1808	3	12	2	24	0	12	16	49	7	27	50	53
1809	8	4	36	3	1	23	3	20	7	8	27	59
1810	0	13	59	6	3	3	43	11	6	19	8	16
1811	4	23	22	10	4	14	23	2	5	29	48	33
1812	9	2	45	13	5	25	2	52	5	10	28	50
1813	1	25	18	51	7	5	49	24	4	21	5	56
1814	6	4	41	51	8	16	29	13	4	1	46	13
1815	10	14	4	58	9	27	9	4	3	12	26	30
1816	2	23	28	2	11	7	48	54	2	23	6	47
1817	7	16	1	41	0	18	35	26	2	3	43	53

The

The Moon's Mean Motion in Years.

	Long. ☽.				Apogee ☽.				Node ☽.			
	∫	°	′	″	∫	°	′	″	∫	°	′	″
1818	11	25	24	44	1	29	15	16	1	14	24	10
1819	4	4	47	48	3	9	55	6	0	25	4	27
1820	8	14	10	51	4	20	34	57	0	5	44	43
1821	1	6	44	30	6	1	21	30	11	16	21	50
1822	5	16	7	33	7	12	1	20	10	27	2	7
1823	9	25	30	37	8	22	41	11	10	7	42	24
1824	2	4	53	40	10	3	21	1	9	18	22	41
1825	6	27	27	19	11	14	7	33	8	28	59	47
1826	11	6	50	22	0	24	47	23	8	9	40	4
1827	3	16	13	26	2	5	27	14	7	20	20	21
1828	7	25	36	29	3	16	7	4	7	1	0	38
1829	0	18	10	8	4	26	53	35	6	11	37	44
1830	4	27	33	11	6	7	33	26	5	22	18	1
1831	9	6	56	15	7	18	13	17	5	2	58	18
1832	1	16	19	18	8	28	53	7	4	13	38	35
1833	6	8	52	56	10	9	39	39	3	24	15	41
1834	10	28	16	0	11	20	19	28	3	4	55	58
1835	2	27	39	3	1	0	59	19	2	15	36	15
1836	7	7	2	7	2	11	39	9	1	26	16	32
1837	11	29	35	46	3	22	25	41	1	6	53	38
1838	4	8	58	49	5	3	5	31	0	17	33	55
1839	8	18	21	53	6	13	45	21	11	28	14	12
1840	0	27	44	56	7	24	25	12	11	8	54	28
1841	5	20	18	35	9	5	11	44	10	19	31	35
1842	9	29	41	38	10	15	51	35	10	0	11	52
1843	2	9	4	42	11	26	31	25	9	10	52	9
1844	6	18	21	45	1	7	11	16	8	21	32	26
1845	11	11	1	24	2	17	57	48	8	2	9	32
1846	3	20	24	27	3	28	37	38	7	12	49	49
1847	7	29	47	31	5	9	17	29	6	23	30	6
1848	0	9	10	34	6	19	57	19	6	4	11	23
1849	5	18	44	13	8	0	43	50	5	14	47	29
1850	9	11	7	16	9	11	23	41	4	25	27	46
1851	1	20	30	20	10	22	3	32	4	6	8	2
1852	5	29	53	21	0	2	43	22	3	16	48	20
1853	10	22	27	1	1	13	29	54	2	27	25	26
1854	3	1	50	5	2	24	9	43	2	8	5	43

Years

Years Compleat.	Long. ☽.				Apogee ☽.				Node ☽.			
	∫	°	′	″	∫	°	′	″	∫	°	′	″
1	4	9	23	3	1	10	39	50	0	19	19	43
2	8	18	46	7	2	21	19	41	1	8	39	26
3	0	28	9	10	4	1	59	31	1	27	59	9
4	5	20	42	49	5	12	46	3	2	17	22	3
5	10	0	5	52	6	23	25	53	3	6	41	46
6	2	9	28	56	8	4	5	44	3	26	1	29
7	6	18	51	59	9	14	45	34	4	15	21	12
8	11	11	25	58	10	25	32	5	5	4	44	6
9	3	20	48	41	0	6	11	56	5	24	3	49
10	8	0	11	45	1	16	51	47	6	13	23	32
11	0	9	34	48	2	27	31	37	7	2	43	15
12	5	2	8	26	4	8	18	9	7	22	6	9
13	9	11	31	30	5	18	57	58	8	15	25	52
14	1	20	54	33	6	29	37	49	9	0	45	35
15	6	0	17	37	8	10	17	39	9	20	5	18
16	10	22	51	16	9	21	4	11	10	9	28	12
17	3	2	14	19	11	1	44	1	10	28	47	55
18	7	11	37	23	0	12	23	51	11	18	7	38
19	11	21	0	26	1	23	3	42	0	7	27	22
20	4	13	34	5	3	3	50	15	0	26	50	15
40	8	27	8	10	6	7	4	30	1	23	40	30
60	1	10	42	15	9	11	30	45	2	20	30	45
80	5	24	16	20	0	15	21	0	3	17	21	0
100	10	7	50	25	3	19	11	15	4	14	11	15
200	8	15	40	50	7	8	22	30	8	28	22	30
300	6	23	31	15	10	27	33	45	1	12	33	45
400	5	1	21	40	2	16	45	0	5	26	45	0
500	3	9	12	5	6	5	56	15	10	10	56	15
600	1	17	2	30	9	25	7	30	2	25	7	30
700	11	24	52	55	1	14	18	45	7	9	18	45
800	10	2	43	20	5	3	30	0	11	23	30	0
900	8	10	33	45	8	22	41	15	4	7	41	15
1000	6	18	24	10	0	11	52	30	8	21	52	30
2000	1	6	48	20	0	23	45	0	5	13	45	0
3000	7	25	12	30	1	5	37	30	2	5	37	30
4000	2	13	36	40	1	17	30	0	10	27	30	0
5000	9	2	0	50	1	29	22	30	7	19	22	30

The

The Moon's Mean Motion in Months and Days.

Days.	January.											
	Long. ☽.				Apog. ☽.				Node ☽.			
	∫	°	′	″	∫	°	′	″	∫	°	′	″
1	0	13	10	35	0	0	6	41	0	0	3	11
2	0	26	21	10	0	0	13	22	0	0	6	21
3	1	9	31	45	0	0	20	3	0	0	9	32
4	1	22	42	20	0	0	26	44	0	0	12	43
5	2	5	52	55	0	0	33	25	0	0	15	53
6	2	19	3	3	0	0	40	6	0	0	19	4
7	3	2	14	5	0	0	46	48	0	0	22	14
8	3	15	24	40	0	0	53	29	0	0	25	25
9	3	28	35	15	0	1	0	10	0	0	28	36
10	4	11	45	50	0	1	6	51	0	0	31	46
11	4	24	56	25	0	1	13	32	0	0	34	57
12	5	8	7	0	0	1	20	13	0	0	38	8
13	5	21	17	35	0	1	26	54	0	0	41	18
14	6	4	28	10	0	1	33	35	0	0	44	29
15	6	17	38	45	0	1	40	16	0	0	47	40
16	7	0	49	20	0	1	46	57	0	0	50	50
17	7	13	59	55	0	1	53	38	0	0	54	1
18	7	27	10	3	0	2	0	19	0	0	57	11
19	8	10	21	5	0	2	7	0	0	1	0	22
20	8	23	31	40	0	2	13	41	0	1	3	33
21	9	6	42	15	0	2	20	23	0	1	6	43
12	9	19	52	50	0	2	27	4	0	1	9	54
23	10	3	3	25	0	2	33	45	0	1	13	5
24	10	16	14	0	0	2	40	26	0	1	16	15
25	10	29	24	35	0	2	47	7	0	1	19	26
26	11	12	35	10	0	2	53	48	0	1	22	37
27	11	25	45	45	0	3	0	29	0	1	25	47
28	0	8	56	20	0	3	7	10	0	1	28	58
29	0	22	6	55	0	3	13	51	0	1	32	9
30	1	5	17	31	0	3	20	32	0	1	35	19
31	1	18	28	6	0	3	27	13	0	1	38	30

The Moon's Mean Motion in Months and Days.

Days.	February.											
	Long. ☽.				Apog. ☽.				Node ☽.			
	♌ : ° : ′ : ″				♌ : ° : ′ : ″				♌ : ° : ′ : ″			
1	2	1	38	41	0	3	33	54	0	1	41	40
2	2	14	49	16	0	3	40	35	0	1	44	50
3	2	27	59	51	0	3	47	16	0	1	48	1
4	3	11	10	26	0	3	53	57	0	1	51	12
5	3	24	21	1	0	4	0	38	0	1	54	23
6	4	7	31	36	0	4	7	19	0	1	57	33
7	4	20	42	11	0	4	14	0	0	2	0	44
8	5	3	52	46	0	4	21	41	0	2	3	54
9	5	17	3	21	0	4	27	22	0	2	7	6
10	6	0	13	56	0	4	34	4	0	2	10	16
11	6	13	24	31	0	4	40	45	0	2	13	17
12	6	26	35	6	0	4	47	26	0	2	16	37
13	7	9	45	41	0	4	54	7	0	2	19	48
14	7	22	56	15	0	5	0	48	0	2	22	59
15	8	6	6	50	0	5	7	29	0	2	26	9
16	8	19	17	26	0	5	14	10	0	2	29	20
17	9	2	28	1	0	5	20	51	0	2	32	30
18	9	15	38	36	0	5	27	32	0	2	35	41
19	9	28	49	11	0	5	34	13	0	2	38	52
20	10	11	59	46	0	5	40	54	0	2	42	0
21	10	25	10	21	0	5	47	36	0	2	45	13
22	11	8	20	56	0	5	54	17	0	2	48	23
23	11	21	31	31	0	6	0	58	0	2	51	34
24	0	4	42	6	0	6	7	39	0	2	54	45
25	0	17	52	41	0	6	14	20	0	2	57	55
26	1	1	3	16	0	6	21	1	0	3	1	6
27	1	14	13	51	0	6	27	42	0	3	4	16
28	1	27	24	26	0	6	34	23	0	3	7	27
29	2	10	35	1	0	6	41	4	0	3	10	38

The Moon's Mean Motion in Months and Days.

Common	March.												Bissex.
	Long. ☽.				Apogee ☽.				Node ☽.				
	∫	°	′	″	∫	°	′	″	∫	°	′	″	
1	2	10	35	1	0	6	41	4	0	3	10	38	0
2	2	23	45	36	0	6	47	45	0	3	13	49	1
3	3	6	56	11	0	6	54	26	0	3	16	59	2
4	3	20	6	46	0	7	1	7	0	3	20	10	3
5	3	3	17	21	0	7	7	48	0	3	23	20	4
6	4	6	27	56	0	7	14	29	0	3	26	31	5
7	4	29	38	31	0	7	21	11	0	3	29	42	6
8	5	12	49	6	0	7	27	52	0	3	32	52	7
9	5	25	59	41	0	7	34	33	0	3	36	3	8
10	6	9	10	16	0	7	41	14	0	3	39	14	9
11	6	22	20	51	0	7	47	55	0	3	42	25	10
12	7	5	31	26	0	7	54	36	0	3	45	36	11
13	7	18	42	1	0	8	1	17	0	3	48	46	12
14	8	1	52	36	0	8	7	58	0	3	51	56	13
15	8	15	3	11	0	8	14	39	0	3	55	7	14
16	8	28	13	46	0	8	21	20	0	3	58	18	15
17	9	11	24	21	0	8	28	1	0	4	1	28	16
18	9	24	34	56	0	8	34	42	0	4	4	39	17
19	10	7	45	31	0	8	41	23	0	4	7	49	18
20	10	20	56	7	0	8	38	4	0	4	11	0	19
21	11	4	6	42	0	8	54	45	0	4	14	11	20
22	11	17	17	17	0	9	1	27	0	4	17	22	21
23	0	0	27	52	0	9	8	8	0	4	20	32	22
24	0	13	38	27	0	9	14	49	0	4	23	43	23
25	0	26	49	2	0	9	21	30	0	4	26	53	24
26	1	9	59	37	0	9	28	11	0	4	30	4	25
27	1	23	10	12	0	9	34	52	0	4	33	15	26
28	2	6	20	47	0	9	41	33	0	4	36	25	27
29	2	19	31	22	0	9	48	14	0	4	39	36	28
30	3	2	41	57	0	9	54	55	0	4	42	47	29
31	3	15	52	32	0	10	1	36	0	4	45	58	30

The Moon's Mean Motion in Months and Days.

Common	April. Long. ☽.				Apogee ☽.				Node ☽.				Bissex.
	ſ	°	′	″	ſ	°	′	″	ſ	°	′	″	
1	3	29	3	7	0	10	8	17	0	4	49	8	0
2	4	12	13	42	0	10	14	58	0	4	52	19	1
3	4	25	24	17	0	10	21	39	0	4	55	29	2
4	5	8	34	52	0	10	28	20	0	4	58	40	3
5	5	21	45	27	0	10	35	2	0	5	1	51	4
6	6	4	56	2	0	10	41	43	0	5	5	1	5
7	6	18	6	38	0	10	48	24	0	5	8	12	6
8	7	1	17	12	0	10	55	5	0	5	11	22	7
9	7	14	27	47	0	11	1	46	0	5	14	33	8
10	7	27	38	22	0	11	8	27	0	5	17	44	9
11	8	10	48	57	0	11	15	8	0	5	20	54	10
12	8	23	59	32	0	11	21	49	0	5	24	5	11
13	9	7	10	10	0	11	28	30	0	5	27	16	12
14	9	20	20	42	0	11	35	11	0	5	30	26	13
15	10	3	31	17	0	11	41	52	0	5	33	37	14
16	10	16	41	52	0	11	48	33	0	5	36	48	15
17	10	29	52	27	0	11	55	14	0	5	39	58	16
18	11	13	3	2	0	12	1	55	0	5	43	9	17
19	11	16	13	37	0	12	8	36	0	5	46	15	18
20	0	9	24	12	0	12	15	18	0	5	49	31	19
21	0	22	34	47	0	12	21	59	0	5	52	41	20
22	1	5	45	22	0	12	28	40	0	5	55	52	21
23	1	18	55	57	0	12	35	21	0	5	59	2	22
24	2	2	6	32	0	12	42	2	0	6	2	13	23
25	2	15	17	7	0	12	48	43	0	6	5	24	24
26	2	28	27	42	0	12	55	24	0	6	8	34	25
27	3	11	38	17	0	13	2	5	0	6	11	45	26
28	3	24	48	58	0	13	8	46	0	6	14	56	27
29	4	7	59	27	0	13	15	27	0	6	18	6	28
30	4	21	10	3	0	13	22	8	0	6	21	17	29
31	5	4	20	37	0	13	28	49	0	6	24	27	30

The

The Moon's Mean Motion in Months and Days.

Common	May. Long. ☽.				May. Apogee ☽.				May. Node ☽.				Bissex.
	∫	°	′	″	∫	°	′	″	∫	°	′	″	
1	5	4	20	37	0	13	28	49	0	6	24	37	0
2	5	17	31	12	0	13	35	30	0	6	27	38	1
3	6	0	41	47	0	13	42	11	0	6	30	48	2
4	6	13	52	22	0	13	48	52	0	6	33	59	3
5	6	27	2	57	0	13	55	34	0	6	37	10	4
6	7	10	13	32	0	14	2	15	0	6	40	20	5
7	7	23	24	7	0	14	8	56	0	6	43	31	6
8	8	6	34	42	0	14	15	37	0	6	46	41	7
9	8	19	45	17	0	14	22	18	0	6	49	52	8
10	9	2	55	53	0	14	28	59	0	6	53	3	9
11	9	16	6	27	0	14	35	40	0	6	56	14	10
12	9	29	17	3	0	14	42	21	0	6	59	24	11
13	10	12	27	38	0	14	49	2	0	7	2	34	12
14	10	25	38	13	0	14	55	43	0	7	5	45	13
15	11	8	48	48	0	15	2	24	0	7	8	56	14
16	11	21	59	34	0	15	9	5	0	7	12	6	15
17	0	5	9	58	0	15	15	46	0	7	15	17	16
18	0	18	20	53	0	15	22	28	0	7	18	27	17
19	1	1	31	8	0	15	29	9	0	7	21	38	18
20	1	14	41	43	0	15	35	50	0	7	24	49	19
21	1	27	52	18	0	15	42	31	0	7	28	0	20
22	2	11	2	53	0	15	49	12	0	7	31	10	21
23	2	24	13	28	0	15	55	53	0	7	34	21	22
24	3	7	24	3	0	16	2	34	0	7	37	32	23
25	3	20	34	38	0	16	9	15	0	7	40	43	24
26	4	8	45	13	0	16	15	56	0	7	43	53	25
27	4	16	55	48	0	16	22	37	0	7	47	4	26
28	5	0	6	23	0	16	29	18	0	7	50	14	27
29	5	13	16	58	0	16	35	59	0	7	53	25	28
30	5	26	27	33	0	16	22	40	0	7	56	36	29
31	6	9	38	8	0	16	49	21	0	7	59	46	30

The Moon's Mean Motion in Months and Days.

Common	June. Long. ☽.				Apog. ☽.				Node ☽.				Bissex.
	∫	°	′	″	∫	°	′	″	∫	°	′	″	
1	6	22	48	43	0	16	56	3	0	8	2	56	0
2	7	5	59	18	0	17	2	44	0	8	6	7	1
3	7	19	9	53	0	17	9	25	0	8	9	18	2
4	8	2	20	28	0	17	16	6	0	8	12	29	3
5	8	15	31	3	0	17	22	47	0	8	15	39	4
6	8	28	41	28	0	17	29	28	0	8	18	50	5
7	9	11	52	13	0	17	36	9	0	8	22	0	6
8	9	25	2	48	0	17	42	50	0	8	25	11	7
9	10	8	13	23	0	17	49	31	0	8	28	22	8
10	10	21	23	58	0	17	56	12	0	8	31	32	9
11	11	4	34	33	0	18	2	53	0	8	34	43	10
12	11	17	45	8	0	18	9	34	0	8	37	54	11
13	0	0	55	43	0	18	16	15	0	8	41	5	12
14	0	14	16	18	0	18	22	56	0	8	44	16	13
15	0	27	16	53	0	18	29	37	0	8	47	26	14
16	1	10	27	28	0	18	36	19	0	8	50	37	15
17	1	23	38	3	0	18	43	0	0	8	53	47	16
18	2	6	48	38	0	18	49	41	0	8	56	58	17
19	2	19	59	13	0	18	56	22	0	9	0	9	18
20	3	3	9	48	0	19	3	3	0	9	3	19	19
21	3	16	20	23	0	19	9	44	0	9	6	30	20
22	3	29	30	58	0	19	16	25	0	9	9	40	21
23	4	12	41	33	0	19	23	6	0	9	12	51	22
24	4	25	52	8	0	19	29	47	0	9	16	2	23
25	5	9	2	43	0	19	36	28	0	9	19	12	24
26	5	22	13	18	0	19	43	9	0	9	22	23	25
27	6	5	23	53	0	19	49	50	0	9	25	34	26
28	6	18	34	28	0	19	56	31	0	9	28	45	27
29	7	1	45	4	0	20	3	12	0	9	31	55	28
30	7	14	55	39	0	20	9	54	0	9	35	6	29
31	7	28	6	14	0	20	16	25	0	9	38	16	30

The

The Moon's Mean Motion in Months and Days.

Common	July. Long. D.				Apog. D.				Node D.				Bissex.
	ſ	°	′	″	ſ	°	′	″	ſ	°	′	″	
1	7	28	6	14	0	20	16	35	0	9	38	16	0
2	8	11	16	49	0	20	23	16	0	9	41	27	1
3	8	24	27	24	0	20	29	57	0	9	44	37	2
4	9	7	37	59	0	20	36	38	0	9	47	48	3
5	9	20	48	34	0	20	43	19	0	9	50	59	4
6	10	3	59	9	0	20	50	0	0	9	54	9	5
7	10	17	9	44	0	20	56	41	0	9	57	20	6
8	11	0	20	19	0	21	3	22	0	10	0	30	7
9	11	13	30	54	0	21	10	3	0	10	3	41	8
10	11	26	41	29	0	21	16	44	0	10	6	51	9
11	0	9	52	4	0	21	23	25	0	10	10	2	10
12	0	23	2	39	0	21	30	6	0	10	13	13	11
13	1	6	13	14	0	21	36	47	0	10	16	24	12
14	1	19	23	49	0	21	43	28	0	10	19	35	13
15	2	2	34	24	0	21	50	9	0	10	22	45	14
16	2	15	44	59	0	21	56	52	0	10	25	56	15
17	2	28	55	34	0	22	3	32	0	10	29	6	16
18	3	12	6	9	0	22	10	13	0	10	32	17	17
19	3	25	16	44	0	22	16	54	0	10	35	28	18
20	4	8	27	19	0	22	23	35	0	10	38	39	19
21	4	21	37	54	0	22	30	15	0	10	41	49	20
12	5	4	48	29	0	22	55	56	0	10	45	0	21
23	5	17	59	4	0	22	43	37	0	10	48	11	22
24	6	1	9	39	0	22	50	19	0	10	51	21	23
25	6	14	20	14	0	22	57	0	0	10	54	32	24
26	6	27	30	49	0	23	3	41	0	10	57	42	25
27	7	10	41	20	0	23	10	22	0	11	0	53	26
28	7	23	51	59	0	23	17	3	0	11	4	3	27
29	8	7	2	34	0	23	23	44	0	11	7	14	28
30	8	20	13	9	0	23	30	25	0	11	10	25	29
31	9	3	23	44	0	23	37	6	0	11	13	36	30

The

The Moon's Mean Motion in Months and Days.

Common	August.												Bissex.
	Long. ☽.				Apogee ☽.				Node ☽.				
	∫	°	′	″	∫	°	′	″	∫	°	′	″	
1	9	16	34	19	0	23	43	47	0	11	16	47	0
2	9	29	44	54	0	23	50	28	0	11	19	58	1
3	10	12	45	29	0	23	57	9	0	11	23	8	2
4	10	26	6	4	0	24	3	51	0	11	26	19	3
5	11	9	16	39	0	24	10	32	0	11	29	29	4
6	11	22	27	14	0	24	17	30	0	11	32	40	5
7	0	5	37	49	0	24	23	54	0	11	35	51	6
8	0	18	48	24	0	24	30	35	0	11	39	2	7
9	1	1	58	59	0	24	37	16	0	11	42	12	8
10	1	15	9	34	0	24	43	57	0	11	45	23	9
11	1	28	20	9	0	24	50	38	0	11	48	33	10
12	2	11	30	44	0	24	57	19	0	11	51	43	11
13	2	24	41	19	0	25	4	0	0	11	54	54	12
14	3	7	51	54	0	25	10	42	0	11	58	5	13
15	3	21	2	29	0	25	17	23	0	12	1	15	14
16	4	4	13	4	0	25	24	4	0	12	4	26	15
17	4	17	23	40	0	25	30	45	0	12	7	36	16
18	5	0	34	15	0	25	37	26	0	12	10	47	17
19	5	13	44	50	0	25	44	7	0	12	13	58	18
20	5	26	55	25	0	25	50	48	0	12	17	8	19
21	6	10	6	0	0	25	57	29	0	12	20	19	20
22	6	23	16	35	0	26	4	10	0	12	23	29	21
23	7	6	27	10	0	26	10	51	0	12	26	40	22
24	7	19	37	45	0	26	17	32	0	12	29	51	23
25	8	2	48	20	0	26	24	13	0	12	33	1	24
26	8	15	58	55	0	26	30	55	0	12	36	12	25
27	8	29	9	3	0	26	37	36	0	12	39	23	26
28	9	12	20	5	0	26	44	17	0	12	42	34	27
29	9	25	30	40	0	26	50	58	0	12	45	44	28
30	10	8	41	15	0	26	57	39	0	12	48	55	29
31	10	21	51	50	0	27	4	20	0	12	52	5	30

The

The Moon's Mean Motion in Months and Days.

Common	September. Long. D.				Apogee D.				Node D.				Biſſex.
	ſ	°	′	″	ſ	°	′	″	ſ	°	′	″	
1	11	5	2	25	0	27	11	1	0	12	55	16	0
2	11	18	13	0	0	27	17	42	0	13	58	27	1
3	0	1	23	35	0	27	24	23	0	13	1	37	2
4	0	14	34	10	0	27	31	4	0	13	4	48	3
5	0	27	44	45	0	27	37	45	0	13	7	58	4
6	1	10	55	20	0	27	44	26	0	13	11	9	5
7	1	24	5	55	0	27	51	7	0	13	14	20	6
8	2	7	16	30	0	27	57	49	0	13	17	31	7
9	2	20	27	5	0	28	4	29	0	13	20	41	8
10	3	3	37	40	0	28	11	11	0	13	23	52	9
11	3	16	48	15	0	28	17	52	0	13	27	3	10
12	3	29	58	50	0	28	24	33	0	13	30	14	11
13	4	13	9	25	0	28	31	14	0	13	33	24	12
14	4	26	20	0	0	28	37	55	0	13	36	35	13
15	5	9	30	35	0	28	44	36	0	13	39	45	14
16	5	22	41	10	0	28	51	17	0	13	42	56	15
17	6	5	51	45	0	28	57	58	0	13	46	7	16
18	6	19	2	20	0	29	4	39	0	13	49	17	17
19	7	2	12	55	0	29	11	20	0	13	52	28	18
20	7	15	23	30	0	29	18	1	0	13	55	38	19
21	7	28	34	5	0	29	24	42	0	13	58	49	20
22	8	11	44	45	0	29	31	23	0	14	2	0	21
23	8	24	55	15	0	29	38	4	0	14	5	10	22
24	9	8	5	50	0	29	44	45	0	14	8	21	23
25	9	21	16	25	0	29	51	27	0	14	11	31	24
26	10	4	27	0	0	29	58	8	0	14	14	42	25
27	10	17	37	35	1	0	4	49	0	14	17	53	26
28	11	0	48	10	1	0	11	30	0	14	21	3	27
29	11	13	58	45	1	0	18	11	0	14	24	14	28
30	11	27	9	20	1	0	24	52	0	14	27	24	29
31	0	10	19	55	1	0	31	33	0	14	30	35	30

The

The Moon's Mean Motion in Months and Days.

Common	October. Long. ☽.				Apog. ☽.				Node ☽.				Bissex.
	∫	°	′	″	∫	°	′	″	∫	°	′	″	
1	0	10	19	55	1	0	31	33	0	14	30	25	0
2	0	23	30	30	1	0	38	14	0	14	33	46	1
3	1	6	41	5	1	0	44	55	0	14	36	56	2
4	1	19	5	40	1	0	51	36	0	14	40	7	3
5	2	3	2	15	1	0	58	17	0	14	43	17	4
6	2	16	12	50	1	1	4	58	0	14	46	28	5
7	2	29	23	26	1	1	11	39	0	14	49	39	6
8	3	12	34	1	1	1	18	20	0	14	52	50	7
9	3	25	44	36	1	1	25	2	0	14	56	0	8
10	4	8	55	11	1	1	31	43	0	14	59	11	9
11	4	22	5	46	1	1	38	24	0	14	2	21	10
12	5	5	16	21	1	1	45	5	0	14	5	32	11
13	5	18	26	56	1	1	51	46	0	14	8	43	12
14	6	1	37	31	1	1	58	27	0	14	11	53	13
15	6	14	48	6	1	2	5	8	0	15	15	4	14
16	6	27	58	41	1	2	11	49	0	15	18	15	15
17	7	11	9	16	1	2	18	30	0	15	21	26	16
18	7	24	19	51	1	2	25	11	0	15	24	36	17
19	8	7	30	26	1	2	31	52	0	15	27	47	18
20	8	20	41	1	1	2	38	33	0	15	30	57	19
21	9	3	51	36	1	2	41	14	0	15	34	8	20
22	9	17	2	11	1	2	55	55	0	15	37	19	21
23	10	0	12	46	1	2	58	36	0	15	40	29	22
24	10	13	23	21	1	3	5	07	0	15	43	40	23
25	10	26	33	56	1	3	11	58	0	15	46	50	24
26	11	9	44	31	1	3	18	39	0	15	50	1	25
27	11	22	55	6	1	3	25	21	0	15	53	12	26
28	0	6	5	41	1	3	32	2	0	15	56	22	27
29	0	19	16	16	1	3	38	43	0	11	59	33	28
30	1	2	26	51	1	3	45	24	0	16	2	43	29
31	1	15	37	26	1	3	52	5	0	16	5	54	30

The Moon's Mean Motion in Months and Days.

Common	November.												Bissex.
	Long. ☽.				Apog. ☽.				Node ☽.				
	∫	°	′	″	∫	°	′	″	∫	°	′	″	
1	1	28	48	01	1	3	58	46	0	16	9	5	0
2	2	11	58	36	1	4	5	27	0	16	12	16	1
3	2	25	9	11	1	4	12	8	0	16	15	26	2
4	3	8	19	36	1	4	18	49	0	16	18	37	3
5	3	21	30	21	1	4	25	30	0	16	21	48	4
6	4	4	40	56	1	4	32	11	0	16	24	59	5
7	4	17	51	31	1	4	38	53	0	16	28	9	6
8	5	1	2	6	1	4	45	34	0	16	31	20	7
9	5	14	12	41	1	4	52	15	0	16	34	30	8
10	5	27	23	16	1	4	58	56	0	16	37	41	9
11	6	10	33	51	1	5	5	37	0	16	40	52	10
12	6	23	44	26	1	5	12	10	0	16	44	2	11
13	7	6	55	1	1	5	18	59	0	16	47	13	12
14	7	20	5	36	1	5	25	40	0	16	50	23	13
15	8	3	16	11	1	5	32	21	0	16	53	34	14
16	8	16	26	46	1	5	39	2	0	16	56	45	15
17	8	29	37	21	1	5	45	43	0	16	59	55	16
18	9	12	47	56	1	5	52	24	0	17	3	6	17
19	9	25	58	31	1	5	59	5	0	17	6	16	18
20	10	9	9	6	1	6	5	46	0	17	9	27	19
21	10	22	19	41	1	6	12	27	0	17	12	38	20
12	11	5	30	16	1	6	19	9	0	17	15	49	21
23	11	18	40	51	1	6	25	50	0	17	18	59	22
24	0	1	51	26	1	6	32	31	0	17	22	10	23
25	0	15	2	2	1	6	39	12	0	17	25	21	24
26	0	28	12	37	1	6	45	53	0	17	28	32	25
27	1	11	23	12	1	6	52	24	0	17	31	42	26
28	1	24	33	47	1	6	59	15	0	17	34	53	27
29	2	7	44	22	1	7	5	56	0	17	38	3	28
30	2	2	54	57	1	7	12	37	0	17	41	14	29
31	3	4	5	32	1	7	19	18	0	17	44	25	30

The Moon's Mean Motion in Months and Days.

Common	December.												Bissex.
	Long. ☽.				Apogee ☽.				Node ☽.				
	∫	°	′	″	∫	°	′	″	∫	°	′	″	
1	3	4	5	32	1	7	19	18	0	17	44	25	0
2	3	17	16	7	1	7	25	59	0	17	47	35	1
3	4	0	26	42	1	7	32	40	0	17	50	46	2
4	4	13	37	17	1	7	39	21	0	17	53	56	3
5	4	26	47	52	1	7	46	2	0	17	57	7	4
6	5	9	58	27	1	7	52	43	0	18	0	18	5
7	5	23	9	2	1	7	59	24	0	18	3	28	6
8	6	6	19	37	1	8	6	6	0	18	6	39	7
9	6	19	30	12	1	8	12	47	0	18	9	49	8
10	7	2	40	47	1	8	19	28	0	18	13	0	9
11	7	15	51	22	1	8	26	9	0	18	16	11	10
12	7	29	1	57	1	8	32	50	0	18	19	21	11
13	8	12	12	30	1	8	39	31	0	18	22	32	12
14	8	25	23	7	1	8	46	12	0	18	25	42	13
15	9	8	33	42	1	8	52	53	0	18	28	53	14
16	9	21	44	17	1	8	59	34	0	18	32	4	15
17	10	4	54	52	1	9	6	15	0	18	35	15	16
18	10	18	5	27	1	9	12	56	0	18	38	25	17
19	11	1	16	2	1	9	19	38	0	18	41	36	18
20	11	14	26	37	1	9	26	19	0	18	44	47	19
21	11	27	37	12	1	9	33	0	0	18	47	58	20
22	0	10	47	47	1	9	39	41	0	18	51	9	21
23	0	23	58	22	1	9	46	22	0	18	54	19	22
24	1	7	8	57	1	9	53	3	0	18	57	30	23
25	1	20	19	32	1	9	59	44	0	19	0	41	24
26	2	3	30	7	1	10	6	25	0	19	3	51	25
27	2	16	40	42	1	10	13	6	0	19	7	2	26
28	2	29	51	17	1	10	19	47	0	19	10	12	27
29	3	13	1	52	1	10	26	28	0	19	13	23	28
30	3	26	12	27	1	10	33	9	0	19	16	33	29
31	4	9	23	2	1	10	39	50		19	19	43	30

The Moon's Mean Motion in Hours, Minutes, and Seconds.

H.	Long. ☽.			Ap. ☽.		No. ☽.	
	°	′	″	′	″	′	″
1	0	32	56	0	17	0	8
2	1	5	53	0	33	0	16
3	1	38	49	0	50	0	24
4	2	11	46	1	7	0	32
4	2	44	42	1	24	0	40
6	3	17	39	1	40	0	48
7	3	50	35	1	57	0	56
8	4	23	32	2	14	1	4
9	4	56	28	2	30	1	12
10	5	29	25	2	47	1	19
11	6	2	21	3	4	1	27
12	6	35	18	3	21	1	35
13	7	8	14	3	37	1	43
14	7	41	10	3	54	1	51
15	8	14	7	4	11	1	59
16	8	47	3	4	27	2	7
17	9	20	0	4	44	2	15
18	9	52	56	5	1	2	23
19	10	25	53	5	18	2	31
20	10	58	49	5	34	2	39
21	11	31	46	5	51	2	47
22	12	4	42	6	8	2	55
23	12	37	39	6	24	3	3
24	13	10	35	6	41	3	11
25	13	43	32	6	58	3	19
26	14	16	28	7	15	3	27
27	14	49	24	7	31	3	34
28	15	22	21	7	48	3	42
29	15	55	17	8	5	3	50
30	16	28	14	8	27	3	58

H.	Long. ☽.			Ap. ☽.		No. ☽.	
	°	′	″	′	″	′	″
31	17	28	14	8	38	4	6
32	17	34	7	8	54	4	14
33	18	7	3	9	11	4	22
34	18	39	59	9	28	4	30
35	19	12	55	9	45	4	38
36	19	45	52	10	2	4	46
37	20	18	48	10	19	4	54
38	20	51	45	10	36	5	2
39	21	24	41	10	52	5	10
40	21	27	38	11	8	5	18
41	22	30	34	11	25	5	26
42	23	3	31	11	42	5	34
43	23	26	27	11	59	5	42
44	24	9	24	12	16	5	50
45	24	42	20	12	32	5	58
46	25	15	17	12	48	6	6
47	25	48	13	13	5	6	14
48	26	21	10	13	22	6	22
49	26	54	6	13	39	6	30
50	27	27	3	13	56	6	38
51	27	59	59	14	13	6	46
52	28	32	56	14	30	6	54
53	29	5	52	14	46	7	1
54	29	38	49	15	2	7	8
55	30	11	45	15	19	7	16
56	30	44	42	15	36	7	24
57	31	17	38	15	53	7	32
58	31	50	34	16	10	7	40
59	32	23	31	16	26	7	48
60	32	56	27	16	43	7	56

A Table of the First Equations of the Moon's Longitude, Apogee, and Node.

	Mean Anomaly of the Sun.												
	Sign . . . 0						Sign . . . 1						
	Lo. ☽.		Ap. ☽.		No. ☽.		Lo. ☽.		Ap. ☽.		No. ☽.		
	Add		Subſtr.		Add		Add		Subſtr.		Add		
	′	″	′	″	′	″	′	″	′	″	′	″	
0	0	0	0	0	0	0	5	56	10	47	4	23	30
1	0	12	0	22	0	9	6	7	11	6	4	31	29
2	0	25	0	44	0	18	6	18	11	25	4	39	28
3	0	37	1	7	0	28	6	28	11	44	4	47	27
4	0	50	1	30	0	37	6	39	12	3	4	54	26
5	1	2	1	52	0	46	6	49	12	22	5	2	25
6	1	14	2	14	0	55	7	0	12	41	5	10	24
7	1	27	2	37	1	4	7	10	12	59	5	17	23
8	1	39	2	59	1	13	7	20	13	18	5	25	22
9	1	51	3	22	1	22	7	29	13	36	5	32	21
10	2	3	3	44	1	31	7	39	13	53	5	39	20
11	2	16	4	6	1	40	7	49	14	10	5	46	19
12	2	28	4	28	1	49	7	58	14	27	5	53	18
13	2	40	4	50	1	58	8	7	14	44	6	0	17
14	2	52	5	13	2	7	8	17	15	1	6	7	16
15	3	4	5	34	2	16	8	26	15	18	6	13	15
16	3	16	5	56	2	25	8	35	15	35	6	20	14
17	3	28	6	17	2	34	8	43	15	49	6	26	13
18	3	40	6	39	2	42	8	52	16	5	6	33	12
19	3	52	7	0	2	51	9	0	16	20	6	39	11
20	4	3	7	22	3	0	9	9	16	35	6	45	10
21	4	15	7	43	3	8	9	17	16	50	6	51	9
22	4	27	8	4	3	17	9	25	17	4	6	57	8
23	4	38	8	25	3	25	9	32	17	18	7	3	7
24	4	50	8	45	3	34	9	40	17	32	7	8	6
25	5	1	9	5	3	42	9	47	17	46	7	14	5
26	5	12	9	26	3	51	9	55	17	59	7	19	4
27	5	23	9	27	3	59	10	2	18	11	7	24	3
28	5	34	10	7	4	7	10	9	18	24	7	29	2
29	5	45	10	27	4	15	10	15	18	36	7	34	1
30	5	56	10	47	4	23	10	22	18	48	7	39	0
	Subſtr.		Add		Subſtr.		Subſtr.		Add		Subſtr.		
	Sign . . 11						Sign . . 10						

A Table of the First Equations of the Moon's Longitude, Apogee, and Node.

	Mean Anomaly of the Sun.												
	Sign . . . 2						Sign . . . 3						
	Lo. ☽.		Ap. ☽.		No. ☽.		Lo. ☽.		Ap. ☽.		No. ☽.		
	Add		Subſtr.		Add		Add		Subſtr.		Add		
	′ : ″		′ : ″		′ : ″		′	″	′	″	′	″	
0	10	22	18	48	7	39	12	6	21	56	8	56	30
1	10	28	19	0	7	44	12	6	21	56	8	56	29
2	10	35	19	11	7	48	12	6	21	56	8	56	28
3	10	41	19	22	7	53	12	5	21	56	8	56	27
4	10	46	19	33	7	57	12	5	21	56	8	55	26
5	10	52	19	43	8	2	12	4	21	54	8	55	25
6	10	57	19	54	8	6	12	3	21	52	8	54	24
7	11	3	20	2	8	10	12	2	21	50	8	53	23
8	11	8	20	11	8	13	12	1	21	46	8	52	22
9	11	12	20	20	8	17	11	59	21	44	8	51	21
10	11	17	20	28	8	20	11	57	21	42	8	50	20
11	11	22	20	36	8	23	11	55	21	38	8	48	19
12	11	26	20	44	8	26	11	53	21	34	8	46	18
13	11	30	20	51	8	29	11	51	21	29	8	44	17
14	11	34	20	58	8	32	11	48	21	24	8	42	16
15	11	37	21	4	8	35	11	45	21	19	8	40	15
16	11	41	21	11	8	37	11	42	21	13	8	38	14
17	11	44	21	17	8	40	11	38	21	7	8	36	13
18	11	47	21	22	8	42	11	35	21	0	8	33	12
19	11	50	21	27	8	44	11	31	20	54	8	30	11
20	11	52	21	32	8	46	11	27	20	46	8	27	10
21	11	54	21	36	8	48	11	23	20	38	8	24	9
22	11	57	21	40	8	49	11	18	20	30	8	21	8
23	11	59	21	43	8	50	11	14	20	22	8	18	7
24	12	0	21	46	8	52	11	9	20	13	8	14	6
25	12	2	21	49	8	53	11	4	20	4	8	10	5
26	12	3	21	52	8	54	10	58	19	54	8	6	4
27	12	4	21	53	8	54	10	53	19	44	8	2	3
28	12	5	21	54	8	55	10	47	19	34	7	58	2
29	12	5	21	56	8	55	10	41	19	23	7	54	1
30	12	6	21	56	8	56	10	35	19	12	7	49	0
	Subſtr.		Add		Subſtr.		Subſtr.		Add		Subſtr.		
	Sign . . . 9						Sign . . . 8						

A Table of the First Equations of the Moon's Longitude, Apogee, and Node.

	Mean Anomaly of the Sun.												
	Sign . . . 4						Sign . . . 5						
	Lo. ☽.		Ap. ☽.		No. ☽.		Lo. ☽.		Ap. ☽.		No. ☽.		
	Add		Subſtr.		Add		Add		Subſtr.		Add		
	′	″	′	″	′	″	′	″	′	″	′	″	
0	10	35	19	12	7	49	6	10	11	11	4	33	30
1	10	29	19	1	7	44	5	58	10	50	4	25	29
2	10	23	18	49	7	39	5	4	10	30	4	16	28
3	10	16	18	37	7	34	5	35	10	9	4	7	27
4	10	9	18	24	7	30	5	24	9	48	3	58	26
5	10	2	18	11	7	24	5	13	9	27	3	51	25
6	9	55	17	55	7	19	5	1	9	6	3	42	24
7	9	48	17	48	7	13	4	49	8	45	3	33	23
8	9	40	17	31	7	7	4	37	8	23	3	24	22
9	9	32	17	17	7	1	4	25	8	1	3	16	21
10	9	24	17	3	6	56	4	13	7	39	3	7	20
11	9	16	16	48	6	50	4	1	7	17	2	58	19
12	9	7	16	33	6	44	3	49	6	55	2	49	18
13	8	59	16	17	6	38	3	36	6	33	2	40	17
14	8	50	16	1	6	31	3	24	6	10	2	31	16
15	8	41	15	45	6	25	3	12	5	48	2	22	15
16	8	32	15	29	6	18	2	59	5	25	2	13	14
17	8	23	15	12	6	11	2	47	5	2	2	3	13
18	8	14	14	55	6	4	2	34	4	39	1	54	12
19	8	4	14	38	5	57	2	21	4	16	1	45	11
20	7	54	14	20	5	50	2	9	3	53	1	35	10
21	7	44	14	2	5	43	1	56	3	30	1	25	9
22	7	34	13	44	5	36	1	43	3	7	1	15	8
23	7	24	13	26	5	28	1	30	2	44	1	6	7
24	7	14	13	7	5	20	1	17	2	20	0	57	6
25	7	4	12	48	5	12	1	5	1	57	0	47	5
26	6	53	12	29	5	4	0	52	1	34	0	37	4
27	6	43	12	10	4	56	0	39	1	11	0	28	3
28	6	32	11	50	4	48	0	26	0	47	0	19	2
29	6	21	11	31	4	41	0	13	0	21	0	9	1
30	6	10	11	11	4	33	0	0	0	0	0	0	0
	Subſtr.		Add		Subſtr.		Subſtr.		Add		Subſtr.		
	Sign . . . 7						Sign . . . 6						

A Table of the Second Equation of the Moon's Apogee.

An. Argument.	ADD.									An. Argument.
	Signs 0 . 6			1 . . . 7			2 . . . 8			
	0 : 1 : 11			0 : 1 : 11			0 : ′ : ″			
0	0	0	0	9	28	24	11	40	42	30
1	0	21	5	9	42	40	11	31	22	29
2	0	42	10	9	56	27	11	20	56	28
3	1	3	13	10	9	43	11	9	25	27
4	1	24	13	10	22	28	10	56	48	26
5	1	45	10	10	34	40	10	43	8	25
6	2	6	3	10	46	19	10	28	28	24
7	2	26	51	10	57	22	10	12	25	23
8	2	47	33	11	7	48	9	55	25	22
9	3	8	8	11	17	38	9	37	21	21
10	3	28	36	11	26	49	9	18	11	20
11	3	48	56	11	35	19	8	58	5	19
12	4	9	6	11	43	3	8	36	44	18
13	4	29	6	11	50	14	8	14	28	17
14	4	48	55	11	56	35	7	51	12	16
15	5	8	33	12	2	12	7	27	0	15
16	5	27	57	12	7	1	7	1	50	14
17	5	47	8	12	11	3	6	35	45	13
18	6	6	4	12	14	15	6	18	51	12
19	6	24	45	12	16	35	5	41	8	11
20	6	43	9	12	18	4	5	12	40	10
21	7	1	16	12	18	40	4	43	28	9
22	7	19	4	12	18	21	4	13	40	8
23	7	36	33	12	17	6	3	43	16	7
24	7	53	42	12	14	54	3	12	22	6
25	8	10	29	12	11	44	2	41	0	5
26	8	26	53	12	7	35	2	9	16	4
27	8	42	54	12	2	24	1	37	13	3
28	8	58	40	11	56	12	1	4	56	2
29	29	13	41	11	49	0	0	32	30	1
30	29	28	24	11	40	42	0	0	0	0
An. Arg.	Signs 11 . 5			10 . . . 4			9 . . . 3			An. Arg.
	Subſtract.									

Excentricities of the Moon.

Apog. ☽. 1st Equat. a ☉.	Signs 0 . . . 6	Signs 1 . . 7	Signs 2 . . 8	Apog. ☽. 1st Equat. a ☉.
0	66777	61754	50224	30
1	66771	61434	49838	29
2	66754	61107	49457	28
3	66724	60772	49082	27
4	66683	60429	48714	26
5	66630	60080	48354	25
6	66566	59725	48001	24
7	66489	59363	47656	23
8	66402	58995	47321	22
9	66302	58621	46995	21
10	66192	58243	46679	20
11	66070	57860	46374	19
12	65936	57472	46081	18
13	65792	57080	45800	17
14	65636	56684	45531	16
15	65469	56285	45275	15
16	65292	55884	45033	14
17	65103	55479	44805	13
18	64905	55073	44592	12
19	64695	54666	44394	11
20	64476	54257	44212	10
21	64246	53848	44046	9
22	64006	53438	43896	8
23	63757	53030	43763	7
24	63498	52622	43647	6
25	63230	52215	43548	5
26	62952	51811	43467	4
27	62665	51409	43404	3
28	62370	51010	43359	2
29	62066	50615	43332	1
30	61754	50224	43323	0
	Signs 11 . . 5	Signs 10 . . 5	Signs 9 . . 3	

A Table of constant Logarithms for the elliptic Equation of the Moon.

An. Arg.	Signs 0 . . . 6	Signs . 1 . . . 7	Signs . . 2 . . 8	An. Arg.
0	9.941912	9.946292	9.956339	30
1	9.941917	9.946571	9.956675	29
2	9.941932	9.946857	9.957007	28
3	9.941958	9.947149	9.957330	27
4	9.941994	9.947448	9.957653	26
5	9.942040	9.947752	9.957968	25
6	9.942096	9.948062	9.958275	24
7	9.942163	9.948377	9.958575	23
8	9.942239	9.948698	9.958867	22
9	9.942326	9.949023	9.959151	21
10	9.942422	9.949353	9.959426	20
11	9.942529	9.949687	9.959691	19
12	9.942645	9.050025	9.959946	18
13	9.942771	9.950367	9.960191	17
14	9.942907	9.950712	9.960425	16
15	9.943053	9.951059	9.960648	15
16	9.943208	9.951409	9.960858	14
17	9.943372	9.951761	9.961056	13
18	9.943545	9.952115	9.961242	12
19	9.943728	9.952470	9.961414	11
20	9.943919	9.952827	9.961573	10
21	9.944120	9.953183	9.961717	9
22	9.944329	9.953540	9.961848	8
23	9.944546	9.953896	9.961963	7
24	9.944772	9.954251	9.662064	6
25	9.945006	9.954605	9.962150	5
26	9.945248	9.954957	9.962221	4
27	9.945498	9.955307	9.962276	3
28	9.945755	9.955655	9.962315	2
29	9.946020	9.955999	9.962339	1
30	9.946292	9.956339	9.962347	0
An. Arg.	Signs. 11 . . 5	Signs . . 10 . 4	Signs . 9 . . 3	An. Arg.

Equation

Equation to be applied to half the Mean Anomaly of the Moon.

Mean Anomaly of the Moon.

Sign . . . . 0 . . . Add

Log.	9·942		9·947		9·952		9·957		9·962	
	′	″	′	″	′	″	′	″	′	″
0	0	0	0	0	0	0	0	0	0	0
1	0	4	0	3	0	3	0	2	0	2
2	0	8	0	7	0	5	0	4	0	3
3	0	12	0	10	0	8	0	6	0	5
4	0	16	0	13	0	11	0	9	0	7
4	0	20	0	16	0	13	0	11	0	8
6	0	24	0	20	0	16	0	13	0	10
7	0	28	0	23	0	19	0	15	0	12
8	0	31	0	26	0	21	0	17	0	13
9	0	35	0	29	0	24	0	19	0	15
10	0	39	0	33	0	27	0	21	0	17
11	0	43	0	36	0	29	0	23	0	18
12	0	46	0	39	0	32	0	25	0	20
13	0	50	0	42	0	34	0	28	0	21
14	0	54	0	45	0	37	0	30	0	23
15	0	57	0	48	0	39	0	32	0	24
16	1	1	0	51	0	43	0	33	0	26
17	1	4	0	54	0	44	0	35	0	27
18	1	8	0	57	0	46	0	37	0	29
19	1	11	0	59	0	48	0	39	0	30
20	1	14	1	2	0	51	0	41	0	32
21	1	18	1	4	0	53	0	42	0	33
22	1	21	1	7	0	55	0	44	0	34
23	1	24	1	9	0	57	0	46	0	36
24	1	26	1	12	0	59	0	47	0	37
25	1	29	1	14	1	1	0	49	0	38
26	1	32	1	17	1	3	0	50	0	39
27	1	34	1	19	1	4	0	52	0	40
28	1	37	1	21	1	6	0	53	0	41
29	1	39	1	23	1	7	0	54	0	42
30	1	42	1	25	1	9	0	55	0	43

Equation to be applied to half the Mean Anomaly of the Moon.

Mean Anomaly of the Moon.

Sign . . . . 1 . . . Add

Log.	9·942		9·947		9·952		9·957		6·962	
	′	″	′	″	′	″	′	″	′	″
0	1	42	1	25	1	9	0	55	0	43
1	1	44	1	27	1	11	0	57	0	44
2	1	46	1	28	1	12	0	58	0	45
3	1	48	1	30	1	14	0	59	0	46
4	1	49	1	31	1	15	1	0	0	46
5	1	51	1	33	1	16	1	1	0	47
6	1	53	1	34	1	17	1	1	0	48
7	1	54	1	35	1	18	1	2	0	48
8	1	56	1	36	1	19	1	3	0	49
9	1	57	1	37	1	20	1	4	0	49
10	1	58	1	38	1	20	1	4	0	50
11	1	59	1	39	1	21	1	5	0	50
12	2	0	1	40	1	22	1	5	0	51
13	2	0	1	40	1	22	1	5	0	51
14	2	1	1	41	1	22	1	6	0	51
15	2	1	1	41	1	23	1	6	0	51
16	2	2	1	41	1	23	1	6	0	51
17	2	2	1	41	1	23	1	6	0	51
18	2	2	1	41	1	23	1	6	0	51
19	2	2	1	41	1	23	1	6	0	51
20	2	2	1	41	1	23	1	6	0	51
21	2	1	1	41	1	22	1	5	0	50
12	2	1	1	40	1	22	1	5	0	50
23	2	0	1	40	1	21	1	4	0	50
24	1	59	1	39	1	21	1	4	0	50
25	1	58	1	38	1	20	1	3	0	49
26	1	57	1	37	1	19	1	3	0	49
27	1	56	1	36	1	19	1	2	0	48
28	1	55	1	35	1	18	1	1	0	48
29	1	54	1	34	1	17	1	1	0	47
30	1	52	1	33	1	15	1	0	0	46

Equation to be applied to half the Mean Anomaly of the Moon.

Mean Anomaly of the Moon.

Sign . . . . 2 . . . Add

Log.	9·942		9·947		9·952		9·957		9·962	
	′	″	′	″	′	″	′	″	′	″
0	1	52	1	33	1	15	1	0	0	46
1	1	51	1	31	1	14	0	59	0	46
2	1	49	2	30	1	13	0	58	0	45
3	1	47	1	28	1	11	0	57	0	44
4	1	45	1	26	1	10	0	56	0	43
5	1	43	1	25	1	8	0	54	0	42
6	1	40	1	23	1	7	0	53	0	41
7	1	38	1	21	1	5	0	52	0	40
8	1	36	1	19	1	4	0	50	0	39
9	1	33	1	17	1	2	0	49	0	38
10	1	30	1	14	1	0	0	47	0	36
11	1	28	1	12	0	58	0	46	0	35
12	1	25	1	9	0	56	0	44	0	34
13	1	22	1	7	0	54	0	42	0	32
14	1	19	1	4	0	52	0	41	0	31
15	1	16	1	2	0	50	0	39	0	30
16	1	12	0	59	0	47	0	37	0	28
17	1	9	0	56	0	45	0	35	0	27
18	1	5	0	53	0	43	0	33	0	25
19	1	2	0	50	0	40	0	31	0	24
20	0	58	0	47	0	38	0	29	0	22
21	0	55	0	44	0	35	0	27	0	20
22	0	51	0	41	0	33	0	25	0	19
23	0	47	0	38	0	30	0	23	0	17
24	0	44	0	35	0	28	0	21	0	16
25	0	40	0	32	0	25	0	19	0	14
26	0	36	0	38	0	22	0	17	0	12
27	0	32	0	25	0	20	0	15	0	11
28	0	28	0	22	0	17	0	12	0	9
29	0	24	0	19	0	14	0	10	0	7
30	0	20	0	15	0	11	0	8	0	6

Equation to be applied to half the Mean Anomaly of the Moon.

Mean Anomaly of the Moon.

Sign . . . . 3 . . . Add

Log.	9·942		9·947		9·952		9·957		9·962	
	′	″	′	″	′	″	′	″	′	″
0	0	20	0	15	0	11	0	8	0	6
1	0	16	0	12	0	9	0	6	0	4
2	0	12	0	9	0	6	0	4	0	2
3	0	8	0	5	0	3	0	2	0	1
4	0	4	0	2	0	0	sub.	0	sub.	1
5	sub.	0	sub.	1	sub.	2	0	2	0	3
6	0	4	0	4	0	5	0	5	0	4
7	0	8	0	8	0	7	0	7	0	6
8	0	12	0	11	0	10	0	9	0	8
9	0	16	0	14	0	13	0	11	0	10
10	0	20	0	18	0	16	0	13	0	11
11	0	24	0	21	0	18	0	16	0	13
12	0	27	0	24	0	21	0	18	0	14
13	0	31	0	27	0	24	0	20	0	16
14	0	35	0	31	0	26	0	22	0	18
15	0	39	0	34	0	29	0	24	0	19
16	0	43	0	37	0	31	0	26	0	21
17	0	46	0	40	0	34	0	28	0	22
18	0	50	0	43	0	36	0	30	0	24
19	0	53	0	46	0	38	0	32	0	25
20	0	57	0	48	0	41	0	33	0	27
21	1	0	0	51	0	43	0	35	0	28
22	1	3	0	54	0	45	0	37	0	29
23	1	6	0	56	0	47	0	39	0	31
24	1	9	0	59	0	49	0	40	0	32
25	1	12	1	1	0	51	0	42	0	33
26	1	15	1	4	0	53	0	43	0	34
27	1	18	1	6	0	55	0	45	0	36
28	1	21	1	9	0	57	0	46	0	37
29	1	23	1	11	0	59	0	48	0	38
30	1	26	1	13	0	0	0	49	0	39

Equation

Equation to be applied to half the Mean Anomaly of the Moon.

Mean Anomaly of the Moon.

Sign . . . . 4. . . . . Subſtract.

Log.	9·942		9·947		9·952		9·957		9·962	
	′	″	′	″	′	″	′	″	′	″
0	1	26	1	13	1	0	0	49	0	39
1	1	29	1	15	1	2	0	50	0	40
2	1	30	1	16	1	3	0	51	0	41
3	1	32	1	18	1	5	0	52	0	42
4	1	34	1	20	1	6	0	53	0	42
4	1	36	1	21	1	7	0	54	0	43
6	1	38	1	23	1	8	0	55	0	44
7	1	39	1	24	1	9	0	56	0	44
8	1	41	1	25	1	10	0	57	0	45
9	1	42	1	26	1	11	0	57	0	45
10	1	44	1	27	1	12	0	58	0	46
11	1	45	1	28	1	13	0	58	0	46
12	1	45	1	29	1	13	0	59	0	46
13	1	46	1	29	1	13	0	59	0	46
14	1	47	1	33	1	14	1	0	0	47
15	1	47	1	30	1	14	1	0	0	47
16	1	47	1	30	1	14	1	1	0	47
17	1	48	1	30	1	15	1	1	0	47
18	1	48	1	30	1	15	1	0	0	47
19	1	48	1	30	1	15	1	0	0	47
20	1	47	1	30	1	14	1	0	0	47
21	1	47	1	30	1	14	0	59	0	47
22	1	46	1	29	1	13	0	59	0	46
23	1	45	1	29	1	13	0	58	0	46
24	1	44	1	28	1	12	0	58	0	45
25	1	43	1	27	1	12	0	57	0	45
26	1	42	1	26	1	11	0	57	0	44
27	1	41	1	25	1	10	0	56	0	44
28	1	40	1	24	1	9	0	55	0	43
29	1	38	1	22	1	8	0	54	0	43
30	1	16	1	21	1	6	0	53	0	42

Equation to be applied to half the Mean Anomaly of the Moon.

Mean Anomaly of the Moon.

Sign . . . . 5. . . . . Subſtract.

Log.	9·942	9·947	9·952	9·957	9·962
	′ ″	′ ″	′ ″	′ ″	′ ″
0	1 36	1 21	1 6	0 53	0 42
1	1 35	1 19	1 5	0 52	0 41
2	1 33	1 18	1 4	0 51	0 40
3	1 31	1 16	1 2	0 50	0 39
4	1 28	1 14	1 1	0 49	0 38
5	1 26	1 12	0 59	0 47	0 37
6	1 24	1 10	0 57	0 46	0 36
7	1 21	1 8	0 56	0 44	0 35
8	1 18	1 6	0 54	0 43	0 34
9	1 16	1 3	0 52	0 41	0 33
10	1 13	1 1	0 50	0 40	0 31
11	1 10	0 58	0 48	0 38	0 30
12	1 7	0 56	0 46	0 37	0 29
13	1 3	0 53	0 43	0 35	0 27
14	1 0	0 50	0 41	0 33	0 26
15	0 57	0 48	0 39	0 31	0 24
16	0 53	0 45	0 37	0 29	0 23
17	0 50	0 42	0 34	0 27	0 21
18	0 46	0 39	0 32	0 25	0 20
19	0 43	0 36	0 29	0 23	0 18
20	0 39	0 33	0 27	0 21	0 17
21	0 35	0 29	0 24	0 19	0 15
22	0 31	0 26	0 21	0 17	0 13
23	0 28	0 23	0 19	0 15	0 12
24	0 24	0 20	0 16	0 13	0 10
25	0 20	0 16	0 13	0 11	0 8
26	0 16	0 13	0 11	0 9	0 7
27	0 12	0 10	0 8	0 7	0 5
28	0 8	0 7	0 5	0 4	0 3
29	0 4	0 3	0 3	0 2	0 2
30	0 0	0 0	0 0	0 0	0 0

A Table

A Table of the Variation, or Third Equation of the Moon's Longitude.

☉ a ☽ 2d Eq.	ADD.						☉ a ☽ 2d Eq.
	0	6	1	7	2	8	
	′	″	″	″	′	″	
0	0	0	30	26	30	26	30
1	1	14	31	3	29	50	29
2	2	27	31	36	29	9	28
3	3	39	32	7	28	27	27
4	4	54	32	35	27	43	26
5	6	6	33	4	26	56	25
6	7	19	33	27	26	8	24
7	8	30	33	48	25	18	23
8	9	40	34	7	24	26	22
9	10	53	34	24	23	32	21
10	12	2	34	39	22	36	20
11	13	10	34	50	21	39	19
12	14	18	34	59	20	40	18
13	15	26	35	5	19	40	17
14	16	30	35	9	18	38	16
15	17	34	35	10	17	34	15
16	18	38	35	9	16	30	14
17	19	40	35	5	15	25	13
18	20	40	34	54	14	18	12
19	21	39	34	50	13	10	11
20	22	36	34	38	12	2	10
21	23	32	34	24	10	32	9
22	24	26	34	7	9	42	8
23	25	18	33	48	8	30	7
24	26	8	33	27	7	20	6
25	26	57	33	3	6	6	5
26	27	43	32	26	4	54	4
27	28	26	32	7	3	40	3
28	29	9	31	36	2	27	2
29	29	49	31	3	1	14	1
30	30	27	30	27	0	0	0
	11	5	10	4	9	3	0
	Subſtract.						

A Table

A Table of Logarithms for the Correction of the Moon's Variation.

	The Sun's Mean Anomaly.						
Sig.	0	1	2	3	4	5	Sig.
Deg.	Logar.	Logar.	Logar.	Logar.	Logar.	Logar.	Deg.
0	0.0242	0.0211	0.0125	0.0004	9.9880	9.9787	30
2	0.0242	0.0207	0.0118	9.9995	9.9872	9.9783	28
4	0.0242	0.0203	0.0110	9.9987	9.9865	9.9779	26
6	0.0241	0.0198	0.0102	9.9978	9.9858	9.9775	24
8	0.0240	0.0193	0.0095	9.9969	9.9851	9.9772	22
10	0.0239	0.0188	0.0087	9.9961	9.9844	9.9769	20
12	0.0237	0.0182	0.0079	9.9952	9.9837	9.9766	18
14	0.0235	0.0177	0.0071	9.9944	9.9830	9.9763	16
16	0.0233	0.0171	0.0063	9.9936	9.9824	9.9760	14
18	0.0231	0.0165	0.0055	9.9928	9.9818	9.9758	12
20	0.0229	0.0159	0.0046	9.9920	9.9812	9.9757	10
22	0.0226	0.0153	0.0038	9.9912	9.9807	9.9755	8
24	0.0223	0.0146	0.0029	9.9904	9.9802	9.9754	6
26	0.0219	0.0139	0.0021	9.9896	9.9797	6.9753	4
28	0.0215	0.0132	0.0012	9.9888	9.9792	9.9753	2
30	0.0211	0.0125	0.0004	9.9880	9.9787	9.9753	0
Sig.	11	10	9	8	7	6	Sig.

A TABLE

A TABLE of the Second Equation of the Node, and Inclination of the Limit above 4° : 59′ : 35″.

☊ ift. Equa. a. ☉.	Signs 0 6			Inclin.		Signs 1 7			Inclin.		Signs 2 8			Inclin.		☊ ift. Equa. a. ☉.
	ADD															
	°	′	″	′	″	°	′	″	′	″	°	′	″	′	″	
0	0	0	0	17	45	1	16	40	13	22	1	18	41	4	27	30
1	0	3	3	17	45	1	18	13	13	6	1	17	6	4	11	29
2	0	6	6	17	44	1	19	41	12	49	1	15	26	3	55	28
3	0	9	8	17	44	1	21	3	12	32	1	13	41	3	40	27
4	0	12	10	17	42	1	22	20	12	15	1	11	49	3	25	26
5	0	15	11	17	40	1	23	31	11	58	1	9	52	3	11	25
6	0	18	11	17	37	1	24	36	11	41	1	7	50	2	57	24
7	0	21	11	17	33	1	25	34	11	24	1	5	42	2	43	23
8	0	24	7	17	28	1	26	27	11	7	1	3	29	2	29	22
9	0	27	3	17	22	1	27	14	10	49	1	1	11	2	16	21
10	0	29	57	17	15	1	27	54	10	31	0	58	49	2	4	20
11	0	32	48	17	8	1	28	28	10	13	0	56	22	1	53	19
12	0	35	38	17	0	1	28	56	9	55	0	53	51	1	42	18
13	0	38	25	16	52	1	29	17	9	37	0	51	15	1	31	17
14	0	41	10	16	44	1	29	32	9	18	0	48	36	1	21	16
15	0	43	51	16	35	1	29	40	8	59	0	45	53	1	11	15
16	0	46	30	16	26	1	29	40	8	40	0	43	6	1	2	14
17	0	49	5	16	17	1	29	37	8	20	0	40	16	0	53	13
18	0	51	37	16	7	1	29	25	8	0	0	37	22	0	45	12
19	0	54	6	15	56	1	29	7	7	40	0	34	26	0	38	11
20	0	56	31	15	44	1	28	42	7	21	0	31	27	0	32	10
21	0	58	52	15	32	1	28	11	7	2	0	28	25	0	26	9
22	1	1	9	15	19	1	27	33	6	44	0	25	22	0	21	8
23	1	3	22	15	6	1	26	49	6	26	0	22	16	0	16	7
24	1	5	30	14	52	1	25	58	6	8	0	19	8	0	12	6
25	1	7	34	14	38	1	25	0	5	50	0	15	59	0	8	5
26	1	9	32	14	24	1	23	57	5	33	0	12	49	0	5	4
27	1	11	27	14	9	1	22	47	5	16	0	9	38	0	3	3
28	1	13	16	13	54	1	21	31	4	59	0	6	26	0	1	2
29	1	15	1	13	38	1	20	9	4	43	0	3	13	0	0	1
30	1	16	40	13	22	1	18	41	4	27	0	0	0	0	0	0
	1 5			Inclin.		10. ... 4			Inclin.		9 ... 3			Inclin.		0
	SUBSTRACT															

A TABLE of the simple Latitude of the Moon, fitted to the least Inclination of its Orbit; with the Increments to the greatest Inclination.

Arg. Lat.	Sig. $\frac{0}{6}$ North. South. Lat. ☽.			A. Add Increm^{t}.		Sig. $\frac{1}{7}$ North. South. Lat. ☽.			A. Add Increm^{t}.		Sig. $\frac{2}{8}$ North. South. Lat. ☽.			A. Add Increm^{t}.		Arg. Lat.
	°	′	″	′	″	°	′	″	′	″	°	′	″	′	″	
0	0	0	0	0	0	2	29	39	8	50	4	19	22	15	21	30
1	0	5	14	0	19	2	34	9	9	7	4	21	58	15	30	29
2	0	10	27	0	37	2	38	37	9	21	4	24	56	15	39	28
3	0	15	40	0	56	2	43	1	9	39	4	26	52	15	47	27
4	0	20	52	1	14	2	47	22	9	54	4	29	12	15	56	26
5	0	26	4	1	32	2	51	41	10	9	4	31	27	16	4	25
6	0	31	17	1	50	2	55	56	10	25	4	33	37	16	12	24
7	0	36	28	2	9	3	0	8	10	39	4	35	43	16	19	23
8	0	41	38	2	28	3	4	18	10	54	4	37	43	16	27	22
9	0	46	48	2	46	3	8	23	11	8	4	39	38	16	34	21
10	0	51	57	3	5	3	12	25	11	21	4	41	28	16	40	20
11	0	57	5	3	23	3	16	24	11	37	4	43	13	16	46	19
12	1	2	13	3	40	3	20	19	11	51	4	44	53	16	52	18
13	1	7	18	3	58	3	24	11	12	4	4	46	27	16	57	17
14	1	12	23	4	16	3	27	58	12	19	4	47	57	17	3	16
15	1	17	27	4	34	3	31	42	12	31	4	49	21	17	8	15
16	1	22	29	4	52	3	35	22	12	45	4	50	39	17	13	14
17	1	27	29	5	11	3	38	58	12	58	4	51	53	17	18	13
18	1	32	28	5	28	3	42	30	13	8	4	53	1	17	22	12
19	1	37	26	5	46	3	45	5	13	21	4	54	4	17	25	11
20	1	42	21	6	4	3	49	22	13	33	4	55	1	17	29	10
21	1	47	14	6	19	3	52	42	13	47	4	55	43	17	32	9
22	1	52	6	6	38	3	55	58	13	58	4	56	39	17	35	8
23	1	56	56	6	56	3	59	9	14	8	4	57	20	17	37	7
24	2	1	43	7	10	4	2	16	14	19	4	57	56	17	39	6
25	2	6	28	7	27	4	5	18	14	30	4	58	26	17	41	5
26	2	11	11	7	44	4	8	16	14	41	4	58	21	17	43	4
27	2	15	52	8	4	4	11	9	14	54	4	59	10	17	44	3
28	2	20	30	8	19	4	13	58	15	2	4	59	24	17	44	2
29	2	25	7	8	35	4	16	42	15	11	4	59	32	17	45	1
30	2	29	39	8	50	4	19	22	15	21	4	59	35	17	45	0
Deg.	Sig. $\frac{5}{11}$ Nor. Sou.			Desc.		Sig. $\frac{4}{10}$ Nor. Sou.			Desc.		Sig. $\frac{3}{9}$ North. South.			Desc.		Deg.

A TABLE of Reduction, with the Excess above the least Inclination $4^{\circ} : 59' : 35''$

Arg. Lat.	Signs (0/0) Reduc. Subst.		Exc.	Signs (1/7) Reduc. Subst.		Exc.	Signs (2/8) Reduc. Subst.		Exc.	Arg. Lat.
	′	″	″	′	″	″	′	″	″	
0	0	0	0	5	40	42	5	40	42	30
1	0	14	2	5	47	43	5	33	41	29
2	0	21	4	5	53	44	5	25	40	28
3	0	41	6	5	59	45	5	17	39	27
4	0	55	8	6	4	46	5	10	38	26
5	1	8	9	6	8	46	5	1	37	25
6	1	22	11	6	13	46	4	52	36	24
7	1	35	12	6	17	47	4	42	35	23
8	1	48	13	6	20	47	4	33	34	22
9	2	1	15	6	23	47	4	23	33	21
10	2	14	17	6	26	47	4	13	31	20
11	2	27	19	6	28	47	4	2	30	19
12	2	40	20	6	30	48	3	51	29	18
13	2	52	22	6	31	48	3	40	28	17
14	3	4	23	6	32	48	3	29	26	16
15	3	16	24	6	32	48	3	17	24	15
16	3	28	26	6	32	48	3	5	23	14
17	3	40	28	6	31	48	2	53	22	13
18	3	51	29	6	30	48	2	40	20	12
19	4	2	30	6	29	47	2	28	19	11
20	4	12	31	6	26	47	2	15	17	10
21	4	23	33	6	23	47	2	2	15	9
22	4	33	34	6	20	47	1	49	13	8
23	4	44	35	6	17	47	1	35	12	7
24	4	52	36	6	12	46	1	22	11	6
25	5	1	37	6	8	46	1	8	9	5
26	5	9	38	6	3	46	0	55	8	4
27	5	18	39	5	58	45	0	41	6	3
28	5	26	40	5	52	44	0	27	4	2
29	5	33	41	5	46	43	0	14	2	1
30	5	40	42	5	40	42	0	0	0	0
Deg.	Signs (5/11) Add			Signs (4/10) Add			Signs (3/9) Add			Deg.

A TABLE

A TABLE of the true hourly Motion, and Semidiamer of the Sun, with the Horizontal Parallax of the Moon, at leaſt and greateſt Excentricities in Eclipſes.

Mean Anomaly of ☉ and ☽.		True Hourly Mot. ☉		Apparent Semidiam. of ☉.		Horizontal Parallax of ☉. 43323		66777		Mean Anomaly of ☉ and ☽.	
ſ	*o*	′	″	′	″	′	″	′	″		
0	0	2	23	15	49	55	7	53	54	12	0
	6	2	23	15	49	55	8	53	55		24
	12	2	23	15	49	55	10	53	57		18
	18	2	23	15	50	55	14	54	3		12
	24	2	23	15	50	55	18	54	10		6
0	0	2	24	15	51	55	24	54	18	11	0
	6	2	24	15	51	55	31	54	29		24
	12	2	24	15	52	55	40	54	41		18
	18	2	24	15	53	55	50	54	57		12
	24	2	25	15	55	56	1	55	11		6
2	0	2	25	15	56	56	13	55	28	10	0
	6	2	26	15	57	56	26	55	47		24
	12	2	26	15	59	56	39	56	7		18
	18	2	27	16	1	56	54	56	29		12
	24	2	27	16	3	57	8	56	52		6
3	0	2	28	16	5	57	24	57	15	9	0
	6	2	28	16	6	57	39	57	39		24
	12	2	29	16	7	57	55	58	3		18
	18	2	29	16	10	58	11	58	28		16
	24	2	30	16	11	58	27	58	53		6
4	0	2	30	16	13	58	42	59	17	8	0
	6	2	31	16	14	58	56	59	40		24
	11	2	31	16	15	59	10	60	1		18
	18	2	32	16	16	59	22	60	22		12
	24	2	32	16	18	59	33	60	41		6
5	0	2	32	16	19	59	43	60	57	7	0
	6	2	33	16	20	59	52	61	11		24
	12	2	33	16	20	59	58	61	23		18
	18	2	33	16	21	60	2	61	31		12
	24	2	33	16	22	60	5	61	35		6
6	0	2	33	16	22	60	6	61	36	6	0
							23454 Difference of Excen.				

A TABLE

A TABLE of the Reduction of the Time between the true Conjunction and Oppoſition of the Moon in her Orbit to the Middle of the Eclipſe.

	Signs. 0 . . . 6 Substract.																					
	True Hourly Motion of ☽ *a.* ☉.																					
	′ 27		′ 28		′ 29		′ 30		′ 31		′ 32		′ 33		′ 34		′ 35		′ 36			
☉	′	″	′	″	′	″	′	″	′	″	′	″	′	″	′	″	′	″	′	″		
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	30	
1	0	36	0	35	0	34	0	32	0	31	0	30	0	29	0	28	0	27	0	26	29	
2	1	13	1	11	1	8	1	6	1	3	1	2	1	0	0	57	0	55	0	53	28	
3	1	50	1	47	1	43	1	39	1	35	1	33	1	30	1	26	1	23	1	21	27	
4	2	27	2	22	2	17	2	12	2	7	2	3	1	59	1	54	1	51	1	48	26	
5	3	4	2	58	2	51	2	45	2	39	2	33	2	29	2	23	2	19	2	15	25	
6	3	40	3	33	3	24	3	17	3	10	3	3	2	58	2	51	2	47	2	41	24	
7	4	16	4	7	3	58	3	49	3	41	3	33	3	27	3	20	3	15	3	8	23	
8	4	52	4	41	4	30	4	21	4	12	4	3	3	55	3	48	3	41	3	35	22	
9	5	27	5	14	5	2	4	52	4	42	4	32	4	23	4	15	4	8	4	2	21	
10	6	1	5	47	5	33	5	22	5	11	5	0	4	50	4	42	4	33	4	28	20	
11	6	34	6	19	6	4	5	51	5	39	5	27	5	16	5	7	4	59	4	52	19	
12	7	5	6	49	6	34	6	19	6	6	5	53	5	42	5	32	5	23	5	16	18	
13	7	36	7	19	7	3	6	47	6	33	6	19	6	7	5	56	5	46	5	39	17	
14	8	7	7	48	7	31	7	14	6	59	6	44	6	31	6	19	6	9	6	1	16	
15	8	36	8	16	7	58	7	41	7	25	7	10	6	56	6	42	6	31	6	23	15	
	Signs. 5 . . . 11 Add.																					

A TABLE

A TABLE of Logiſtical Logarithms, being in its Uſe the moſt ready Way for finding the Proportional Parts in all Aſtronomical Calculations; was invented by THOMAS STREET, and publiſhed in his *Aſtronomia Carolina.*

The Manner of Conſtructing it, is thus:

RULE.

To the Logarithm of $1^{\circ} = 60' = 3600''$, add the Complement-Arithmetical of the Logarithm of the given Minutes, or Minutes and Seconds when reduced to Seconds; and the Sum (rejecting Radius) will be the Logiſtical Logarithm required.

EXAMPLE 1.

LET it be required to find the Logiſtical Logarithm of 8 Minutes.

The Logarithm of the abſolute Number 3600 is — —	3·5563025
And the Comp. Arith. of the Logarithm of $8' = 480''$ is —	7·3187588
The Sum, rejecting Radius, is the Logiſtical Logarithm required. —	·8750613

EXAMPLE 2.

WHAT is the Logiſtical Logarithm of $24' : 26''$.

The Logarithm of $3600''$ — — — —	3·5563025
The Comp. Arith. of the Logarithm of $24' : 26'' = 1466$, is —	6.8338660
The Sum is the Logiſtical Logarithm of $24' : 26'' = 3902$ —	·3901685

EXAMPLE 3.

To find the Logiſtical Logarithm of more than 60′, as of $86' : 28''$.

Reduce $86' : 28''$ into Seconds, and it is $5188''$, the Log. is —	3·7150000
To this add the conſtant Logarithm — — —	6.4436975
The Sum rejecting Radius is the Logiſt. Logar. of $86' : 28''$ —	·1586975

Note, THAT the conſtant Logarithm 6.4436975 is the Complement-Arithmetical of 3·5563025, which is the Logarithm of 3600, as above.

IF, from the Logiſtical Logarithm thus found, you cut off the three Figures towards the Right Hand, thoſe that remain towards the Left will be ſufficient; ſo, in the firſt Example 8751 may paſs for the Logiſtical Logarithm of 8′, and, in the ſecond Example, 3902 for that of $24' : 26''$.

STREET's Logiſtical Logarithms.

′	0	1	2	3	4	5	6	7	8	9
″	0	60	120	180	240	300	360	420	480	540
0		17782	14771	13010	11761	10792	10000	9331	8751	8239
1	35563	17710	14735	12986	11743	10777	9988	9320	8742	8231
2	32553	17639	14699	12962	11725	10763	9976	9310	8733	8223
3	30792	17570	14664	12939	11707	10749	9964	9300	8724	8215
4	29542	17501	14629	12915	11689	10734	9952	9289	8715	8207
5	28573	17434	14594	12891	11671	10720	9940	9279	8706	8199
6	27782	17368	14559	12868	11654	10706	9928	9269	8697	8191
7	27112	17302	14525	12845	11636	10692	9916	9259	8688	8183
8	26532	17238	14491	12821	11619	10678	9905	9249	8679	8175
9	26021	17175	14457	12789	11601	10663	9893	9238	8670	8167
10	25563	17112	14424	12775	11584	10649	9881	9228	8661	8159
11	25149	17050	14390	12753	11566	10635	9869	9218	8652	8152
12	24771	16990	14357	12730	11549	10621	9858	9208	8643	8144
13	24424	16930	14325	12707	11532	10608	9846	9198	8635	8136
14	24102	16871	14292	12685	11515	10594	9834	9188	8626	8128
15	23802	16812	14260	12663	11498	10580	9823	9178	8617	8120
16	23522	16755	14228	12640	11481	10566	9811	9168	8608	8112
17	23259	16698	14196	12618	11464	10552	9800	9158	8599	8104
18	23010	16642	14165	12596	11447	10539	9788	9148	8591	8097
19	22775	16587	14133	12574	11430	10525	9777	9138	8582	8089
20	22553	16532	14102	12553	11413	10512	9765	9128	8573	8081
21	22341	16478	14071	12531	11397	10498	9754	9119	8565	8073
22	22139	16425	14042	12510	11380	10484	9742	9109	8556	8066
23	21946	16372	14010	12488	11363	10471	9731	9099	8547	8058
24	21761	16320	13979	12467	11347	10458	9720	9089	8539	8050
25	21584	16269	13949	12445	11331	10444	9708	9079	8530	8043
26	21413	16218	13919	12424	11314	10431	9697	9070	8522	8035
27	21249	16168	13890	12403	11298	10418	9686	9060	8513	8027
28	21091	16118	13860	12382	11282	10404	9675	9050	8504	8020
29	20939	16069	13831	12362	11266	10391	9664	9041	8496	8012
30	20792	16021	13802	12341	11249	10378	9652	9031	8487	8004

STREET's Logistical Logarithms.

′	0	1	2	3	4	5	6	7	8	9
″	0	60	120	180	240	300	360	420	480	540
30	20792	16021	13802	12341	11249	10378	9652	9031	8487	8004
31	20649	15973	13773	12320	11233	10365	9641	9021	8479	7997
32	20512	15925	13745	12300	11217	10352	9630	9012	8470	7989
33	20378	15878	13716	12279	11201	10339	9619	9002	8462	7981
34	20248	15832	13688	12259	11186	10326	9608	8992	8453	7974
35	20122	15786	13660	12239	11170	10313	9597	8983	8445	7966
36	20000	15740	13632	12218	11154	10300	9586	8973	8437	7959
37	19881	15695	13604	12198	11138	10287	9575	8964	8428	7951
38	19765	15651	13576	12178	11123	10274	9564	8954	8420	7944
39	19652	15607	13549	12159	11107	10261	9553	8945	8411	7936
40	19542	15563	13522	12139	11091	10248	9542	8935	8403	7929
41	19435	15520	13495	12119	11076	10235	9532	8926	8395	7921
42	19331	15477	13468	12099	11061	10223	9521	8917	8386	7914
43	19228	15435	13441	12080	11045	10210	9510	8907	8378	7906
44	19128	15393	13415	12061	11030	10197	9499	8898	8370	7899
45	19031	15351	13388	12041	11015	10185	9488	8888	8361	7891
46	18935	15310	13362	12022	10999	10172	9478	8879	8353	7884
47	18842	15269	13336	12003	10984	10160	9467	8870	8345	7877
48	18751	15229	13310	11984	10969	10147	9456	8861	8337	7869
49	18661	15189	13284	11965	10954	10135	9446	8851	8328	7862
50	18573	15149	13259	11946	10939	10122	9435	8842	8320	7855
51	18487	15110	13233	11927	10924	10110	9425	8833	8312	7847
52	18403	15071	13208	11908	10909	10098	9414	8824	8304	7840
53	18320	15032	13183	11889	10894	10085	9404	8814	8296	7832
54	18229	14994	13158	11871	10880	10073	9393	8805	8288	7825
55	18159	14956	13133	11852	10865	10061	9383	8796	8279	7818
56	18081	14918	13108	11834	10850	10049	9372	8787	8271	7811
57	18004	14881	13083	11816	10835	10036	9362	8778	8263	7803
58	17929	14844	13059	11797	10821	10024	9351	8769	8255	7796
59	17855	14808	13034	11779	10806	10012	9341	8760	8247	7789
60	17782	14771	13010	11761	10792	10000	9331	8751	8239	7782

STREET's Logistical Logarithms.

′	10	11	12	13	14	15	16	17	18	19
″	600	660	720	780	840	900	960	1020	1080	1140
0	7782	7368	6990	6642	6320	6021	5740	5477	5229	4994
1	7774	7361	6984	6637	6315	6016	5736	5473	5225	4990
2	7767	7354	6978	6631	6310	6011	5731	5469	5221	4986
3	7760	7348	6972	6625	6305	6006	5727	5464	5217	4983
4	7753	7341	6966	6620	6300	6001	5722	5460	5213	4979
5	7745	7335	6960	6614	6294	5997	5718	5456	5209	4975
6	7738	7328	6954	6609	6289	5992	5713	5452	5205	4971
7	7731	7322	6948	6603	6284	5987	5709	5447	5201	4967
8	7724	7315	6942	6598	6279	5982	5704	5443	5197	4964
9	7717	7309	6936	6592	6274	5977	5700	5439	5193	4960
10	7710	7302	6930	6587	6269	5973	5695	5435	5189	4956
11	7703	7296	6924	6581	6264	5968	5691	5430	5185	4952
12	7696	7289	6918	6576	6259	5963	5686	5426	5181	4949
13	7688	7283	6912	6570	6254	5958	5682	5422	5177	4945
14	7681	7276	6906	6565	6248	5954	5677	5418	5173	4941
15	7674	7270	6900	6559	6243	5949	5673	5414	5169	4937
16	7667	7264	6894	6554	6238	5944	5669	5409	5165	4933
17	7660	7257	6888	6548	6233	5939	5664	5405	5161	4930
18	7653	7251	6882	6543	6228	5935	5660	5401	5157	4926
19	7646	7244	6877	6538	6223	5930	5655	5397	5153	4922
20	7639	7238	6871	6532	6218	5925	5651	5393	5149	4918
21	7632	7232	6865	6527	6213	5920	5646	5389	5145	4915
22	7625	7225	6859	6521	6208	5916	5642	5384	5141	4911
23	7618	7219	6853	6516	6203	5911	5637	5380	5137	4907
24	7611	7212	6847	6510	6198	5906	5633	5376	5133	4903
25	7604	7206	6841	6505	6193	5902	5629	5372	5129	4900
26	7597	7200	6836	6500	6188	5897	5624	5368	5125	4896
27	7590	7193	6830	6494	6183	5892	5620	5364	5122	4892
28	7583	7187	6824	6489	6178	5888	5615	5359	5118	4889
29	7577	7181	6818	6484	6173	5883	5611	5355	5114	4885
30	7570	7175	6812	6478	6168	5878	5607	5351	5110	4881

STREET's Logistical Logarithms.

′	10	11	12	13	14	15	16	17	18	19
″	600	660	720	780	840	900	960	1020	1080	1140
30	7570	7175	6812	6478	6168	5878	5607	5351	5110	4881
31	7563	7168	6807	6473	6163	5874	5602	5347	5106	4877
32	7556	7162	6801	6467	6158	5869	5598	5343	5103	4874
33	7549	7156	6795	6462	6153	5864	5594	5339	5098	4870
34	7542	7149	6789	6457	6148	5860	5589	5335	5094	4866
35	7535	7143	6784	6451	6143	5855	5585	5331	5090	4863
36	7528	7137	6778	6446	6138	5850	5580	5326	5086	4859
37	7522	7131	6772	6441	6133	5846	5576	5322	5082	4855
38	7515	7124	6766	6435	6128	5841	5572	5318	5079	4852
39	7508	7118	6761	6430	6123	5836	5567	5314	5075	4848
40	7501	7112	6755	6425	6118	5832	5563	5310	5071	4844
41	7494	7106	6749	6420	6113	5827	5559	5306	5067	4841
42	7488	7100	6743	6414	6108	5823	5554	5302	5063	4837
43	7481	7093	6738	6409	6103	5818	5550	5298	5059	4833
44	7474	7087	6732	6404	6099	5813	5546	5294	5055	4830
45	7467	7081	6726	6398	6094	5809	5541	5290	5051	4826
46	7461	7075	6721	6393	6089	5804	5537	5285	5048	4822
47	7454	7069	6715	6388	6084	5800	5533	5281	5044	4819
48	7447	7063	6709	6383	6079	5795	5528	5277	5040	4815
49	7441	7057	6704	6377	6074	5790	5524	5273	5036	4811
50	7434	7050	6698	6372	6069	5786	5520	5269	5032	4808
51	7427	7044	6692	6367	6064	5781	5516	5265	5028	4804
52	7421	7338	6687	6362	6059	5777	5511	5261	5025	4800
53	7414	7032	6681	6357	6055	5772	5507	5257	5021	4797
54	7407	7026	6676	6351	6050	5768	5503	5253	5017	4793
55	7401	7020	6670	6346	6045	5763	5498	5249	5013	4789
56	7394	7014	6664	6341	6040	5758	5494	5245	5009	4786
57	7387	7008	6659	6336	6035	5754	5490	5241	5005	4782
58	7381	7002	6653	6331	6030	5749	5486	5237	5002	4778
59	7374	6996	6648	6325	6025	5745	5481	5233	4998	4775
60	7368	6990	6642	6320	6021	5740	5477	5229	4994	4771

STREET's Logistical Logarithms.

′	20	21	22	23	24	25	26	27	28	29
″	1200	1260	1320	1380	1440	1500	1560	1620	1680	1740
0	4771	4559	4357	4164	3979	3802	3632	3468	3310	3158
1	4768	4556	4354	4161	3976	3799	3629	3465	3307	3155
2	4764	4552	4351	4158	3973	3796	3626	3463	3305	3153
3	4760	4549	4347	4155	3970	3793	3623	3460	3302	3150
4	4757	4546	4344	4152	3967	3791	3621	3457	3300	3148
5	4753	4542	4341	4149	3964	3788	3618	3454	3297	3145
6	4750	4539	4338	4145	3961	3785	3615	3452	3294	3143
7	4746	4535	4334	4142	3958	3782	3612	3449	3292	3140
8	4742	4532	4331	4139	3955	3779	3610	3446	3289	3138
9	4739	4528	4328	4136	3952	3776	3607	3444	3287	3135
10	4735	4525	4324	4133	3949	3773	3604	3441	3284	3133
11	4732	4522	4321	4130	3946	3770	3601	3438	3282	3130
12	4728	4518	4318	4127	3943	3768	3598	2436	3279	3128
13	4724	4515	4315	4124	3940	3765	3596	3433	3276	3125
14	4721	4511	4311	4120	3937	3762	3593	3431	3274	3123
15	4717	4508	4308	4117	3934	3759	3590	3428	3271	3120
16	4714	4505	4305	4114	3931	3756	3587	3425	3269	3118
17	4710	4501	4302	4111	3928	3753	3585	3423	3266	3115
18	4707	4498	4298	4108	3925	3750	3582	3420	3264	3113
19	4703	4494	4295	4105	3922	3747	3579	3417	3261	3110
20	4699	4491	4292	4102	3919	3745	3576	3415	3259	3108
21	4696	4488	4289	4099	3917	3742	3574	3412	3256	3105
22	4692	4484	4285	4096	3914	3739	3571	3409	3253	3103
23	4689	4481	4282	4092	3911	3736	3568	3407	3251	3101
24	4685	4477	4279	4089	3908	3733	3565	3404	3248	3098
25	4682	4474	4276	4086	3905	3730	3563	3401	3246	3096
26	4678	4471	4273	4083	3902	3727	3560	3399	3243	3093
27	4675	4467	4269	4080	3899	3725	3557	3396	3241	3091
28	4671	4463	4266	4077	3896	3722	3555	3393	3238	3088
29	4668	4460	4263	4074	3893	3719	3552	3391	3236	3086
30	4664	4457	4260	4071	3890	3716	3549	3388	3233	3083

STREET's Logistical Logarithms.

′	20	21	22	23	24	25	26	27	28	29
″	1200	1260	1320	1380	1440	1500	1560	1620	1680	1740
30	4664	4457	4260	4071	3890	3716	3549	3388	3233	3083
31	4660	4454	4256	4068	3887	3713	3546	3386	3231	3081
32	4657	4450	4253	4065	3883	3710	3544	3383	3228	3078
33	4653	4447	4250	4062	3881	3708	3541	3380	3225	3076
34	4650	4444	4247	4059	3878	3705	3538	3378	3223	3073
35	4646	4440	4244	4055	3875	3702	3535	3375	3220	3071
36	4643	4437	4240	4052	3872	3699	3533	3372	3218	3069
37	4639	4434	4237	4049	3869	3696	3530	3370	3215	3066
38	4636	4430	4234	4046	3866	3693	3527	3367	3213	3064
39	4632	4427	4231	4043	3863	3691	3525	3365	3210	3061
40	4629	4424	4228	4040	3860	3688	3522	3362	3208	3059
41	4625	4420	4224	4037	3857	3685	3519	3359	3205	3056
42	4622	4417	4221	4034	3855	3682	3516	3357	3203	3054
43	4618	4414	4218	4031	3852	3679	3514	3354	3200	3052
44	4615	4410	4215	4028	3849	3677	3511	3351	3198	3049
45	4611	4407	4212	4025	3846	3674	3508	3349	3195	3247
46	4608	4404	4209	4022	3843	3671	3506	3346	3193	3044
47	4604	4400	4205	4019	3840	3668	3502	3344	3190	3042
48	4601	4397	4202	4016	3837	3665	3500	3341	3188	3039
49	4597	4394	4199	4013	3834	3663	3497	3338	3185	3037
50	4594	4390	4196	4010	3831	3660	3495	3336	3183	3034
51	4590	4387	4193	4007	3828	3657	3492	3333	3180	3032
52	4587	4384	4189	4004	3825	3654	3489	3331	3178	3030
53	4584	4380	4186	4001	5822	3651	3487	3328	3175	3027
54	4580	4377	4183	3998	3820	3649	3484	3325	3173	3025
55	4577	4374	4180	3995	3817	3646	3481	3323	3170	3022
56	4573	4370	4177	3991	3814	3643	3479	3320	3168	3020
57	4570	4367	4174	3988	3811	3640	3476	3318	3165	3018
58	4566	4364	4171	3985	3808	3637	3473	3315	3163	3015
59	4563	4361	4167	3982	3805	3635	3471	3313	3160	3013
60	4559	4357	4164	3979	3802	3632	3468	3310	3158	3010

STREET's Logistical Logarithms.

′	30	31	32	33	34	35	36	37	38	39
″	1800	1860	1920	1980	2040	2100	2160	2220	2280	2340
0	3010	2868	2730	2596	2467	2341	2218	2099	1984	1871
1	3008	2866	2728	2594	2465	2339	2216	2098	1982	1869
2	3005	2863	2725	2592	2462	2337	2214	2096	1980	1867
3	3003	2861	2723	2590	2460	2335	2212	2094	1978	1865
4	3001	2859	2721	2588	2458	2333	2210	2092	1978	1863
5	2998	2856	2719	2585	2456	2331	2208	2090	1974	1862
6	2996	2854	2716	2583	2454	2328	2206	2088	1972	1160
7	2993	2852	2714	2581	2452	2326	2204	2086	1970	1858
8	2991	2849	2712	2579	2450	2324	2202	2084	1968	1856
9	2989	2847	2710	2577	2448	2322	2200	2082	1967	1854
10	2986	2845	2707	2574	2445	2320	2198	2080	1965	1852
11	2984	2842	2705	2572	2443	2318	2196	2078	1963	1850
12	2981	2840	2703	2570	2441	2316	2194	2076	1961	1849
13	2979	2838	2701	2568	2439	2314	2192	2074	1959	1847
14	2977	2835	2698	2566	2437	2312	2190	2072	1957	1845
15	2974	2833	2696	2564	2435	2310	2188	2070	1955	1843
16	2972	2831	2694	2561	2433	2308	2186	2068	1653	1841
17	2969	2828	2692	2559	2431	2306	2184	2066	1951	1839
18	2967	2826	2689	2557	2429	2304	2182	2064	1950	1838
19	2965	2824	2687	2555	2426	2302	2180	2062	1948	1836
20	2962	2821	2685	2553	2424	2300	2178	2061	1946	1834
21	2960	2819	2683	2551	2422	2298	2176	2059	1944	1832
22	2958	2817	2681	2548	2420	2296	2174	2057	1942	1830
23	2955	2815	2678	2546	2418	2294	2172	2055	1940	1828
24	2953	2812	2676	2544	2416	2291	2170	2053	1938	1827
25	2950	2810	2674	2542	2414	2289	2169	2051	1936	1825
26	2948	2808	2672	2540	2412	2287	2167	2049	1934	1823
27	2946	2805	2669	2538	2410	2285	2165	2047	1933	1821
28	2943	2803	2667	2535	2408	2283	2163	2045	1931	1819
29	2941	2801	2665	2533	2405	2281	2161	2043	1929	1817
30	2939	2798	2663	2531	2403	2279	2159	2041	1927	1816

STREET's Logiſtical Logarithms.

′	30	31	32	33	34	35	36	37	38	39
″	1800	1860	1920	1980	2040	2100	2160	2220	2280	2340
30	2939	2798	2663	2531	2403	2279	2159	2041	1927	1816
31	2936	2796	2660	2529	2401	2277	2157	2039	1925	1814
32	2934	2794	2658	2527	2399	2275	2155	2037	1923	1812
33	2931	2792	2656	2525	2397	2273	2153	2035	1921	1810
34	2929	2789	2654	2522	2395	2272	2151	2033	1919	1808
35	2927	2787	2652	2520	2393	2269	2149	2032	1918	1806
36	2924	2785	2649	2518	2391	2267	2147	2030	1916	1805
37	2922	2782	2647	2516	2389	2265	2145	202	1914	1803
38	2920	2780	2645	2514	2387	2263	2143	2026	1912	1801
39	2917	2778	2643	2512	2384	2261	2141	2024	1910	1799
40	2915	2775	2640	2510	2382	2259	2139	2022	1908	1797
41	2912	2773	2638	2507	2380	2257	2137	2020	1906	1795
42	2910	2771	2636	2505	2378	2255	2135	2018	1904	1794
43	2908	2769	2634	2503	2376	2253	2133	2016	1903	1792
44	2905	2766	2632	2501	2374	2251	2131	2014	1901	1790
45	2903	2764	2629	2499	2372	2249	2129	2012	1899	1788
46	2901	2762	2627	2497	2370	2247	2127	2010	1897	1786
47	2898	2760	2625	2494	2368	2245	2125	2009	1895	1785
48	2896	2757	2623	2492	2366	2243	2123	2007	1893	1783
49	2894	2755	2621	2490	2364	2241	2121	2005	1891	1781
50	2891	2753	2618	2488	2362	2239	3119	2003	1889	1779
51	2889	2750	2616	2486	2359	2237	2117	2001	1888	1777
52	2887	2748	2614	2484	2357	2235	2115	1999	1886	1775
53	2884	2746	2612	2482	2355	2233	2113	1997	1884	1774
54	2882	2744	2610	2480	2353	2231	2111	1995	1882	1772
55	2880	2741	2607	2477	2351	2229	2109	1993	1880	1770
56	2877	2739	2605	2475	2349	2227	2107	1991	1878	1768
57	2875	2737	2603	2473	2347	2225	2105	1989	1876	1766
58	2873	2735	2601	2471	2345	2223	2103	1987	1875	1765
59	2870	2732	2599	2469	2343	2220	2101	1986	1873	1763
60	2868	2730	2596	2467	2341	2218	2099	1984	1871	1761

STREET's Logistical Logarithms.

′	40	41	42	43	44	45	46	47	48	49
″	2400	2460	2520	2580	2640	2700	2760	2820	2880	2940
0	1761	1654	1549	1447	1347	1249	1154	1061	969	880
1	1759	1652	1547	1445	1345	1248	1152	1059	968	878
2	1757	1650	1546	1443	1344	1246	1151	1057	966	877
3	1755	1648	1544	1442	1342	1245	1149	1056	965	875
4	1754	1647	1542	1440	1340	1243	1148	1054	963	874
5	1752	1645	1540	1438	1339	1241	1146	1053	962	872
6	1750	1643	1539	1437	1337	1240	1145	1051	960	871
7	1748	1641	1537	1435	1335	1238	1143	1050	959	869
8	1746	1640	1535	1433	1334	1237	1141	1048	957	868
9	1745	1638	1534	1432	1332	1235	1140	1047	956	866
10	1743	1636	1532	1430	1331	1233	1138	1045	954	865
11	1741	5634	1530	1428	1329	1232	1137	1044	953	863
12	1739	1633	1528	1427	1327	1230	1135	1042	951	862
13	1737	1631	1527	1425	1326	1229	1134	1041	950	860
14	1736	1629	1525	1423	1324	1227	1132	1039	948	859
15	1734	1627	1523	1422	1322	1225	1130	1047	947	857
16	1733	1626	1522	1420	1321	1224	1129	1036	945	856
17	1730	1624	1520	1418	1319	1222	1127	1034	944	855
18	1728	1622	1518	1417	1317	1221	1126	1033	942	853
19	1727	1620	1516	1415	1316	1219	1124	1031	941	852
20	1725	1619	1515	1413	1314	1217	1123	1030	939	850
21	1723	1617	1513	1412	1313	1216	1121	1028	938	849
22	1721	1615	1511	1410	1311	1214	1119	1027	936	847
23	1719	1613	1510	1408	1309	1213	1118	1025	935	846
24	1718	1612	1508	1407	1308	1211	1116	1024	933	844
25	1716	1610	1506	1405	1306	1209	1115	1022	932	843
26	1714	1608	1504	1403	1304	1208	1113	1021	930	841
27	1712	1606	1503	1402	1303	1206	1112	1019	929	840
28	1711	1605	1501	1400	1301	1205	1110	1018	927	838
29	1709	1603	1499	1398	1300	1203	1109	1016	926	837
30	1707	1601	1498	1397	1298	1201	1107	1015	924	835

STREET's Logiſtical Logarithms.

′	40	41	42	43	44	45	46	47	48	49
″	2400	2460	2520	2580	2640	2700	2760	2820	2880	2940
30	1707	1601	1498	1397	1298	1201	1107	1015	924	835
31	1705	1599	1496	1395	1296	1200	1105	1013	923	834
32	1703	1598	1494	1393	1295	1198	1104	1012	921	833
33	1702	1596	1493	1392	1293	1197	1102	1010	920	831
34	1700	1594	1491	1390	1291	1195	1101	1008	918	830
35	1698	1592	1489	1388	1290	1193	1099	1007	917	828
36	1696	1591	1487	1387	1288	1192	1098	1005	915	827
37	1694	1589	1486	1385	1287	1190	1096	1004	914	825
38	1693	1587	1484	1383	1285	1189	1095	1002	912	824
39	1691	1585	1482	1382	1283	1187	1093	1001	911	822
40	1689	1584	1481	1380	1282	1186	1091	999	909	821
41	1687	1582	1479	1378	1280	1184	1090	998	908	819
42	1686	1580	1477	1377	1278	1182	1088	996	906	818
43	1684	1578	1476	1375	1277	1181	1087	995	905	816
44	1682	1577	1475	1373	1275	1179	1085	993	903	815
45	1680	1575	1472	1372	1274	1178	1084	992	902	814
46	1678	1573	1470	1370	5272	1176	1082	990	900	812
47	1677	1571	1469	1368	1270	1174	1081	989	899	811
48	1675	1570	1467	1367	1269	1173	1079	987	897	809
49	1673	1568	1465	1365	1267	1171	1078	986	896	808
50	1671	1566	1464	1363	1266	1170	1076	984	894	806
51	1670	1565	1462	1362	1264	1168	1074	983	893	805
52	1668	1563	1460	1360	1262	1167	1073	981	891	803
53	1666	1561	1459	1359	1261	1165	1071	980	890	802
54	1664	1559	1457	1357	1259	1163	1070	978	888	801
55	1663	1558	1455	1355	1257	1162	1068	977	887	799
56	1661	1556	1454	1354	1256	1160	1067	975	885	798
57	1659	1554	1452	1352	1254	1159	1065	974	884	796
58	1657	1552	1450	1350	1253	1157	1064	972	883	795
59	1655	1551	1449	1349	1251	1156	1062	971	881	793
60	1654	1549	1447	1347	1249	1154	1061	966	880	792

STREET's Logiſtical Logarithms.

′	50	51	52	53	54	55	56	57	58	59
″	3000	3060	3120	3180	3240	3300	3360	3420	3480	3540
0	792	706	621	539	458	378	300	223	147	73
1	790	704	620	537	456	377	298	221	146	72
2	789	703	619	536	455	375	297	220	145	71
3	787	702	617	535	454	374	296	219	143	69
4	786	700	616	533	452	373	294	218	142	68
5	785	699	615	532	451	371	293	216	141	67
6	783	697	613	531	450	370	292	215	140	66
7	782	696	612	529	448	369	291	214	139	64
8	780	694	610	528	447	367	289	213	137	63
9	779	693	609	526	446	366	288	211	136	62
10	777	692	608	525	444	365	287	210	135	61
11	776	690	606	524	443	363	285	209	134	60
12	774	689	605	522	442	362	284	208	132	58
13	773	687	603	521	440	361	283	206	131	57
14	772	686	602	520	439	359	282	205	130	56
15	770	685	601	518	438	358	280	204	129	55
16	769	683	599	517	436	357	279	202	127	53
17	767	682	598	516	435	256	278	201	126	52
18	766	680	596	514	434	354	276	200	125	51
19	764	679	595	513	432	353	275	199	124	50
20	763	678	594	512	431	352	274	197	122	49
21	762	676	592	510	430	350	273	196	121	47
22	760	675	591	509	428	349	271	195	120	46
23	759	673	590	507	427	348	270	194	119	45
24	757	672	588	506	426	346	269	192	117	44
25	756	670	587	505	424	345	267	191	116	42
26	754	669	585	503	423	344	266	190	115	41
27	753	668	584	502	422	342	265	189	114	40
28	751	666	583	501	420	341	264	187	112	39
29	750	665	581	499	419	340	262	186	111	38
30	749	663	580	498	418	339	261	185	110	36

STREET's Logiſtical Logarithms.

′	50	51	52	53	54	55	56	57	58	59
″	3000	3060	3120	3180	3240	3300	3360	3420	3480	3540
30	749	663	580	498	418	339	261	185	110	36
31	747	662	579	497	416	337	260	184	109	35
32	746	661	577	495	415	336	258	182	107	34
33	744	659	576	494	414	335	257	181	106	33
34	743	658	574	493	412	333	256	180	105	31
35	741	656	573	491	411	332	255	179	104	30
36	740	655	572	490	410	331	253	177	103	29
37	739	654	570	489	408	329	252	176	101	28
38	737	652	569	487	407	328	251	175	100	27
39	736	651	568	486	406	327	250	174	99	25
40	734	649	566	484	404	326	248	172	98	24
41	733	648	565	483	403	324	247	171	96	23
42	731	647	563	482	402	323	246	170	95	22
43	730	645	562	480	400	322	244	169	94	21
44	729	644	561	479	399	320	243	167	93	19
45	727	642	559	478	398	319	242	166	91	18
46	726	641	558	476	396	318	241	165	90	17
47	724	640	557	475	395	316	239	163	89	16
48	723	638	555	474	394	315	238	162	88	15
49	721	637	554	472	392	314	237	161	87	13
50	720	635	552	471	391	313	235	160	85	12
51	719	634	551	470	390	311	234	158	84	11
52	717	633	550	468	388	310	233	157	83	10
53	716	631	548	467	387	309	232	156	82	8
54	714	630	547	466	386	307	230	155	80	9
55	713	628	546	464	384	306	229	153	79	6
56	711	627	544	463	383	305	228	152	78	5
57	710	626	543	462	382	304	227	151	77	4
58	709	624	541	460	381	302	225	150	75	3
59	707	623	540	459	379	301	224	148	74	1
60	706	621	539	458	378	300	223	147	73	0

The Deſcription and Uſe of the TABLE of Logiſtical Logarithms.

THE Numbers in the Top, or uppermoſt Line of each Page alter their Denomination, as Occaſion requires; and ſo do the Figures of the firſt Column, accordingly: So, if you call the uppermoſt Figures Degrees, then thoſe in the firſt Column are Minutes: If you call the Figures in the uppermoſt Line Minutes, then thoſe in the firſt Column are Seconds. When the uppermoſt are Hours, the firſt Column are Minutes, *&c.*

THE Line next to the uppermoſt, are the Number of Minutes in thoſe Degrees that ſtand over them, when they are Degrees; but when they are Minutes, then thoſe under them are Seconds in ſo many Minutes. All the Numbers that are placed in each Column under theſe, are the Logiſtical Logarithms.

To find the Logiſtical Logarithm of Degrees and Minutes, or of Minutes under 60, *and Seconds.*

EXAMPLE.

LET the Logiſtical Logarithm of 36° : 18', or 36' : 18'' be requir'd.

UNDER 36 in the Top-Line, and againſt 18 in the firſt Column you will find 2182, the Anſwer.

The Uſe of the TABLE *of Logiſtical Logarithms.*

EXAMPLES.

1. IF 60 Minutes be in the firſt Place, and the Minutes in the ſecond and third Places are each leſs than 60; add the Logiſtical Logarithm of the ſecond and third Terms together, and the Sum will be the Logiſtical Logarithm of the fourth Term.

Thus, As	——	60' — —	0
Is to	——	2' : 27''	13890
So is	———	57 : 30	185
To	———	2 : 21	14075

2. IF 60' be the firſt Term, the ſecond leſs than 60', and the third more than 60'; to the Logiſtical Logarithm of the ſecond Term, add the Logiſtical Logarithm of half the third Term, the Sum will be the Logiſtical Logarithm of half the fourth Term; which double, and you'll have the Anſwer.

As 60'	——	——	0
Is to 2' : 24''	———	———	13979
So is 75' : 9'', half is 37' : 34''	——		2033
To 1' : 30'', which being doubled is 3' : 0'', the Anſwer			16012

3. IF

3. If 60′ be the firſt Term, the ſecond more than 60′, and the third leſs; add the Logiſtical Logarithm of half the ſecond Term to that of the third, the Sum will be the Logiſtical Logarithm of half the fourth Term; which double, and you will have the Anſwer.

As 60′	——	0
Is to 63′ : 41″, half 31′ : 50″	——	2753
So is 58 : 48	——	88
To 31′ : 12″, which being doubled is 62′ : 24″, the Anſwer	—	2841

4. If 60′ be the firſt Term, and each of the other Terms above 60′, then take half each and work as below:

As 60′ — half = 30′ : 0″	——	3010	ſubſt.
Is to 72′ : 24″ half = 36 : 12	——	2194	Add
So is 83 : 12 half = 41 : 36	——	1591	Add
		3785	Sum
To 50′ : 12″, which being doubled is 100′ : 24″, the Anſwer		775	

CHAP. XVII.

To calculate the SUN's Place.

1. From the Table of the Sun's Middle or mean Motion take out the Longitude and Anomaly anſwering to the given Year, Month, Day, Hour, Minute, and Second.

2. Add the mean Longitudes together, and alſo the mean Anomalies; ſo you will have the Sun's mean Longitude and Anomaly for the Time given.

3. With the Sun's mean Anomaly enter the Table of his Equation, where, if you find the Sign at the Top, then the Degrees are to be found in the firſt Column; but if the Sign be found at the Bottom, then you muſt find the Degrees in the laſt or Right-hand Column; and, in the Column Angle, or Place of Meeting, you will have the Sun's Equation, which you muſt either add to the Sun's mean Longitude, or ſubſtract from it, according as the Title at the Bottom or Top of the Table will direct you.

EXAMPLE 1.

EXAMPLE 1.

LET it be required to find the Sun's Place at *London*, upon *Auguſt* the 24th at 32 Minutes paſt 6 in the Afternoon, equal Time 1743.

	Long. ☉.	Anom. ☉.
1743 . . .	9ˢ : 20° : 33′ : 18″	6ˢ : 12° : 11′ : 42″
Auguſt 24 . . .	7 : 22 : 36 : 45	7 : 22 : 36 : 36
Hours 6 . . .	14 : 47	14 : 47
Min. 32 . . .	1 : 19	1 : 19
Mean Long. of ☉. . .	5 : 13 : 26 : 09	2 : 5 : 03 : 52
Equation ſubſtract. . .	1 : 44 : 32	
Remains the Sun's true Place . .	5 : 11 : 41 : 37	

THE mean Anomaly of the Sun being 2ˢ : 5° : 3′ : 52″, look for 2ˢ and you will find it at the Top of the Table of the Sun's Equation; under which, and againſt 5 Deg. in the firſt Column you have 1° : 44′ : 29″, which if the mean Anomaly had been only 2ˢ : 5°, then this would have been the Equation to be ſubſtracted from the Sun's mean Longitude; but becauſe there is 3′ : 52″ more, therefore you muſt find the proportional Parts agreeing thereto.

THUS, For . . .	2ˢ : 5°	you have	1° : 44′ : 29″
And for	2 : 6	—	1 : 45 : 21
Their Difference is		—	0 : 52
NOW, If	1° = 60′	—	0
Gives a Difference of	0′ : 52″	—	18403
What will	— 3 : 52	—	11908
Anſwer — —	0 : 03	—	30311 L. L.

Then becauſe the Equations are increaſing, this 0′ : 3″ muſt be added to 1° : 44′ : 29″, and the Sum is 1° : 44′ : 32″; which, according to the Title, you muſt ſubſtract from the Sun's mean Longitude, as at the Example.

Note, THAT Aſtronomers reckon their Time from Noon to Noon, ſo that at 9 in the Morning of (ſuppoſe) the 25th Day of any Month is 21 Hours after the Noon of the 24th Day, as at the next Example.

EXAMPLE 2.

LET the Sun's Place be required upon *May* 15th at 18′ and 42″ paſt 10 in the Morning, 1743, equal Time.

	Long. ☉.				Anomaly ☉.			
	∫	°	′	″	∫	°	′	″
1743,	9	20	33	18	6	12	11	42
May 14 D.	4	12	4	36	4	12	4	14
Hours 22			54	13			54	13
Min. 18			0	44			0	44
Sec. 42			0	02			0	02
Mean Longitude of ☉	2	3	32	53	10	25	10	55
Equation add — — —	—	1	05	17				
The Sum is the Sun's true Place — —	2	4	38	10				

EXAMPLE 3.

LET the Sun's Place be required *July* 18^{d} : 5^{h} : 12′ : 20″ *P. M.* 1728, apparent Time.

	Long. ☉.				Anom. ☉.			
	∫	°	′	″	∫	°	′	″
1728,	9	20	11	43	6	12	5	52
July 18 Biſſex.	6	17	7	45	6	17	7	11
Hours 5 — —			12	19			12	19
Min. 12 — —			0	30			0	30
Sec. 20 — —			0	01			0	01
Mean Longitude of ☉	4	7	32	18	0	29	25	53
Equation ſubſtract			56	07				
Sun's Place at the apparent Time	4	6	36	11				

THE Sun's Place and mean Anomaly being now found for the given apparent Time, then to reduce it to equal Time you muſt take the Sun's Place to the Table of the firſt Part of the Equation of Time, and thence take out the Equation 9′ : 36″ with its Title. Add to the apparent Time.

AGAIN, with the Sun's mean Anomaly 0^{s} : 29° : 25′ : 53″ enter the ſecond Part of the Table, and thence take out the Equation 3′ : 42″, with its Title ſubſtract from the apparent Time. Now, becauſe theſe two Equations are of different Denominations, you muſt take their Difference, and give that the Title of the greater.

THUS, The greater is — —	9′ : 36″	Add		
The leſs — —	3 : 42	Subſtract		
Their Difference is the Equation — —	5 : 54	Add	}	
The apparent Time given was — —	18^{d} : 5^{h} : 12 : 20			
The Sum is the equal Time — —	18 : 5 : 18 : 14			

If these two Equations of Time had been of one Denomination, that is, both add, or both subſtract, then their Sum would have been the Equation of Time, and to be applied to the apparent Time, according to the common Title of add, or ſubſtract.

Having thus obtained the equal Time, you muſt find the Sun's Place for that Time: For this Purpoſe, take the Sun's mean Anomaly to the Table of the hourly Motion, and Semidiameter of the Sun, *&c.* and thence take out the hourly Motion of the Sun 2′ : 23″. Then ſay,

If 1 Hour, or 60′		0
gives a Motion of 2′ : 23″		14010 } L.L.
what will the Equation 5′ : 54″ give		10073 }
Anſwer	0′ : 14″	24083

Add theſe 14″ (according to the Title of the Equation of Time) to the Sun's Place before found, *viz.* 4ˢ : 6° : 36′ : 11″, and the Sum is 4ˢ : 6° : 36′ : 25″, his Place at the equal Time.

If the equal Time had been given, as at the firſt and ſecond Examples, you might reduce it to apparent Time in the ſame Manner.

To calculate the Sun's Place for any given Year before Chriſt.

1. Set down the mean Longitude and Anomaly of the Sun for the firſt Radical Year of *Chriſt*, and from theſe ſubſtract the mean Longitude and Anomaly which you find in the Table of Years compleat, againſt the next Number of Years given.

2. To the Remainder, add the Motions for the Number of Years that the given Year wants of thoſe Years whoſe Motions you ſubſtracted from the firſt Radical Year, and the Sum will be the mean Longitude and Anomaly for the Time given.

EXAMPLE.

Let the Sun's Place be required for *Auguſt* 2ᵈ : 22ʰ : 14′ : 39″ in the Year of the *Julian* Period 4283, or 431 Years before the firſt Radical Year of *Chriſt*, being the mean Time of an Eclipſe of the Sun.

The next Number of Years in the Table of Years compleat above 431 is	500
From which take	431
Remains	69

Long.

	Long. ☉.				Anom. ☉.			
	ſ	°	′	″	*ſ*	°	′	″
The firſt Radical Year of *Chriſt* — — —	9	7	53	10	7	0	0	40
From which take the Motions for 500 — —	0	3	46	40	11	25	1	40
Remains — — —	9	4	06	30	7	4	59	00
60 — — —	0	0	27	12	11	29	24	12
9 — — —	11	29	49	18	11	29	39	51
Auguſt 2 — —	7	0	55	42	7	0	55	6
Hours 22 — —			54	13			54	13
Min. 14 — —			0	35			0	35
Sec. 39 — —			0	02			0	02
Mean Longitude of ☉ — —	4	6	13	32	2	05	52	59
Equation ſubſtract — —		1	45	20				
Remains the Sun's true Place — —	4	4	28	12				

To calculate the Moon's true Place in her Orb (which is what you are to compare with the Sun's Place in calculating the Eclipſes of either the Sun or Moon) and to reduce it from her Orb to the Ecliptic.

IT has been obſerved by many very ingenious Men, and taken Notice of by Mr. *Brent*, in his *Compendious Aſtronomer* (a very uſeful Book), that tho' according to the late Sir *Iſaac Newton*'s Theory of the Moon, ſeven Equations have been introduced; yet it has been found, that when only the Annual, Elliptic, and Variation Equations are uſed in finding the Moon's Place, in her Orb, the Calculation has agreed better with Obſervations, either in or out of the Syzygys, than when the reſt are brought in.

ACCORDING to this Mr. *Brent*, in that Book, gives an Example at Page 164; where he leaves out all but theſe Three, except that of reducing her Place in her Orb to the Ecliptic.

UPON examining the Tables of the ſeveral Equations, that I ſhall make Uſe of, I find that he has done Juſtice to the Theory, eſpecially as they have been improved by Mr. *Machin*; for which Reaſon (to ſave myſelf a great deal of Trouble, and more than my Time and Infirmities will permit me to go through) I have borrowed what I thought moſt uſeful for my preſent Purpoſe; and refer the Reader to the Book above-mentioned, where he will find Tables of all the Equations, and conciſe Ways of uſing them.

Now, to proceed:

1. FIND the Sun's true Place, and mean Anomaly to the given Time; and alſo the mean Longitude, Apogee, and Node of the Moon to the ſame Time: Then take the Sun's mean Anomaly to the Table of the Firſt, or Annual Equations of their Longitude, Apogee, and Node; and thence take out the Equations, reſpectively, according to their Titles in the Table: So will you have the Moon's mean Longitude, Apogee, and Node, equated the firſt Time.

2. *To find the Annual Argument.*

FROM the Sun's true Place ſubſtract the firſt equated Place of the Moon's Apogee, what remains is called the *Annual Argument*, which take to the Table of the ſecond Equation of the Moon's Apogee, and thence take out the ſecond Equation, which apply according to its Title to the firſt equated Place, and you will have the ſecond equated Place of the Moon's Apogee.

3. To

3. *To find the Moon's mean Anomaly.*

FROM the Moon's Longitude firſt equated, ſubſtract the ſecond equated Place of her Apogee, what remains is her mean Anomaly, of which you muſt take half; and if that half be above three Signs, take it out of ſix Signs, and reſerve the Remainder.

4. *To find the true Elliptic Equation.*

WITH the Annual Argument enter the Table of conſtant Logarithms for the Elliptic Equation of the Moon, and thence take out the conſtant Logarithm anſwering thereto; which being found, take it to the Table of Equations to half the Moon's mean Anomaly, and ſeek at the Head thereof for the firſt four Figures on the Left-hand of the conſtant Logarithm, and alſo for the Moon's mean Anomaly if it is under ſix Signs, or its Complement to twelve Signs, if above; and, againſt the Degrees in the Left-hand Column, you will have an Equation; which, as the Table directs, muſt be added to, or ſubſtracted from half the mean Anomaly, if under ſix Signs; or what you reſerv'd, if more. So you will have half the mean Anomaly equated, to the Tangent of which add the conſtant Logarithm before found, and the Sum will be the Tangent of an Arch. Subſtract this from half the mean Anomaly before it was equated, then double the Remainder ſhall be the Elliptic Equation.

Now, if the mean Anomaly of the Moon be 0. 1. 2. 3. 4. 5 Signs, ſubſtract this Elliptic Equation from the firſt equated Place of the Moon's Longitude; but if it be 6. 7. 8. 9. 10. 11 then add it, and you will have her ſecond equated Place.

5. *To find the Variation, or Third Equation of the Moon's Longitude.*

SUBSTRACT the Sun's true Place from the ſecond equated Place of the Moon, then take the Remainder to the Table of Variation, and take out the Variation; and find the Logarithm of the Seconds contained therein (as if an abſolute Number).

AGAIN, Take the Sun's mean Anomaly to the Table of Logarithms, to correct the Moon's Variation; and thence take out a Logarithm, which ſubſtract from the former Logarithm (borrowing Radius, if Subſtraction cannot be made without), the Remainder is the Logarithm of the correct Variation in Seconds; which you muſt add to, or ſubſtract from the ſecond equated Place of the Moon, as the Title of the Variation directs. So will you have the Moon's Place in her Orb.

6. *To find the true Place of the Node.*

FROM the Sun's true Place ſubſtract the firſt equated Place of the Node, and take the Remainder to the Table of the ſecond Equation of the Node, and thence take out the Equation; which add to, or ſubſtract from the firſt equated Place, as the Title directs, and you will have its true Place; and, at the ſame time, take out the Inclination of Limit.

7. *To find the Argument of Latitude.*

SUBSTRACT the true Place of the Node from the Moon's Place in her Orb, and what remains is the Argument of Latitude.

8. *To find the Reduction.*

TAKE the Argument of Latitude to the Table of Reduction, and take out the Reduction with its Title of Add. or Subſtract. and alſo the Exceſs; then ſay, as the greateſt Inclination of Limit above the leaſt (always $17' : 45''$) is to the preſent Exceſs, ſo is the preſent Inclination of Limit to a proportional Part of the Exceſs: Add this to the Reduction before found, and the Sum will be the true Reduction; which, according to the Title of Reduction, add it to, or ſubſtract it from the Moon's Place in her Orb, and you will have her true Place in the Ecliptic.

9. *To*

9. *To find the Moon's true Latitude.*

TAKE the Argument of Latitude to the Table of the ſimple Latitudes of the Moon, and take out the Latitude and alſo the Increment; then ſay, as the greateſt Inclination of Limit 17′ : 45″ is to the preſent Increment, ſo is the preſent Inclination of Limit to a proportional Part of the Increment: Add this to the ſimple Latitude before found, and the Sum will be the true Latitude, which the Table will let you know whether it is North or South, Aſcending or Deſcending.

10. *To find the preſent Excentricity of the Moon.*

TAKE the Annual Argument to the Table of Excentricities, and take out the preſent Excentricity.

11. *To find the Horizontal Parallax of the Moon in Eclipſes.*

TAKE the Moon's mean Anomaly to the Table of the Horizontal Parallax, and take out the Horizontal Parallax under the leaſt and greateſt Excentricities, and find their Difference; and alſo the Difference of the leaſt and greateſt Excentricities, which is always 23454. Then ſay,

As the Difference between the leaſt and greateſt Excentricities,
Is to the Difference between the leaſt and preſent Excentricity,
So is the Difference between the Horizontal Parallaxes,
To the preſent Difference.

Which being added to, or ſubſtracted from that found under the leaſt Excentricity, as the Nature of the Numbers require, will give the Horizontal Parallax requir'd.

12. *To find the apparent Semidiameter of the Moon.*

THE mean Horizontal Parallax of the Moon is —— —— 57′ : 30″
And her mean Semidiameter —— —— —— 15 : 45
Then, having found the preſent Horizontal Parallax of the Moon ſay (by the Logiſtical Logarithms),

			L. L.
As the mean Horixontal Parallax	—	57′ : 30″	185 —
Is to the mean Semidiameter	——	15 : 45	5809
So is the preſent Horizontal Parallax	(ſuppoſe	53 : 38)	487 +
To the preſent apparent Semidiameter		14′ : 49″	6111 Rem.

FOR the Horizontal Semidiameter, you muſt add the Logiſtical Logarithm of 16′ : 26″ 24 = 5623 to the Logiſtical Logarithm of the preſent Horizontal Parallax of ☽, and the Sum will be the Logiſtical Logarithm of the Horizontal Semidiameter.

Note, That the Sun's Horizontal Parallax is always 10″.

EXAMPLE I.

LET the Moon's Place be requir'd for *October* the 25th Day at Noon, in the Year 1743.

The Sun's true Place at this Time is ——— ——— 7ˢ : 12° : 41′ : 16″
And his mean Anomaly ——— ——— ——— 4 : 5 : 54 : 15

	Longit. ☽.	Apogee ☽.	Node.	
	∫ : ° : ′ : ″	∫ : ° : ′ : ″	∫ : ° : ′ : ″	
1743.	4 1 14 17	8 7 20 11	1 25 3 24	
Octob. 25.	10 26 33 56	1 3 11 58	0 15 46 50	Subst.
Mean Long. ☽. — —	2 27 48 13	9 10 32 9	1 9 16 34	
Annual Equat. ———	+ 9 56	— 17 56	+ 7 20	
☽ 1st equated ———	2 27 58 09	9 10 14 13	1 9 23 54	☊
Elliptic Equation ———	— 1 50	7 12 41 16 ☉	7 12 41 16 ☉	
☽ 2d equated ———	2 27 56 19	10 2 27 3 *a*.	6 3 17 22	☊ *a* ☉
Variation add ———	+ 36 20	— 11 59 0	2d Equat. Apog.	
☽ in Orb ——— ———	2 28 32 39	8 28 15 13	Apog. 2d equated.	
Reduction and Excentr.	— 7 16			
☽ in the Ecliptic —	2 28 25 23	Constant Log. ———	9·955498	
True Lat. N. A. —	3 58 22	Half M. Anom. 89° : 51′ : 28″	12.605901	Tang.
		Half Tr. Anom. 89 : 50 : 33	12.561399	Tang.
		Diff. ———	0 : 55 } +	
			0 : 55 }	
		Elliptic Equat. —	1 : 50	
Mean Anomaly of ☽ ———	———	———	5ˢ : 29° : 42′ : 56″	
True Place of the Node ———	———	———	1 : 09 : 33 : 55	
Inclination of Limit ———	———	———	17 : 44	
Reduction subtract ———	———	———	6 : 29	
Excess ———	———	———	0 : 47	
Present Excentricity ———	———	———	51190	
Horizontal Parallax ———	———	———	60 : 36	
Semidiameter ☽ ———	———	———	16 : 36	

EXAMPLE

EXAMPLE 2.

LET the Moon's Place be required for *November* the 23d, at 20′ and 30″ paſt 5 in the Afternoon, 1743.

The Sun's Place at this Time is —— —— 8ˢ : 12° : 14′ : 13″
And his mean Anomaly —— —— 5 : 4 : 42 : 18

	Long. ☽.	Apog. ☽.	Node.
	ſ : ° : ′ : ″	ſ : ° : ′ : ″	ſ : ° : ′ : ″
1743.	4 1 14 17	8 7 20 11	1 25 3 24
Novem. 23.	11 18 40 51	1 6 25 50	0 17 18 59
Hours 5	2 44 42	1 24	0 40
Min. 20	11 59	0 06	0 03
Sec. 30	0 16		
			0 17 19 42
Mean Long. ☽ ——	3 22 52 05	9 13 47 31	1 7 43 42
Annual Equat. ——	+ 5 16	— 9 33	+ 3 53
☽ 1ſt equated ——	3 22 57 21	9 13 37 58	1 7 47 35 ☊
Elliptic Equation ——	+ 2 28 44	8 12 14 13 ☉	8 12 14 13 ☉
☽ 2d equated ——	3 25 26 05	10 28 36 15	7 4 26 38 ☊ *a.* ☉.
Variation add ——	+ 37 07	— 9 48 07	+ 1 22 52 2d Eq. ☊
☽ in Orb ——	3 26 3 12	9 3 49 51	Apog. 2d equated
Reduction and Exceſs —	— 3 09	Conſtant Log.	9.946684
			10.772748
☽ in the Ecliptic ——	3 26 0 03	T.M. Anom. Eq.	
True Latitude ☽. N. A.	5 3 32	80° : 25′ : 17″	

Preſent Excentricity 61305 —— —— —— { 79 : 11 : 53 —— 10.719432 Tangent.
{ 80 : 26 : 15 = half Anom. before equated.

Horizontal Parallax ☽ — 61′ : 1″ { 1 : 14 : 22 }
Semidiameter —— 16 : 10 { 1 : 14 : 22 }

Mean Anom. ☽ 6ˢ : 19° : 07′ : 30″ + 2 : 28 : 44 Ellipt. Equation.
Inclinat. Lun. —— 12 : 07 True Place of ☊ 1ˢ : 9° : 10′ : 27″ }
Reduction ſubſt. —— 2 : 54 } Argument Lat. 2 : 16 : 52 : 45 }
Exceeſs 0′ : 22″ Prop. Part — 0 : 15 }

TABLES

TABLES

TO FIND THE

MEAN TIMES

OF THE

FULLS and CHANGES of the MOON:

And alſo, to find when an

ECLIPSE

OF EITHER

The SUN or MOON

May be expected in any Month and Year,

Before and after CHRIST's NATIVITY.

A TABLE of Radical Years before CHRIST.

Radical Years.	Epacts.				Mot. Lat.			
	d	h	′	″	☌	°	′	″
1	16	17	56	9	11	21	54	57
101	10	10	55	15	6	18	25	6
201	14	19	5	43	2	4	58	3
301	19	3	16	11	9	2	17	13
401	23	11	26	41	4	26	3	53
501	27	19	37	7	0	6	36	49
601	2	15	3	37	8	17	49	58
701	6	23	14	5	3	28	23	3
801	11	7	24	33	11	8	55	49
901	15	14	35	57	6	20	0	17
1001	19	23	6	7	2	0	23	33
1101	24	7	55	59	9	10	34	35
1201	28	16	6	15	4	21	7	35
1301	3	11	32	53	1	2	20	42
1401	7	19	43	21	8	12	53	36
1501	12	3	53	51	3	23	26	32
1601	16	12	4	19	11	3	59	28
1701	20	20	18	43	6	14	30	14
1801	25	4	25	17	1	25	5	19
1901	29	12	35	45	9	5	38	14
2001	4	8	2	9	5	16	51	25
2101	8	16	12	37	0	27	24	22
2201	13	0	23	7	8	17	59	16
2301	17	8	33	35	3	18	21	13
2401	21	16	44	3	10	29	3	8
2501	26	0	54	33	6	9	26	3
2601	0	20	21	0	2	20	49	12
2701	5	4	31	29	10	1	22	7
2801	9	12	41	57	5	11	55	3
2901	13	20	52	25	0	22	27	58
3001	18	5	2	55	8	3	0	54
3101	22	13	13	23	3	13	33	50
3201	26	21	23	51	10	24	6	46
3301	1	16	50	17	7	5	19	58
3401	6	1	0	47	2	15	52	50
3501	10	9	11	15	9	26	25	45
3601	14	17	21	39	5	6	58	45
3701	19	1	32	11	0	17	31	38
3801	23	9	42	41	7	28	4	32
3901	27	17	53	9	3	8	37	23
4001	2	13	21	33	11	19	59	32
4101	6	21	30	3	7	0	23	34
4201	11	5	40	31	2	10	56	30
4301	15	13	51	0	9	21	29	24
4401	19	21	1	51	5	2	2	8
4501	24	6	11	57	0	12	35	17
4601	28	14	22	25	7	23	8	13
4701	3	9	48	53	4	5	4	25
4801	7	17	59	21	11	14	54	16
4901	12	2	9	51	6	25	27	11
5001	16	10	20	17	2	6	0	9
5101	20	18	30	47	9	16	33	4

A TABLE of Radical Years after CHRIST.

Radical Years.	Epacts.				Mot. Lat.			
	d	h	′	″	♌	°	′	″
1	16	17	56	9	11	21	54	57
101	11	11	14	55	4	10	32	51
201	8	1	35	9	9	0	49	8
301	3	7	22	47	1	20	17	14
401	28	21	58	15	5	9	3	4
501	24	13	47	49	9	28	30	6
601	20	5	37	19	2	17	57	11
701	15	21	26	53	7	7	24	14
801	11	13	16	21	11	26	51	20
901	7	5	5	55	4	16	18	23
1001	2	20	55	27	9	5	45	27
1101	28	1	28	59	0	24	32	20
1201	23	17	18	29	5	13	59	25
1301	19	9	8	3	9	20	12	40
1401	15	0	57	35	2	22	53	31
1501	10	16	47	5	7	12	20	36
1601	6	8	38	25	0	1	47	35
1621	17	7	18	5	0	17	23	55
1641	28	5	55	39	1	3	2	20
1661	9	15	51	9	2	19	19	51
1681	20	14	30	40	3	14	57	11
1701	2	0	26	9	4	21	14	44
1721	12	23	5	39	5	6	52	15
1741	23	21	45	11	5	22	29	24
1761	5	7	40	41	7	8	46	55
1781	16	6	20	9	7	24	44	17
1801	27	4	59	43	8	10	1	34
1821	8	14	55	9	9	26	19	9
1841	19	13	34	43	10	11	56	27
1861	0	23	30	11	11	28	14	11
1881	11	22	9	43	0	13	51	19
1901	22	20	49	15	0	29	28	39
1921	4	6	44	41	2	15	46	14
1941	15	5	24	15	3	1	23	32
1961	26	4	3	45	3	17	0	50
1981	7	13	59	13	5	3	18	26
2001	18	12	38	35	5	18	55	49
2101	14	4	28	7	10	8	22	54
2201	9	20	17	39	2	27	49	59
2301	5	12	7	11	7	17	17	4
2401	1	3	56	43	0	6	44	9
2501	26	8	30	25	3	25	31	6
2601	22	0	19	50	8	14	58	5
2701	17	16	9	22	1	4	25	10
2801	13	7	58	54	5	23	52	15
2901	7	23	48	26	10	13	19	20
3001	4	15	38	2	3	2	46	25
3101	0	7	27	34	7	22	13	30
3201	25	12	1	9	11	11	0	21
3301	21	3	50	41	4	0	27	26
3401	16	19	40	13	8	19	54	31
3501	12	11	29	45	1	9	21	36
3601	8	3	19	17	5	28	48	41
3701	3	19	8	49	10	17	55	18
3801	28	23	32	24	2	7	2	37
3901	24	15	31	56	6	26	29	42
4001	20	7	21	21	11	15	56	29
4101	15	23	10	53	4	5	23	34
4201	11	15	0	25	8	24	50	39
4301	7	6	49	57	1	14	17	44
4401	2	22	39	29	6	3	44	49
4501	28	3	12	59	9	22	31	40
4601	23	19	2	36	2	11	58	45
4701	19	10	52	8	7	1	25	50
4801	15	2	41	40	11	20	52	55
4901	10	18	31	12	4	10	20	0
5001	6	10	20	44	8	29	47	5

A TABLE

A TABLE of Years Complete.

Years.	Epacts. d	h	′	″	Mot. Lat. ſ	°	′	″
1	10	15	11	23	0	8	2	46
2	21	6	22	47	0	16	5	34
3	2	8	50	7	1	24	48	35
4	14	0	1	31	2	2	51	23
5	24	15	12	53	2	10	54	10
6	5	17	40	17	3	19	37	10
7	16	8	51	39	3	27	39	58
8	28	0	3	3	4	5	42	45
9	9	2	30	23	5	14	25	45
10	19	17	41	47	5	22	28	32
11	0	20	9	7	7	1	11	33
12	12	11	20	29	7	9	14	22
13	23	2	31	55	7	17	17	8
14	4	4	59	13	8	26	0	9
15	14	20	10	37	9	4	2	57
16	26	11	22	3	9	12	5	45
17	7	13	49	23	10	20	48	43
18	18	5	0	49	10	28	55	30
19	28	20	12	9	11	6	54	20
20	10	22	39	31	0	15	37	19

Years.	Epacts. d	h	′	″	Mot. Lat. ſ	°	′	″
40	21	21	19	2	1	1	14	38
60	3	7	14	30	2	17	32	11
80	14	5	54	1	3	3	9	30
100	25	4	33	32	3	18	46	49
200	20	20	23	20	8	8	12	52
300	16	12	12	28	0	27	40	55
400	12	4	1	56	5	17	7	58
500	7	19	51	24	10	6	35	1
600	3	11	40	52	2	26	2	4
700	28	16	14	24	6	14	48	53
800	24	8	3	52	11	4	15	56
900	19	23	53	20	3	23	42	59
1000	15	15	42	48	8	13	10	2
2000	1	18	41	32	5	27	0	18
3000	17	10	24	20	2	10	10	20
4000	3	13	23	4	11	24	0	36
5000	19	5	5	52	8	7	10	38
6000	5	8	34	37	5	21	0	54
7000	21	0	17	25	2	4	10	56
8000	7	3	16	10	11	18	01	12

Months.

Jan.	0	0	0	0	0	0	0	0
Febr.	1	11	15	57	1	0	43	25
Mar.	29	11	15	57	1	0	43	25
April	1	9	47	2	3	2	4	19
May	1	21	3	55	4	2	44	13
June	3	8	19	45	5	3	24	25
July	3	19	35	43	6	4	4	32
Auguſt	5	6	51	39	7	4	44	49
Sept.	6	18	9	35	8	5	27	56
October	7	5	23	29	9	6	5	6
Novem.	8	16	39	25	10	6	44	26
Decem.	9	3	55	25	11	7	24	45

A Canon of Fulls and Changes of the Moon.

Full	14	18	22	1	6	15	20	7
Change	29	12	44	3	1	0	40	14
Full	44	7	6	4	7	16	0	21
Change	59	1	28	5	2	1	20	28
Full	73	19	50	6	8	16	40	35
Change	88	14	12	8	3	2	0	42
Full	103	8	34	9	9	17	20	49
Change	118	2	56	10	4	2	40	56

A TABLE

A TABLE from FLAMSTEAD's *Doctrine of the Sphere*, to find the Interval between the mean and true Time of the Conjunctions and Oppositions of the Sun and Moon.

H. ′ ″	° ′ ″ ‴	H. ′ ″	° ′ ″ ‴	H. ′ ″	° ′ ″ ‴
1	0 30 29	21	10 40 1	41	20 49 33
2	1 0 57	22	11 10 30	42	21 20 2
3	1 31 26	23	11 40 58	43	21 50 31
4	2 1 54	24	12 11 27	44	22 20 59
5	2 32 23	25	12 41 55	45	22 51 28
6	3 2 54	26	13 12 24	46	23 21 56
7	3 33 20	27	13 42 53	47	23 52 25
8	4 3 49	28	14 13 21	48	24 22 54
9	4 34 18	29	14 43 50	49	24 53 22
10	5 4 46	30	15 14 19	50	25 23 51
11	5 35 15	31	15 44 47	51	25 54 19
12	6 5 43	32	16 15 16	52	26 24 48
13	6 36 12	33	16 45 44	53	26 55 17
14	7 6 41	34	17 16 13	54	27 25 45
15	7 37 9	35	17 46 42	55	27 56 13
16	8 7 38	36	18 17 10	56	28 26 43
17	8 38 6	37	18 47 39	57	28 57 11
18	9 8 35	38	19 18 7	58	29 27 40
19	9 39 4	39	19 48 36	59	29 58 8
20	10 9 32	40	20 19 5	60	30 28 37

EXAMPLE.

SUPPOSE the Moon at the mean Time of an Oppoſition to be in — 4′ 18° 15′ 22″
And the Sun's Place at the ſame Time — 10 22 46 35

Then the Moon is ſhort of the Oppoſition — 4 31 13
Againſt the next leſs than this in the 2d Column you have 8 Hours — 4 3 49

Remains — 27 24
Againſt the next leſs than the Rem. in the 6th Col. you have 55 Min. — 26 55

Remains — 0 29
Againſt the next leſs than the laſt Rem. in the 6th Column is 57 Sec. — 0 29

Remains — 0 00

HENCE the Interval is found to be $8^h : 53' : 57''$; which, becauſe the Moon was ſhort of the Oppoſition, muſt be added the mean Time, *&c.*

THE

THE Table of Years before the Birth of CHRIST, was chiefly calculated as an Aſſiſtant to Chronology, ſo far as relates to the Eclipſes of the Sun and Moon, from the Creation of the World; which is ſuppoſed to be in the Year of the *Julian* Period 4764, to the Birth of *Chriſt*: Which, according to the vulgar Account, was in the Year of the *Julian* Period 4713. But, according to the Account of ſome of the beſt Authors, our Saviour was born on the 7th Day of *October*, in the Year of the *Julian* Period 4710.

Now, if any thing remarkable did happen in any Year of the *Julian* Period before 4713, ſubſtract it from 4713, and what remains will be the Number of Years before CHRIST (0). But, if you take the given Year of the *Julian* Period out of 4714, then there will remain ſo many Years before the firſt Year after the Birth of CHRIST.

THE firſt and fourth Columns of the Table for Years before or after CHRIST's Nativity are Radical Years, and the ſecond, third, and ſixth, are the Epacts and Motions of Latitude for thoſe Years, as the Titles over them may inform you.

To find the mean Time of a Conjunction or Oppoſition of the Sun and Moon in a given Year and Month before CHRIST (1), *or the Year of the* Julian *Period* 4714.

1. SUBSTRACT the given Year of the *Julian* Period from 4714, and if the Remainder be a Radical Year, ſuch as you find in the firſt and fourth Columns, then take out the Epact and Motion of Latitude for the ſame, to which add the Epact and Motion of Latitude for the given Month; as you ſee is done in the firſt and ſecond Examples next following.

BUT, if the Remainder is not a Radical Year, you muſt ſubſtract it out of the Radical Year that is next above it, as at the third Example, where the Remainder is 431; but this not being a Radical Year, I have taken it out of the Radical Year next above it, *viz.* 501, and the Remainder is 70: This done, write down 501 and 70, in ſuch Parts as you will find in the Table of Years complete, as 60 and 10, and alſo the given Month, and againſt each ſet down the Epacts and Motions of Latitude (as you ſee at the third and fourth Examples); then collect the Epacts and Motions of Latitude into one Sum, reſpectively, and take the Sum of the Epacts to the Canon of Fulls and Changes.

Now, if you would know the mean Time of the Full Moon in the given Month, take the Sum of the Epacts out of the next Number of Days, Hours, Minutes, *&c.* above it, that ſtand againſt the Word Full; but if you would know the mean Time of the New Moon, then take the ſaid Sum out of the Days, Hours, *&c.* that are next above it, that ſtand againſt the Word Change; and what remains will be the mean Time of the Full or Change, according to the Tables that thoſe Numbers were extracted from. Here obſerve, that in either Caſe you muſt add the Motion of Latitude, found in the Canon, againſt the Word Full or Change to the Sum of the Motions of Latitude belonging to the Year and Month given, and the Sum will be the Motion of Latitude for that Time.

To find whether a given Year of the Julian *Period be a Biſſextile or Common Year.*

SUBSTRACT it from 4713, and divide the Remainder by 4; if nothing remains, it is a Leap-Year; but if 1, 2, or 3 remains, it is a Common Year.

Or, thus: Divide any Year of the *Julian* Period by 28, and ſeek the Remainder in the following Table; then, if you find but one Letter againſt it, that ſhews it to be a Common Year, and the Letter is the Dominical Letter for that Year; but if you find two Letters ſtand againſt the Remainder, that ſhews it to be a Leap-Year, and that the firſt of thoſe Letters is the *Sunday* Letter from the Beginning of the Year to the 24th of *February*, and the other thence to the End of the Year.

1	GF	8	E	15	C	22	A
2	E	9	DC	16	B	23	G
3	D	10	B	17	AG	24	F
4	C	11	A	18	F	25	ED
5	BA	12	G	19	E	26	C
6	G	13	FE	20	D	27	B
7	F	14	D	21	CB	28	A

By having the mean Time of a Full or Change of the Moon given, and the Motion of Latitude, to find whether the Sun or Moon may be Eclipſed or not.

THE Moon may be Eclipſed when at the Time of the mean Oppoſition the Motion of Latitude is between $11^s : 14^\circ : 48'$ and $0^s : 15^\circ : 12'$; and alſo between $5^s : 14^\circ : 48'$ and $6^s : 15^\circ : 12'$.

THE Sun may be Eclipſed when at the Time of the mean Conjunction the Motion of the Moon's Latitude is between $11^s : 18^\circ : 38'$, and $0^s : 20^\circ : 41'$; and alſo between $5^s : 9^\circ : 19'$ and $6^s : 11^\circ : 12'$.

HERE you muſt obſerve, that theſe Tables do rarely give the true equal Time of a New Moon, or a Full, but only according to the Middle or mean Motion; for tho' ſometimes it may happen, that by them you may hit upon the true equal Time, yet at other Times it may be ſooner or later; and tho' it ſeldom exceeds 14 Hours, yet the Difference may be 19 Hours; but the Interval may be moſt ſpeedily found by the laſt Table before-going, as will be ſeen hereafter.

EXAMPLE 1.

IN the ſixth Year of the War between the *Medes* and *Lydians*, in the Year of the *Julian* Period 4113, there was an Eclipſe of the Sun upon the 20th Day of *September*, which was foretold by *Thales* of *Miletus*. This was the Year before *Jehoiakim* King of the *Jews* was ſlain by *Nebuchadnezzar*.

The firſt Year of the *Julian* Period after the Birth of *Chriſt* was —— ——	4714
From which ſubſtract the given Year of the *Julian* Period —— ——	4113
And the Remainder is a Radical Year —— —— ——	601

Which is what you muſt calculate for: Thus,

	Epacts.				Mot. Lat.			
	d	h	′	″	ſ	°	′	″
601 —	2	15	3	37	8	17	49	58
Sept. —	6	18	9	35	8	5	27	56
Sum ſubſtract. — — —	9	9	13	12	4	23	17	54
From the Canon (for a Change) — — — —	29	12	44	3	1	0	40	14
Remains — — — — *September* — —	20	3	30	51	5	23	58	8

HENCE we find that the true mean Time of the Conjunction was upon the Twentieth Day of *September* at $30' : 51''$ paſt Three in the Afternoon.

AND the Motion of Latitude being $5^s : 23^\circ : 58' : 8''$, ſhews that an Eclipſe of the Sun did happen at that Time; which confirms the Truth of Hiſtory in this Point.

EXAMPLE

EXAMPLE 2.

In the 20th Year of *Darius* on the 7th Day of *November*, in the Year of the *Julian* Period 4213, there was obſerved an Eclipſe of the Moon: Take this from 4717, there will remain the Radical Year 501.

	Epacts.				Mot. Lat.			
	d : h : ′ : ″				ſ : ° : ′ : ″			
Radical Year 501 — —	27	19	37	07	0	6	36	4
November — — —	8	16	39	25	10	6	44	26
Sum ſubſtract. — —	36	12	16	32	10	13	20	30
A full Moon from the Canon — — —	44	7	06	04	7	16	0	21
Remains the mean Time of the Full *November* —	7	18	49	32	5	29	20	51

And the Mot. of Lat. 5ˢ : 29° : 20′ : 51″, ſhew that the ☽ was eclipſed at this Time.

MORE EXAMPLES.

In Page 141, of the late Sir *Iſaac Newton*'s Obſervations upon the Prophecies of *Daniel*, he ſays, That the Reigns of the two Kings *Cambyſes* and *Darius Hyſtapis*, were determined by three Eclipſes of the Moon, obſerved at *Babylon*, and recorded by *Ptolemy*, viz.

One was in the Year of the *Julian* Period 4191, *July* 16ᵈ : 11ʰ *P. M.*
Another in the Year of the *Julian* Period 4212, *Novem.* 19 : 11 : 45′ *P. M.*
And the third in the Year of the *Julian* Period 4223 *April* 25 : 11 : 30 *P. M.*

The firſt 4714 — 4191 = 523 before *Chriſt* (1) and 601 — 523 = 78.

	Epacts.				Mot. Lat.			
	d : h : ′ : ″				ſ : ° : ′ : ″			
A Radical Year 601 — —	2	15	3	37	8	17	49	58
{ 60 — —	3	7	14	30	2	17	32	11
{ 18 — —	18	5	0	49	10	28	55	30
July — — —	3	19	35	43	6	4	4	32
Sum — — —	27	22	54	39	4	08	22	11
From the Canon (a full ☽) — —	44	7	6	04	7	16	00	21
Full Moon *July* — — —	16	8	11	25	11	24	22	32

The Motion of Latitude is 11ˢ : 24° : 22′ : 32″ ſhews an Eclipſe at this Oppoſition.

The

THE ſecond, 4714 — 4212 = 502 before *Chriſt* (1), and 601 — 502 = 92.

	Epacts.				Mot. Lat.			
	d :	h :	′ :	″	ſ :	° :	′ :	″
601 — —	2	15	3	37	8	17	49	58
80 — —	14	5	54	1	3	3	9	30
19 — —	28	20	12	9	11	6	54	20
November — —	8	16	39	25	10	6	44	26
Sum — — —	54	9	49	12	9	4	38	14
From the Canon (a full ☽) —— ——	73	19	50	06	8	16	40	35
Full Moon *November* —— ——	19	10	00	54	5	21	18	49

THE Motion of Lat. being 5ˢ : 21° : 18′ : 49″ ſhews that an Eclipſe of the ☽ was at this Time.

THE third 4714 — 4223 = 491 before *Chriſt* (1), and 501 — 491 = 10.

	Epacts.				Mot. Lat.			
	d :	h :	′ :	″	ſ :	° :	′ :	″
501 — —	27	19	37	7	0	6	36	49
10 — —	19	17	41	47	5	22	28	32
April — —	1	9	47	02	3	2	4	19
Sum — — —	48	23	05	56	9	01	09	40
From the Canon (a full ☽) — — — —	73	19	50	6	8	16	40	35
Full Moon *April* — —	24	20	44	10	5	17	49	15

To the foregoing Examples we will add three more, as we find them in the Rev. Mr. *Bedford*'s *Horæ Mathematicæ Vacuæ*.

AMOS, Chap. viii. Ver. 9 and 10.

It ſhall come to paſs in that Day, ſaith the Lord God, that I will cauſe the Sun to go down at Noon, and I will darken the Earth in the clear Day, and I will turn your Feaſts into Mourning, and all your Songs into Lamentation.

THIS Prophecy the Chriſtian Fathers have accommodated in an allegorical Senſe to the Darkneſs which happened at the Paſſover, when our Saviour ſuffered; and others think, that it may be fulfilled in a litteral Senſe by three great Eclipſes of the Sun, which darkned their Days in the Time of their ſolemn Feaſts, when all their Males were obliged to appear at *Jeruſalem* before the Lord. And thus, as *Thales* of *Miletus* was the firſt who foretold Eclipſes, by his Skill in Aſtronomy, ſo *Amos* amongſt the *Jews* foretold the Eclipſes of the Sun, by the Inſpiration of the Holy Ghoſt.

THE firſt of theſe happened in the Year of the *Julian* Period 3923, *June* 24th, at the Feaſt of Pentecoſt.

THE ſecond happened in the Year of the *Julian* Period 3943, *November* the 8th, in the Feaſt of Tabernacles.

AND the third in the Year of the *Julian* Period 3944, *May* 5th in the Feaſt of Unleavened Bread (all common Years).

THE

THE first 4714 — 3923 = 791 before *Chriſt* (1), and 801 — 791 = 10.

	Epacts. d : h : ′ : ″	Mot. Lat. ſ : ° : ′ : ″
801 — —	11 7 24 23	11 8 55 49
10 — —	19 17 41 47	5 22 28 32
June — —	3 8 19 45	5 3 24 25
Sum ſubſtract. — — —	34 9 26 05	10 4 48 46
From the Canon (a Change) —— ——	59 1 28 05	2 1 20 28
New Moon, *June* —— —— ——	24 16 02 00	0 6 09 14

And the Motion of Latitude 0ˢ : 6° : 9′ : 14″ ſhews that the Sun was eclipſed at that Time.

THE ſecond 4714 — 3943 = 771 Years before *Chriſt* (1), and 801 — 771 = 30.

	Epacts. d : h : ′ : ″	Mot. Lat. ſ : ° : ′ : ″
801 — —	11 7 24 33	11 8 55 49
20 — —	10 22 39 31	0 15 37 19
10 — —	19 17 41 47	5 22 28 32
November — —	8 16 39 25	10 6 44 26
Sum ſubſtract. — — —	50 16 25 16	3 23 46 [illegible]
From the Canon (a Change) —— ——	59 1 28 05	2 1 20 28
New Moon, *November* —— ——	8 9 2 49	5 25 06 34

The Motion of Latitude 5ˢ : 25° : 6′ : 34″ ſhews that the Sun was eclipſed at this New Moon.

THE third 4714 — 3944 = 770 Years before *Chriſt* (1), and 801 — 770 = 31.

	Epacts. d : h : ′ : ″	Mot. Lat. ſ : ° : ′ : ″
801 — —	11 7 24 33	11 8 55 49
20 — —	10 22 39 31	0 15 37 19
11 — —	0 20 9 7	7 1 11 33
May — —	1 21 3 55	4 2 44 13
Sum ſubſtract. — — —	24 23 17 6	10 28 28 54
From the Canon (a Change) —— ——	29 12 44 3	1 0 40 14
New Moon and ☉ eclipſed, *May* —— ——	4 13 26 57	11 29 09 08

By the foregoing Calculations we find that there were Eclipſes of the Sun and Moon at ſuch Times, as Hiſtory informs us; and this may ſhew how much Chronology is obliged to Aſtronomy.

To prove whether the mean Times thus found are true or ſufficiently near the true Means, you muſt find the mean Longitudes of the Sun and Moon, and if it be for a full Moon the Difference ſhould be 6 Signs; but if it be for a Change, then the Difference ſhould be nothing but a ſmall Matter, ſuch as a few Minutes or Seconds may be excuſed in this Part of Aſtronomy. Since, if it was the true mean Time it may differ from the true equal Time conſiderably, as aforeſaid.

For an EXAMPLE.

We will take the mean Time laſt found, *viz.* *May* $4^d : 13^h : 26' : 57''$ of the Year of the *Julian* Period 3944, which was 770 Years before *Chriſt* (1), as above.

Then $800 - 770 = 30$.

1. From the mean Longitude of the Sun and Moon for the firſt Year of *Chriſt* ſubſtract the mean Longitudes of 800 (from the Tables of compleat Years reſpectively) and to the Remainders add the mean Longitudes for the Overplus, ſuch as 30 above, and alſo the mean Longitudes of the Months, Days, Hours, Minutes, *&c.* as follows:

	Long. ☉.					Long. ☽.			
	ſ	°	′	″		*ſ*	°	′	″
First Radical Year of *Chriſt*	9	7	53	10	*Chriſt* 1ſt	4	2	2	55
From the Tab. of comp. Years 800	0	6	2	40	800	10	2	43	20
Remains	9	1	50	30	Remains	5	29	19	35
30 { 20	0	0	09	04	30 { 20	4	13	34	05
30 { 10	11	29	34	58	30 { 10	8	0	11	45
May 4th	4	2	13	12	*May* 4	6	13	52	22
Hours 13			32	02	Hours 13			7	8	14
Min. 26			1	04	Min. 26			14	16
Sec. 57			0	02	Sec. 57			0	31
Mean Long. of ☉	1	4	20	52	Long. of ☽	1	4	20	48

Here we may ſee that the Difference between the mean Longitude of the Sun and Moon is but 4 Seconds, therefore we may conclude the mean Time is found ſufficient near the Truth.

To find the mean Times of the Fulls and Changes of the Moon for any Year of Chriſt.

Collect the Epacts and Motions of Latitude for the given Year and Month into one Sum reſpectively, and take the Sum of the Epacts to the Canon of Fulls and Changes, and then go on as you did for Years before *Chriſt*; but if it be a Leap-Year take 1 from the Number of Days thus found.

EXAMPLE.

EXAMPLE.

LET it be required to find the mean Time of the Oppoſition of the Sun and Moon in *December* 1749.

		Epacts.				Mot. Lat.			
		d : h :	′ :	″		ſ :	° :	′ :	″
1741	— —	23	21	45	11	5	22	29	24
8	— —	28	0	3	3	4	5	42	45
December	— —	9	3	55	25	11	7	24	45
Sum ſubſtr.	—— ——	61	1	43	39	9	5	36	54
From the Canon (a Full)	—— ——	73	19	50	06	8	16	40	35
Full Moon *December*	—— —— ——	12	18	06	27	5	22	17	29

THE Motion of Latitude is 5ˢ : 22° : 17′ : 29″ ſhews that the Moon will be eclipſed at this Oppoſition.

To find the mean Times of the Fulls and Changes of the Moon, and alſo when an Eclipſe of either the Sun or Moon may be expected at ſuch Times throughout the whole Year.

To do this you muſt firſt find either the Full or Change that happens next to the Beginning of the Year, and alſo the Motion of Latitude.

For an EXAMPLE.

WE will take the Year 1749, and, calculating as above, you will find that a new Moon will happen next the Beginning of the Year, *viz.* upon *January* the 7th at 39′ : 51″ paſt 3 in the Afternoon; and that the Motion of Latitude will be 11ˢ : 29° : 32′ : 37″.

Now, if to theſe Times you add 14ᵈ : 18ʰ : 22′ : 2″ continually, and to the Motion of Latitude 6ˢ : 15° : 20′ : 7″ to the Year's End, you may form ſuch a Table as follows; where you may obſerve the ſeveral Motions of Latitude, and how many are within the Bounds of Eclipſes.

January

	Times. d : h : ′ : ″	Mot. Lat. ∫ : ° : ′ : ″		
January	7 3 39 51	11 29 32 37	New ☽	☉ Eclipſed.
	21 22 1 53	6 14 52 44	Full ☽	
February	5 16 23 55	1 0 12 51	New ☽	
	20 10 45 57	7 15 32 58	Full ☽	
March	7 5 7 59	2 0 53 05	New ☽	
	21 23 30 1	8 16 13 12	Full ☽	
April	5 17 52 3	3 1 33 19	New ☽	
	20 12 14 5	9 16 53 26	Full ☽	
May	5 6 36 7	4 2 13 33	New ☽	
	20 0 58 9	10 17 33 40	Full ☽	
June	3 19 20 11	5 2 53 47	New ☽	
	18 13 42 13	11 18 13 54	Full ☽	☽ Eclipſed.
July	3 8 4 15	6 3 34 1	New ☽	☉ Eclipſed.
	18 2 26 17	0 18 54 8	Full ☽	
Auguſt	1 20 48 19	7 4 14 15	New ☽	
	16 15 10 21	1 19 34 22	Full ☽	
	31 9 32 23	8 4 54 29	New ☽	
September	15 3 54 25	2 20 14 36	Full ☽	
	29 22 16 27	9 5 34 43	New ☽	
October	14 16 38 29	3 20 54 50	Full ☽	
	29 11 0 31	10 6 14 57	New ☽	
November	13 5 22 33	4 21 35 4	Full ☽	
	27 23 44 35	11 6 55 11	New ☽	
December	12 18 6 37	5 22 15 18	Full ☽	☽ Eclipſed.
	27 12 28 39	0 7 35 25	New ☽	☉ Eclipſed.

To find the Time of the true Conjunction or Oppoſition of the Luminaries.

To the Time of the mean Conjunction or Oppoſition of the Sun and Moon, found according to the preceding Rules and Examples, calculate the true Place of the Sun, and that of the Moon in her Orb; and if the Moon's Place is directly the ſame or oppoſite to the Place in which the Sun was found, then you have the true equal Time of the Conjunction or Oppoſition; but if at this Time they differ (as moſt commonly they do), take the Difference to *Flamſtead*'s Table of the Hourly Motion of the Moon from the Sun, and by that find what Time anſwers to the ſaid Difference; which being obtained, add it to, or ſubſtract it from the mean Time, according as the Moon is ſhort of the true Conjunction or Oppoſition, or is paſt it. Then again, find their Places for the Time given by the Sum or Remainder, when,

when, if they are in the ſame Sign, Degree, *&c.* or juſt oppoſite, you have the true equal Time; if they yet differ, proceed with the Difference as before, until you have the true equal Time.

For an EXAMPLE.

WE will take the full Moon that is to be in *December*, 1749, the mean Time being

December			$12^d : 18^h : 6' : 37''$	
Interval ſubſtract			$10 : 6 : 3$	
Remains the equal Time of the true Oppoſition			$12 : 8 : 0 : 34$	
At which Time the Sun's Place is			$9^s : 2^o : 14' : 55''$	
And the Moon's Place in her Orb			$3 : 2 : 14 : 53$	
True Place of the Node ſubſtract.			$9 : 10 : 13 : 57$	
Remains the Argument of Latitude			$5 : 22 : 0 : 56$	
True Latitude N. D.				$43 : 56$
Reduction add				$1 : 48$
The Sum is the Moon's true Place in the Ecliptic			$3 : 2 : 16 : 41$	
Hourly Motion of ☽ *a.* ☉				$31 : 46$
Time of Reduct. add				$3 : 49$
The Time of Reduction apply'd to the true equal Time according to its firſt Title			$12^d : 8^h : 0' : 34''$	
Give the equal Time of the Middle of the Eclipſe			$12 : 8 : 4 : 23$	
And apply'd contrary to its firſt Title, gives the equal Time of the true Ecliptic Oppoſition			$12 : 7 : 56 : 45$	
Equation of Time ſubſtract.				$0 : 16$
Remains the apparent Time of the Middle of the Oppoſition			$12 : 8 : 4 : 07$	
Apparent Time of the Ecliptic Oppoſition			$12 : 7 : 56 : 29$	
Mean Anomaly of ☉			$5^s : 23^o : 58' : 28''$	
of ☽			$9 : 13 : 37 : 09$	
The Horizontal Parallax of ☽				$56 : 49$
of ☉ (always)				$0 : 10$
Their Sum				$56 : 59$
Semidiameter of ☉ ſubſtract				$16 : 22$
Remains the Semidiameter of the Earth's Shadow				$40 : 37$
Semidiameter of ☽ add				$18 : 32$
Sum of the Semidiameters of the Moon and Earth's Shadow				$59 : 09$
True Latitude of ☽ ſubſtract				$43 : 56$
Remains the Parts deficient				$15 : 13$

HERE obſerve, that becauſe the Parts deficient are leſs than the Moon's Diameter, *viz.* $37' : 4''$ it ſhews the Eclipſe not to be total; but when they are equal, then the Eclipſe will be total without Continuance. But if the Parts deficient are more than the Moon's Diameter, then the Eclipſe will be total with Continuance.

To find the Digits Eclipſed.

SAY,		L. L.
As the Semidiameter of ☽	$38' : 32''$	5102
Is to 6 Digits		10000
So are the Parts deficient	$15 : 13$	5958
To the Digits Eclipſed	$4^d : 55 : 34$	10856

To find the Scruples of Incidence, or Half-Duration.

RULE.

TAKE the Logarithms of the Sum and Difference of the Semidiameters of the Moon and Earth's Shadow, and of the Latitude of the Moon; the Half Sum of the two Logarithms is the Logarithm of the Scruples of Incidence or Half Duration.

		Log.
Sum of the Semidiameters	$59' : 9' = 3549''$	
Latitude of ☽	$43 : 56 = 2636$	
Sum	6185	3·791339
Difference	913	2.960471
Sum of the Logarithms		6.751810
Half is the Logarithm of $= 2377'' = 39' : 37''$		3·375905

FOR the Time of Incidence, say by *Street*'s L. L. Logarithms;

As the Hourly Motion of ☽ *a* ☉ $= 31' : 46''$	2762
Is to 1 Hour, or 60 Min.	0
So is the Scruples of Incidence $39' : 37''$	1803
To the Time of Incidence, or Half Duration $1^h : 14' : 50''$	959

SUBSTRACT this from the apparent Time of the Middle of the Eclipse, what remains will be the Beginning; and, being added to it, will give the End of the Eclipse. Thus,

Apparent Time of the Middle of the Eclipse, *December*	$12^d : 8^h : 4' : 7''$
Time of Incidence subſtract, and add	$1 : 14 : 49$
Remains the Beginning	*December* $12 : 6 : 49 : 18$
The Sum is the End	$12 : 9 : 18 : 56$
Whole Duration	$2 : 29 : 38$
Digits Eclipſed	$40 : 55' : 34''$.

To delineate the Eclipse.

1. TAKE the Argument of Lat. and Hourly Motion of ☽ *a* ☉ to the Table of the Angle of the Moon's Way with the Ecliptic, and thence take out the Angle which you will find to be $5' : 39''$.

2. DRAW the Line E B for an Arch of the Ecliptic, and upon C erect the Perpendicular C *e* for the Axis.

3. TAKE in your Compaſſes, from a Line of equal Parts the Sum of the Semidiameters of the Moon and Earth's Shadow $= 59' : 9''$, and ſetting one Foot at C, with the other draw E *e* B.

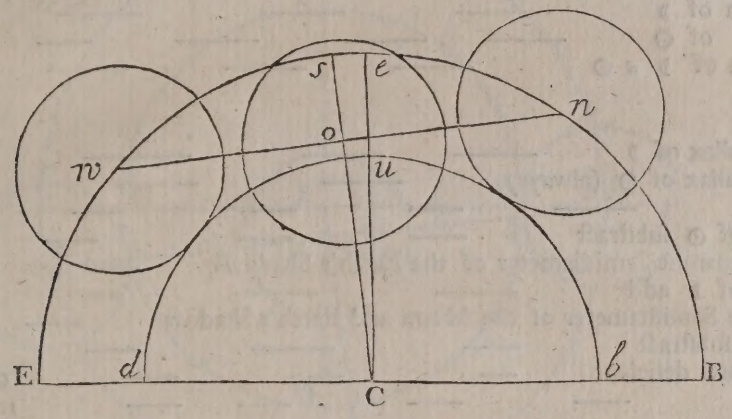
4. FROM the ſame Line take the Semidiameter of the Earth's Shadow $= 40' : 37''$, and ſetting one Foot in the Centre C, with the other draw the Semicircle *dub*, which will repreſent half the Shadow.

5. MAKING C *e* the Radius of a Line of Chords, by a Sector lay off the Angle of the Moon's Way $5° : 39'$ from *e* to *s* on the Left-hand from the Axis of the Ecliptic (becauſe the Moon's Latitude is deſcending) and draw the Line C *s*.

6. TAKE the Moon's Latitude $43' : 56''$, and ſet that from C to *c*, and through *o* at Right Angles to C *s* draw the Line *w o n* for the Moon's Way.

7. TAKE

7. TAKE the Semidiameter of the Moon 18′ : 32″ in your Compaſſes, and upon the Centers *n*, *o*, *w*, draw three Circles to repreſent the Moon at the Beginning, Middle, and End of the Eclipſe.



Note, That this was drawn from a Line of 35 equal Parts in an Inch.

An EXAMPLE of a total Eclipſe of the Moon, that is to be in February, 1747, *according to the following Calculation.*

	Epacts.				Mot. Lat.			
	d :	h :	′ :	″	ſ :	° :	′ :	″
1741 — —	23	21	45	11	5	22	29	24
6 — —	5	17	40	17	3	19	37	10
February — —	1	11	15	15	1	0	43	25
Sum — — — — —	31	2	41	25	10	12	49	59
From the Canon (a Full ☽) ——— ———	44	7	6	4	7	16	0	21
Mean Time of the Oppoſition, *February* — —	13	4	24	39	5	28	50	20

THE Motion of Latitude 5ˢ : 28° : 50′ : 20″ being between the Limits 5ˢ : 9° : 19′ and 6ˢ : 11° : 12′ ſhews that an Eclipſe may be expected at this Full.

The mean Time 1747, *February* ——— ———	13ᵈ :	4ʰ :	24′ :	39″
Interval add ——— ———		13 :	2 :	56
The Sum is the equal Time of the true Oppoſition ———	13 :	17 :	27 :	35
At which Time the mean Anomaly of ☽ is ——— ———	3ˢ :	21° :	22 :	08
of ☉ ——— ———	7 :	26 :	14 :	38
The Sun's true Place ——— ———	11 :	6 :	18 :	04 ♋
The Moon's Place in her Orb ——— ——— ———	5 :	6 :	18 :	04 ♌
True Place of the Node ſubſtract ——— ——— ———	11 :	5 :	14 :	58
Remains the Argument of Latitude ——— ———	6 :	1 :	3 :	06
True Latitude of ☽ South deſcending ——— ———			5 :	49
Time of Reduction ſubſtract and add ——— ———			0 :	28
Remains the true equal Time of the Middle ——— ———		17 :	27 :	07

The

The Sum is the equal Time of the Ecliptic Oppoſition	17 : 28 : 03
Equation of Time ſubſtract	13 : 32
Remains the apparent Time of the Middle	17 : 13 : 35
Apparent Time of the Ecliptic Oppoſition	17 : 14 : 31
Hourly Motion of ☽	34 : 37
of ☉	2 : 31
Hourly Motion of ☽ *a* ☉	32 : 06

Horizontal Parallax of ☽	58 : 20
Horizontal Parallax of ☉ (always)	0 : 10
Their Sum is	58 : 30
Semidiameter of ☉ ſubſtract	16 : 14
Remains the apparent Semidiameter of the Earth's Shadow	42 : 16
Semidiameter of ☽ add	15 : 59
The Sum is the Semidiameter of the Moon and Earth's Shadow	58 : 15
Latitude of ☽ ſubſtract	5 : 49
Remains the Parts deficient	0 : 52 : 26
Digits Eclipſed	19 : 41 . 0

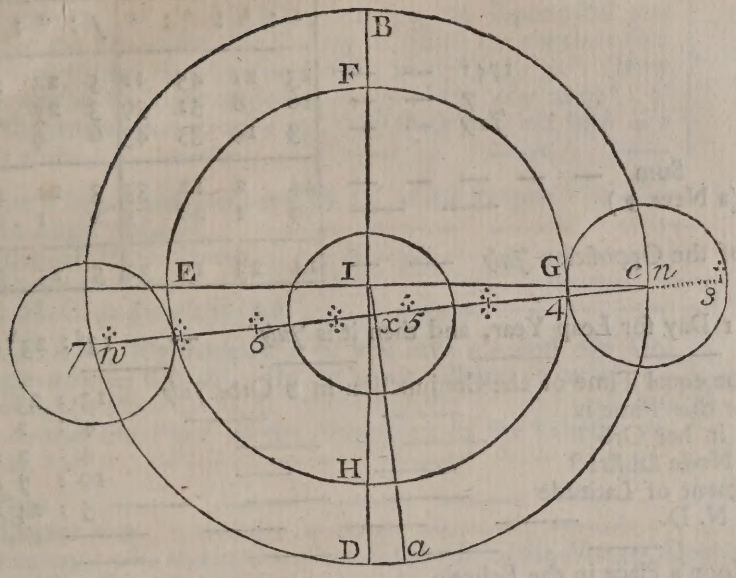
HERE the Parts deficient being more than the Moon's Diameter 31′ : 58″ ſhews that the Eclipſe will be total with Continuance.

Scruples of Incidence or Half Duration	57′ : 58″
Time of Incidence (found, as in the laſt Example)	1ʰ : 48 : 20
In Total Eclipſes, inſtead of the Sum of the Semidiameters of the Moon and Earth's Shadow, you muſt take their Difference, and with that and the Moon's Latitude proceed to find the Scruples of Half Duration in Total Darkneſs, as you did the Scruples of Half Duration of the whole Eclipſe, that will be	23 : 21
And the Time of Half Duration in Total Darkneſs will be	43 : 38
Theſe Times being ſubſtracted from the apparent Time of Middle of the Eclipſe, and added to it, will ſhew the Beginning to be at	15ʰ : 25 : 15
Beginning of Total Darkneſs	16 : 29 : 57
End of Total Darkneſs	17 : 57 : 13
End of the Eclipſe	18 : 38 : 55
Continuance in Total Darkneſs	1 : 27 : 16
Continuance of the whole Eclipſe	3 : 16 : 40

Sum of the Semidiameter of the Moon and Earth's Shadow	I B =	58′ : 15″
Semidiameter of the Earth's Shadow	I F =	42 : 16
Semidiameter of the Moon	G *n* =	15 : 59
The Angle of the Moon's Way with the Ecliptic 5° : 42′ = D J *a* ſet from the Axis of the Eclipt. at D to *a* towards the Right-hand, becauſe the Moon's Latitude is deſcending		
The Latitude of the Moon is SD	I *x* =	5 : 49

Drawn from a Line of 40 equal Parts in an Inch.

Obſerve



Obſerve the Directions for Drawing the laſt Figure.

To find at what Point of the Moon's Way her Center is at any Time during the Eclipſe.

1. THE Middle, or Time of greateſt Obſcuration, is at 13′ : 35″ after 5 in the Morning of the 14th Day of *February*, and the Hourly Motion of ☽ *a* ☉ is 32′ : 16″. Now ſay,

If 1 Hour, or 60 Minutes,
Gives a Motion of 32′ : 6″ —— 2716
What will —— 13 : 35 —— 6451 } L. L.
Anſwer —— 7 : 16 —— 9167

2. TAKE 7′ : 16″ in your Compaſſes, from the ſame Line of equal Parts, and ſet that from *x* (towards *n*) to 5, for the Place of the Moon's Center at that Time; this Point being found, take 32′ : 6″ the Hourly Motion of ☽ *a* ☉, and lay that Extent both Ways, from 5 to 4, and 6, and, if you pleaſe, from 4 to 3, and from 6 to 7, for whole Hours; and if you would put on Half Hours, then take 16′ : 3″, and lay that from 5 both Ways; ſo will you have Half an Hour after 4, and Half an Hour after 5; and, in like manner, you may lay down any Time leſs than Half an Hour.

To calculate an Eclipſe of the Sun.

OUR Example ſhall be that in *July*, 1748.

	Epacts.				Mot. Lat.			
	d	h	′	″	*s*	°	′	″
1741	23	21	45	11	5	22	29	24
7	16	8	51	39	3	27	39	58
July	3	19	35	43	6	4	4	32
Sum	44	2	12	33	3	24	13	54
From the Canon (a New ☽)	59	1	28	5	2	1	20	28
The mean Time of the Oppoſition *July*	14	23	15	32	5	25	34	22

From which take 1 Day for Leap Year, and then it is *July* — 13d : 23h : 15′ : 32″
Interval add — 4 : 46
The Sum is the true equal Time of the Conjunction in ☽ Orb, *July* 13 : 23 : 20 : 18
The Sun's Place for this Time is — 4s : 2 : 42 : 2
The Moon's Place in her Orb — 4 : 2 : 42 : 2
True Place of the Node ſubſtract — 10 : 7 : 46 : 26
Remains the Argument of Latitude — 5 : 24 : 55 : 36
The true Latitude N. D. — 28 : 0
Reduction add — 1 : 18
The Sum is the Moon's Place in the Ecliptic 4 : 2 : 43 : 20
Hourly Motion of ☉ — 2 : 23
Horizontal Parallax of ☽ — 53 : 57

Now to find the Hourly Motion of the Moon, according to Mr. *Brent*.

The mean Hourly of ☽ is — 33′ : 33″
And the mean Horizontal Parallax of ☽ is — 57 : 30
The Difference between this and the preſent Horizontal Parallax is — 3 : 33 = 213″

Multiply this Difference in Seconds 213 by 1.12, and the Product is 238.56; which divide by 60, and the Quotient 4′ *fere*. Now becauſe the preſent Horizontal Parallax is leſs than the Mean, this 4′ muſt be ſubſtracted from the mean Hourly Motion 33′ : 33″, and the Remainder is 29′ : 33″ for the Hourly Motion of ☽. From which take the Hourly Motion of ☉ — — — — } 2 : 23

And the Remainder is the Hourly Motion of ☽ *a* ☉ — 27 : 10
By which the Time of Reduction (contrary to its firſt Title) is ſubſtract. 2 : 52
The equal Time of the true Conjunction is — 23h : 20 : 18
Remains the true equal Time of the Ecliptic Conjunction — 23 : 17 : 26
Equation of Time ſubſtract — 6 : 00
Remains the apparent Time of the Ecliptic Conjunction — 23 : 11 : 26
Which in Degrees of the Equinoctial is — 347° : 51 : 30
The Sun's Place for this Time is 4s : 2° : 41′ : 55″, and R. A. 124 : 59 : 0
The Sum (want. 360) is the R. A. of M. C. — 112 : 50 : 30

The

The R. A. of M. C. being more than 90°, and under 180°, ſhews that this Caſe falls under the *Second Variety*, where the Ecliptic interſects the South Part of the Meridian between the Equinoctial and Zenith. See the Figure for that Variety in which $r æ$ may be called 112° : 50′ : 30″ and $\triangle æ$ its Supplement = 67° : 9′ : 30″. Now in the Triangle $\triangle b æ$ Right-angled at $æ$, you have $\triangle æ$ = 67° : 9′ 30″, and the Angle $b \triangle æ$ = 23° : 29′. By theſe you will find $\triangle b$ = 68° : 53′ = — — — — } 2ˢ : 8° : 53′

Take this from 6 Signs, and you have M. C. in the Ecliptic — — 3 : 21 : 7
The Meridian Angle $\triangle b æ$ = — — — — — 81 : 6
The Declination of M. C. North = $b æ$ ſubſtract — — — 21 : 40
Latitude of *London* = $z æ$ = — — — — — 51 : 32
Remains the M. C. a Zenith = $z b$ — — — — — 29 : 52

In the Triangle $z b v$, Right-angled at v, you have $z b$ and the Meridian Angle $z b v$ = 81° : 6′. By theſe you will find $b v$ = 5° : 5′, take this from the Place of M. C. in the Ecliptic 3ˢ : 21° : 7′, and there will remain the Place of the *Nonageſſima* in the Ecliptic, repreſented by $r v$ — — — — — } 3 : 16 : 02

The Sun's Place — — — — — — 4 : 02 : 41 : 55
Nonageſſima Degree $a \odot$ — — — — — 16 : 39 : 55
The Side $z v$ = 29° : 28′, and its Complem. is the Alt. of the *Nonageſ.* Degree 60 : 32
The Horizontal Parallax of $\mathfrak{D}\ a \odot$ is — — — — — 53 : 47

To find the Moon's Parallax of Longitude.

ADD together the Sine of the Altitude of the *Nonageſſima* Degree, and the Sine of the Diſtance of the Sun from the *Nonageſſima* Degree, then ſubſtract their Sum from the Logiſtical Logarithm of the Horizontal Parallax of the Moon from the Sun, and there will remain the Logiſtical Logarithm of the Parallax of Longitude.

	L. L.
The Horizontal Parallax of $\mathfrak{D}\ a \odot$ 53′ : 47″ — — —	475
Altitude of the *Nonageſſima* Degree 6° : 32′ *ſine* — — —	9·939840
Diſtance of the Sun from the *Nonageſſima* 16° : 40′ *ſine* — — —	9·457584
Sum — —	9·397424
Remains the L. L. of the Parallax of Longitude 13′ : 26″ — —	6501

To find the Moon's viſible Place.

IF the Moon be between the Aſcendant and the *Nonageſſima* Degree, the Parallax of Longitude muſt be added to her true Longitude in the Ecliptic. But if ſhe be between the *Nonageſſima* Degree and the Deſcendent, the Parallax of Longitude muſt be ſubtracted, the Sum or Difference will be the viſible Longitude of the Moon.

The Moon in the Ecliptic — — — 4ˢ : 2° : 43′ : 20″
Parallax of Longitude add — — — — 13 : 26
The Sum is the Moon's viſible Place — — 4 : 2 : 56 : 46

To

To find the Parallax of Latitude.

FROM the Logiſtical Logarithm of the Horizontal Parallax of the Moon from the Sun ſubſtract the Coſine of the Altitude of the *Nonageſſima* Degree, the Remainder will be the Logiſtical Logarithm of the Parallax of Latitude.

The Horizontal Parallax of the Moon from the Sun 53′ : 47″ ——	475 L. L.
Altitude of the *Nonageſſima* Degree 60° : 32′ Co-ſine —— ——	9.6919
Parallax of the Latitude (South) 26′ : 28″ —— —— ——	3556

To find the viſible Latitude.

HAVING found the true Latitude of the Moon, and Parallax of Latitude (which Parallax in theſe Northern Parts is always South); then if they are both of one Name, that is both North, or both South, add the true Latitude and Parallax of Latitude together.

BUT, if one be North, and the other South, ſubſtract the leſſer from the greater, and the Sum or Difference will be the viſible Latitude of the Moon, either Northward or Southward, which you may know by the Name of the greater, for the viſible Latitude will have the ſame Name.

In this Caſe the true Latitude is N. D. —— —— ——	28′ : 00″
And the Parallax of Latitude South —— —— ——	26 : 28
The Difference is the viſible Latitude N. —— —— —— ——	1 : 32

Note, That when the Sun's Place is leſs than the *Nonageſſima* Degree the Luminaries appear more Weſt than the Truth; but if his Place be greater, then they appear more Eaſt, according to the Quantity of the Parallax of Longitude.

WHEN the Sun's Place is greater than the *Nonageſſima* Degree, the Eclipſe happens in the Oriental Quadrant; but if leſs, then in the Occidental.

IN this Ecliptic the Sun is in the Oriental Quadrant of the Ecliptic, or on the Eaſt Side of the *Nonageſſima* Degree; therefore find the Requiſites for 1 Hour before the apparent Time of the true Ecliptic Conjunction.

BUT, if it had happened in the Occidental Quadrant, or on the Weſt Side of the *Nonageſſima* Degree, then for 1 Hour after.

One Hour before the apparent Time of the Ecliptic Oppoſition ——	22ʰ : 11′ : 26″
The ſame in Degrees of the Equinoctial —— —— ——	332° : 51 : 30
The Sun's Place then 4ˢ : 2° : 39′ : 32″ and his R. A. ——	124 : 56 : 52
The Sum want. 360 is the RA of MC —— —— ——	97 : 48 : 22

See the Fig. for the *Second Variety*, in which let $r æ$ be 97° : 48′ : 22″, then its Supplement will be $\triangleq b = 82° : 11′ : 38″$. By this and the Angle $b \triangleq æ = 23° : 29′$, you will find $\triangleq b = 82° : 50′ = 2ˢ : 22° : 50′$; take this from 6 Signs and the Remainder is the Place of the M. C. in the Ecliptic —— —— —— 3ˢ : 7° : 10′

The Declination of M. C. North $= b æ =$ —— ——	23 : 17
Altitude of the Equinoctial at *London* $= t æ$ —— —— —— ——	38 : 28
Altitude of M. C. $= t b$ —— —— —— —— —— ——	61 : 45
M. C. a Zenith $b z =$ —— —— —— —— —— ——	28 : 15
Meridian Angle $\triangleq b æ =$ —— —— —— —— —— ——	86 : 54

IN

In the Triangle bzv we have $bz = 28° : 15'$ and the Meridian Angle $Zbv = 86° : 54'$. By theſe we find $bv = 1° : 39'$, take this from the Place of M. C. in the Ecliptic, and there will remain the Place of the *Nonageſſima* Degree ——— ——— $3^s : 5° : 31'$

The Sun in ——— ——— ———	$4 : 2 : 39 : 32''$
Diſtance of the Sun from the *Nonageſſima* Degree ———	$22 : 08 : 32$
The Side zv is $28° : 13'$, its Complement is vx the Altitude of the *Nonageſſima* Degree ——— ——— ——— ——— ———	$61° : 47'$
The Horizontal Parallax of ☽ *a* ☉ ——— ——— ———	$53' : 47''$
The Parallax of Longitude at this Time is ——— ———	$21 : 38$
The Parallax of Longitude before found was ——— ———	$13 : 26$
Their Difference (the Parallax of Long. decreaſing) ——— ———	$8 : 12$

To find the viſible Hourly Motion of the Moon from the Sun.

1. If the Eclipſe happens in the Oriental Quadrant of the Ecliptic from the *Nonageſſima* Degree, and the Parallax of Longitude decreaſe, ſubſtract the Difference of the Parallax of Longitude (in one Hour or any leſs Time, as you think convenient) from the true Motion of the Moon from the Sun in that Time. But if the Parallax of Longitude increaſe, add the ſaid Difference.

2. If the Eclipſe happens in the Occidental Quadrant, and the Parallax of Longitude decreaſe, add the Difference of Parallaxes to the true Motion of the Moon from the Sun. But if the Parallax of Longitude increaſe, ſubſtract it.

3. If the Eclipſe happens in the *Nonageſſima* Degree, ſo that the Beginning falls in the Oriental Quadrant, and the End in the Occidental; ſubſtract the Difference from the Motion of the Moon from the Sun, and you will have the viſible Motion of the Moon from the Sun.

The Hourly Motion of ☽ *a* ☉ is ——— ——— ———	$27' : 10''$
And the Difference of the Parllaxes ſubſtract ——— ——— ———	$8 : 12$
Remains the viſible Hourly Motion of ☽ *a* ☉ ——— ———	$18 : 58$

To find the Interval between the apparent Time of the true Ecliptic Conjunction, and the apparent Time of the viſible Conjunction.

Say, As the viſible Hourly Motion of ☽ *a* ☉	$18' : 58''$	L.L. ———	5002
Is to 1 Hour or 60 Minutes ——— ———		———	0
So is the Parallax of Longitude firſt found	$13 : 26$	L.L. ———	6500
To the Interval requir'd ———	$42 : 30$	L.L. ———	1498

When the Eclipſe happens in the Oriental Quadrant, as in this Example, you muſt ſubſtract the Interval from the apparent Time of the true Ecliptic Conjunction; but, in the Occidental, add it.

The apparent Time of the true Ecliptic Conjunction *July* ———	$13^d : 23^h : 11' : 26''$
Interval ſubſtract ——— ——— ——— ——— ——— ———	$42 : 30$
Remains the apparent Time of the viſible Conjunction ———	$22 : 28 : 56$

The apparent Time of the visible Conjunction in Degrees	337° : 14′ : 0″
Sun's Place 4ˢ : 2° : 40′ : 14″, and his R. A.	124 : 57 : 36
The Sum want. 360. is the R. A. of M. C.	102 : 11 : 36

Let this be represented by $r æ$ in the Figure for the *Second Variety*, then its Supplement is $\triangle æ = 77° : 48′ : 24″$, the Base of the Triangle $\triangle b æ$, in which you have also the Angle $b \triangle æ = 23° : 29′$. By these you will find $\triangle b = 78° : 47′ = 2ˢ : 18° : 47′$, which take from 6 Signs, and the Remainder is the Place of M. C. in the Ecliptic — 3ˢ : 11° : 13′

The Declination of M. C. $= b æ$ North	23 : 01
Altitude of the Equinoctial at *London* $= t æ =$	38 : 28
Altitude of M. C. $= t b$	61 : 29
M. C. a Zenith $= z b$	28 : 31
The Meridian Angle $\triangle b æ = z b v$	85 : 10

Now in the Right-angled Triangle $z b v$ you have $z b = 28° : 15′$, and the Angle $z b v = 86° : 54′$; by these you will find $b v$ (subst.) — 2° : 37′

M. C. in the Ecliptic	3ˢ : 11 : 13
The Place of the *Nonagessima* Degree in the Ecliptic	3 : 8 : 36
The Sun in	4 : 2 : 40
The Distance of the *Nonagessima* Degree from the Sun	24 : 04
And the Side $z v = 28° : 24′$ the Complement is the Altitude of the *Nonagessima* Degree $= v x$	61 : 36

The Horizontal Parallax of ☽ *a* ☉ is	53′ : 47″
Parallax of Longitude	19 : 17
Parallax of Latitude	25 : 35

Now say,	As 60 Min. or 1 Hour		0
	Is to the Hourly Motion of ☽ *a* ☉	= 27′ : 10″	3441
	So is the Interval	42 : 30	1498
	To	19 : 15	4939

This should have been the same with the Parallax of Longitude 19′ : 17″; but as the Difference is only 2″, you may conclude that the apparent Time of the visible Conjunction is found sufficiently near the Truth.

To find the visible Latitude of the Moon at the Time of the visible Conjunction.

Add the Parallax of Longitude at the visible Conjunction to the Motion of the Sun, agreeing to the Interval between the apparent Time of the Ecliptic Conjunction and the apparent Time of the visible Conjunction; then, if the Eclipse happens in the Oriental Quadrant, subtract the Sum from the Argument of Latitude at the Time of the true Conjunction. But, if it happens in the Occidental Quadrant, you must add it, and the Difference or Sum will be the Argument of Latitude at the Time of the visible Conjunction, by which find her true Latitude. According to this you will find her true Latitude to be — 30′ : 3″

Parallax of Latitude	25 : 35
Tne Difference is the visible Latitude at 22ʰ : 28′ : 56″	4 : 28

The visible Latitude at the apparent Time of the Ecliptic Conjunction 23ʰ : 11′ : 26″ was 1′ : 32″. Hence we find that this visible Latitvde 4′ : 28″ is N. D.

To

To find the Quantity of a Solar Eclipſe.

By Help of the mean Anomaly of the Sun at the viſible Conjunction (0ˢ : 25° : 1′ : 5″)
you'll find his Semidiameter to be ——— ——— — 15′ : 49″

And the Semidiameter of the Moon ——— ——— ——— 14 : 47

Their Sum is ——— ——— ——— ——— ——— 30 : 36
The viſible Latitude at the viſible Conjunction ſubſtract ——— 4 : 28

Remains the Parts Deficients ——— ——— ——— 26 : 08

Now ſay, As the Semidiameter of the Sun 15′ : 49″ ——— L. L. 5790

Is to 6 Digits ——— ——— ——— 10000
So are the Parts deficient —— 26 : 08 —— 3610

13610

To the Digits Eclipſed 9° : 55′ ——— ——— ——— 9820

To find the Scruples of Incidence.

To the Semidiameter of the Moon add the Semidiameter of the Sun, both in Seconds; and to that Sum add the viſible Latitude of the Moon (at the viſible Conjunction) in Seconds, and alſo ſubſtract it from the ſame. Then take the Logarithm of the Sum, and alſo of the Difference; ſo ſhall Half the Sum of theſe two Logarithms be the Logarithm of the Scruples of Incidence. But, if the Eclipſe be Total, then for the Scruples in Total Darkneſs, you muſt take the Difference of the Semidiameters of the Sun and Moon, inſtead of the Sum, and go on with that and the viſible Latitude, as above.

Semidiameter of the Sun — 15′ : 49″ = 949″
Semidiameter of the Moon — 14 : 47 = 887

Sum ——— 1836
Viſible Latitude of the Moon 4 : 28 = 268

Sum ——— ——— ——— 2104 Logar. — — 3.323046
Difference ——— ——— 1568 Logar. — — 3.195346

6.518392 Sum

Half the Sum is the Logarithm of 1816″ = 30′ : 16″ Scrup. of Inciden. 3.259196

To

To find the Middle, Beginning, and End of the Eclipſe.

TAKE the viſible Latitude of the Moon 4′ : 28″ to the Table next following, and thence take the Numbers anſwering thereto, which you'll find to be 22″; theſe, becauſe the Latitude is deſcending, muſt be divided by the viſible Hourly Motion of the Moon from the Sun for one Hour after the viſible Conjunction (elſe by the viſible Hourly Motion of the Moon from the Sun for one Hour before).

A TABLE of the Diſtance, or Motion of the Moon from the Conjunction to the greateſt Obſcuration.

Viſible Latitude of the Moon.		Diſtan. from the Conjunction.	
°	′	′	″
0	1	0	5
0	2	0	10
0	3	0	15
0	4	0	20
0	5	0	25
0	10	0	51
0	15	1	17
0	20	1	43
0	25	2	9
0	30	2	34
0	35	3	0
0	40	3	25
0	45	3	50
0	50	4	15
0	55	4	41
1	0	5	7
1	5	5	32
1	10	5	57
1	15	6	24
1	20	6	49
1	25	7	14
1	30	7	39
1	35	8	4
1	40	8	30
1	45	8	55

{ North, South } Aſcending ſubſtract

{ North, South } Deſcending add

ONE

One Hour before the visible Conjunction $21^h : 28' : 56''$ in Degrees	$322° : 14' : 0''$
Sun in — $4^s : 2° : 37' : 51''$ and R. A.	$124 : 55 : 8$
The Sum want. 360 is the R. A. of M. C.	$87 : 9 : 8$

See the Figure for the *First Variety*, in which let $r æ$ be $87° : 9' : 8''$, the Base of the Right-angled Triangle $r æ b$; by which, and the Angle $b r æ = 23° : 29'$, you will find $r b = 87° : 23'$ the Place of the M. C. in the Ecliptic, which is $2^s : 27° : 23'$

The Declination of M. C. North $= b æ$	$23 : 28$
Altitude of the Equinoctial at *London* $= t æ$ add	$38 : 28$
Altitude of M. C. $= t b$	$61 : 56$
M. C. *à* Zenith $= b z$	$28 : 04$
Meridian Angle $r b æ = z b v =$	$88 : 52$

Now, in the Right-angle Triangle $z v b$ you have $z b = 28° : 4'$, and the Meridian Angle $z b v = 88° : 52'$ to find

1. The Distance of the *Nonagesſima* Degree from M. C. that is $b v =$	$0° : 36'$
M. C. in the Ecliptic	$2^s : 27 : 23$
The *Nonagesſima* Degree at v in	$2 : 27 : 59$
The Sun's Place	$4 : 02 : 38$
Nonagesſima Degree *à* ☉ $= 34° : 39'$	$1 : 4 : 39$

2. The Side $z v = 28° : 4'$, then the Altitude of the *Nona.* Deg. is $v m =$	$61° : 56'$
The Horizontal Parallax of ☽ *à* ☉ is	$53' : 47''$
Parallax of Latitude	$25 : 18$
Parallax of Longitude	$26 : 59$
The Parallax of Longitude at the visible Conjunction was found to be	$19 : 17$
Difference of Parallaxes (decreasing) subtract	$7 : 42$
Hourly Motion of ☽ *à* ☉	$27 : 10$
Remains the visible Hourly Motion of ☽ *a* ☉	$19 : 28$

One Hour after the visible Conjunction $23^h : 28' : 56''$ in Deg.	$352° : 14' : 0''$
☉ in — $4^s : 2° : : 42' : 37''$ and his R. A.	$125 : 00 : 14$
Their Sum want. 360°, is the R. A. of M. C.	$117 : 14 : 14$

Let this be represented by $r æ$ in the Figure for the *Second Variety*, the Supplement of which is $\triangle æ$ the Base of the Triangle $b \triangle æ$ $62° : 45' : 46''$

By this and the Angle $b \triangle æ = 23° : 29'$ you'll find $b \triangle = 64° : 43' = 2^s : 4° : 43'$, which take from 6 Signs, and you have the Place of M. C. in the Ecliptic $= r b$ $3^s : 25° : 17'$

The Declination of M. C. North $= b æ$	$21 : 07$
Altitude of the Equinoctial at *London* $= t æ$	$38 : 28$
Altitude of the M. C. $= t b$	$59 : 35$
M. C. *à* Zenith $= z b$	$30 : 25$
The Meridian Angle $\triangle b æ = z b v$	$79 : 30$

In the Triangle $z b v$ you have $z b = 30° : 25'$, and the Meridian Angle $z b v = 79° : 30'$, by theſe $b v$ is found to be —— —— —— ſubſt. $6° : 7'$

The Place of M. C. —— —— ——	$3^s : 25 : 17$
Remains the Place of the *Nona.* Degree in v —— ——	$3 : 19 : 10$
The Sun's Place —— —— —— ——	$4 : 2 : 42 : 37''$
The Diſtance of the Sun from the Nonageſſima Degree ——	$13 : 32 : 37$
The Side $z v$ is $29° : 51'$, whoſe Complement is $v x$ the Altitude of the Nonageſſima Degree —— —— ——	$60 : 9$

The Horizontal Parallax of ☽ *à* ☉ —— —— ——	$53' : 47''$
Parallax of Latitude —— —— —— ——	$26 : 46$
Parallax of Longitude —— —— —— ——	$10 : 56$
Parallax of Longitude at the viſible Conjunction —— ——	$19 : 17$
Difference (decreaſing) —— —— ——	$8 : 21$
Hourly Motion of ☽ *à* ☉ —— —— ——	$27 : 10$
The viſible Hourly Motion of ☽ *à* ☉ —— —— ——	$18 : 49$

To Reduce the Moon's Motion from the true Conjunction to the greateſt Obſcuration, *viz.* $0' : 22''$ before found, into Time.

As the viſible Hourly Motion $18' : 49''$ —— —		L. L. ——	5036
Is to 1 Hour,			
So is ——	$0 : 22$ —— ——	L. L. —	22139
To —— ——	$1 : 10$ —— ——	L. L. —	17103
The apparent Time of the viſible Conjunction —— ——			$22^h : 28' : 56''$
To which add —— —— —— ——			$1 : 10$
The Sum is the Middle of the Eclipſe, or Time of greateſt Obſcuration —			$22 : 30 : 06$

To find the Beginning of the Eclipſe.

As the viſible Hourly Motion 1 Hour before the viſible Conjunction $19' : 28''$	L. L.	4889
Is to Half an Hour, or 30 Min. —— ——	L. L.	3010
So is the Scruples of Incidence $30' : 16''$ —— ——	L. L.	2972
		5982
To $46' : 39''$, which doubled is $1^h : 33' : 18''$ Time of Incidence ——		1093
The Time of greateſt Obſcuration is —— ——		$22^h : 30' : 6''$
Time of Incidence ſubſtract —— —— ——		$1 : 33 : 18$
Remains the Beginning of the Eclipſe —— ——		$20 : 56 : 48$

To

To find Time of Repletion and End of the Eclipſe.

As the viſible Hourly Motion 1 Hour after the viſible Conjunction 18′ : 49″ —	5036
Is to 30 Min. or Half an Hour — — —	3010
So is the Scruples of Incidence 30′ : 16″ — —	2972
To 48′ : 16″, which being doubled is 1ʰ : 36′ : 32″ Time of Repletion —	5982

The apparent Time of the greateſt Obſcuration —	22ʰ : 30′ : 6′
Time of Repletion add — — —	1 : 36 : 32
End of the Eclipſe — — —	24 : 06 : 38
Total Duration — — —	3 : 09 : 50

By the preceding Calculation we find,

That the Beginning of the Eclipſe will be at — —	8ʰ : 56′ : 48″	A.M.
The Middle — — —	10 : 30 : 06	A.M.
End — — —	0 : 6 : 38	P.M.
Total Duration — — —	3 : 9 : 50	
Digits Eclipſed — — —	9 : 55 : 0	

July 14th. 1748.

To find the Latitude at the Beginning and End of the Eclipſe.

Add to the Scruples of Incidence, the Motion of the Sun agreeing to the Time of Incidence, ſubſtract the Sum from the Argument of Latitude at the Time of the viſible Conjunction, and you will have the Argument of Latitude at the Beginning of the Eclipſe; by which you will find the true Latitude to be 33′ : 31″.

Again, For the Argument of Latitude at the End, add to the Scruples of Incidence, the Motion of the Sun, agreeing to the Time of Repletion, and add the Sum to the Argument of Latitude at the Time of the viſible Conjunction; ſo will you have the Argument of Latitude at the End, by which you will find the true Latitude to be 26′ : 54″.

Beginning 20ʰ : 56′ : 48″ in Degrees — —	314° : 12′ : 0″
Sun in 4ˢ : 2° : 36′ : 34″ R. A. — —	124 : 53 : 50
The Sum want. 360 is the R. A. of M. C. — —	79 : 5 : 50
See the Figure for the *Firſt Variety*, where let $r æ$ be 79° : 5′ : 50″, and the Angle $b r æ = 23° : 29′$; by theſe you will find $r b = 79° : 59′$ = the Place of the M. C. in the Ecliptic — —	2ˢ : 19° : 59′

The Declination of M. C. North $b æ$ —	23° : 6′
The Altitude of the Equinoctial at *London* — — —	38 : 28
Altitude of the M. C. $t b$ = — — —	61 : 34
M. C. *à* Zenith $z b$ = — — —	28 : 26
Meridian Angle $r b æ = z b v$ — — —	85 : 41

Now,

Now, in the Triangle $z\,b\,v$, you have $z\,b\,v$ and $z\,b$, by theſe you will find $z\,v$ = 28° : 21′ whoſe Complement is the Altitude of the *Nonageſſima* Degree — 61° : 39′

And $v\,b$ = 2° : 20′, which add to 2ˢ : 19′ : 59″, and the Sum is	2ˢ : 22° : 19′	
Sun's Place	4 : 02 : 36	: 34″
Remains Diſtance of *Nonageſſima* Degree $a\,\odot$ = 40° : 17′ : 34″ —	1 : 10 : 17	: 34
The Horizontal Parallax of ☽ $a\,\odot$	53′ : 47″	
Parallax of Latitude	25 : 32	
The true Latitude at the Beginning N. D.	33 : 31	
Viſible Latitude at the Beginning North	7 : 59	

End of the Eclipſe 24ʰ : 6′ : 38″ in Deg.	361° : 39′ :	30″
The Sun in 4ˢ : 2° : 44′ : 7″ his R. A. is	125 : 1 :	36
The Sum want. 360 is the R. A. of M. C. =	126 : 41 :	06

See the Figure for the *Second Variety*, in which let $r\,æ$ repreſent 126° : 41′ : 6″, then its Supplement is $♎\,æ$ — 53 : 18 : 54

By which and the Angle $b\,♎\,æ$ = — 23 : 29

You will find $b♎$ = 55° : 39′ = 1ˢ : 25° : 39′, take this from ſix Signs and the Remainder is the Place of the M. C. — 4ˢ : 4 : 21

The Declination of M. C. North $b\,æ$ =	19	: 13
Altitude of the Equinoctial at *London*	38	: 28
Altitude of M. C.	57	: 41
M. C. a Zenith = $z\,b$	32	: 19
Meridian Angle $♎\,b\,æ$ = $z\,b\,v$ =	76	: 14

Now, in the Triangle $z\,b\,v$ you have $z\,b$ = 32° : 19′, and the Angle $z\,b\,v$; by theſe you will find $b\,v$ = — 80 : 34′ ſubſt.

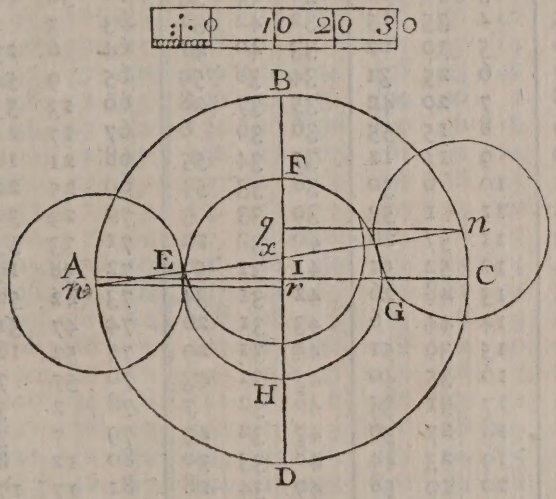
M. C. in the Ecliptic	4ˢ : 4	: 21
The *Nonageſſima* Degree in the Ecliptic	3 : 25	: 47
The Sun in	4 : 02	: 44 : 7″
Nonageſ. Degree $a\,\odot$	6 : 57	: 7
Altitude of the *Nonageſ.* Degree	58 : 43	
Parallax of Latitude, South	27 : 56	
True Latitude, North	26 : 54	
Viſible Latitude at the End, South	1 : 02	

To

To delineate an Eclipſe of the Sun.

THIS differs nothing at all from that of the Moon; but only, that inſtead of the Semidiameter of the Shadow of the Earth, you uſe the Semidiameter of the Sun, and the viſible Latitude for the true.

Sum of the Semidiameters of ☽ and ☉	30′ : 36″	= BI
Semidiameter of the Sun	15 : 49	= FI
Semidiameter of the Moon	14 : 47	= BF
Viſible Latitude at the Beginning, North	7 : 59	= I*q*
Viſible Latitude at the End, — South	1 : 02	= I*r*
Digits Eclipſed	9° : 55′	



A TABLE of the Right Aſcenſion to each Degree of the Ecliptic, the greateſt Declination being 23° : 29′.

Signs	0			1			2			Signs
	°	′	″	°	′	″	°	′	″	°
0	0	0	0	27	54	09	57	48	36	0
1	0	55	2	28	51	32	58	51	9	1
2	1	50	4	29	49	3	59	53	52	2
3	2	45	7	30	46	44	60	56	46	3
4	3	40	10	31	44	33	61	59	49	4
5	4	35	15	32	42	32	63	3	1	5
6	5	30	22	33	40	41	64	6	23	6
7	6	25	31	34	38	59	65	9	54	7
8	7	20	42	35	37	28	66	13	33	8
9	8	15	55	36	36	6	67	17	21	9
10	9	11	11	37	34	55	68	21	18	10
11	10	6	30	38	33	54	69	25	22	11
12	11	1	53	39	33	3	70	29	33	12
13	11	57	20	40	32	22	71	33	53	13
14	12	52	51	41	31	53	72	38	19	14
15	13	48	26	42	31	34	73	42	52	15
16	14	44	6	43	31	26	74	47	31	16
17	15	39	51	44	31	29	75	52	16	17
18	16	35	40	45	31	43	76	57	7	18
19	17	31	35	46	32	7	78	2	3	19
20	18	27	36	47	32	43	79	7	3	20
21	19	23	44	48	33	30	80	12	8	21
22	20	19	58	49	34	28	81	17	17	22
23	21	16	18	50	35	36	82	22	30	23
24	22	12	46	51	36	55	83	27	45	24
25	23	9	20	52	38	25	84	33	4	25
26	24	6	2	53	40	6	85	38	24	26
27	25	2	52	54	41	58	86	43	46	27
28	25	59	49	55	44	0	87	49	10	28
29	26	56	55	56	46	13	88	54	35	29
30	27	54	9	57	48	36	90	0	0	30

When the Sign exceeds 6, ſubſtract 6 from it, and to the Right Aſcenſion of the Remainder add 180°.

A TABLE

A TABLE of the Right Aſcenſion to each Degree of the Ecliptic, the greateſt Declination being 23° : 29′.

Signs	3			4			5			Signs
	°	′	″	°	′	″	°	′	″	
0	90	0	0	122	11	24	152	5	51	0
1	91	5	25	123	13	47	153	3	5	1
2	92	10	50	124	16	0	154	0	11	2
3	93	16	14	125	18	2	154	57	8	3,
4	94	21	36	126	19	54	155	53	58	4
5	95	26	56	127	21	35	156	50	40	5
6	96	32	15	128	23	5	157	47	14	6
7	97	37	30	129	24	24	158	43	42	7
8	98	42	43	130	25	32	159	40	2	8
9	99	47	52	131	26	30	160	36	16	9
10	100	52	57	132	27	17	161	32	24	10
11	101	57	57	133	27	53	162	28	25	11
12	103	2	53	134	28	17	163	24	20	12
13	104	7	44	135	28	31	164	20	9	13
14	105	12	29	136	28	34	165	15	54	14
15	106	17	28	137	28	25	166	11	34	15
16	107	21	21	138	28	7	167	7	9	16
17	108	26	7	139	27	38	168	2	40	17
18	109	30	27	140	26	57	168	58	7	18
19	110	34	38	141	26	6	169	53	30	19
20	111	38	42	142	25	5	170	48	49	20
21	112	42	39	143	23	54	171	44	5	21
22	113	46	27	144	22	32	172	39	18	22
23	114	50	6	145	21	1	173	34	29	23
24	115	53	37	146	19	19	174	29	38	24
25	116	56	59	147	17	28	175	24	45	25
26	118	0	11	148	15	27	176	19	50	26
27	119	3	14	149	13	16	177	14	53	27
28	220	6	8	150	10	57	178	9	56	28
29	121	8	51	151	8	28	179	4	58	29
30	122	11	24	152	5	51	180	0	0	30

When the Sign exceeds 6, ſubſtract 6 from it, and to the Right Aſcenſion of the Remainder add 180°.

A TABLE for converting Hours and Minutes into Degrees, and the contrary.

Time.	Motion.	Time.	Motion.	Time.	Motion.
H.	°	′	° ′	′	° ′
′	′	″	′ ″	″	′ ″
″	″	‴	″ ‴	‴	″ ‴
1	15	1	0 15	31	7 45
2	30	2	0 30	32	8 0
3	45	3	0 45	33	8 15
4	60	4	1 00	34	8 30
5	75	5	1 15	35	8 45
6	90	6	1 30	36	9 0
7	105	7	1 45	37	9 15
8	120	8	2 00	38	9 30
9	135	9	2 15	39	9 45
10	150	10	2 30	40	10 0
11	165	11	2 45	41	10 15
12	180	12	3 00	42	10 30
13	195	13	3 15	43	10 45
14	210	14	3 30	44	11 0
15	225	15	3 45	45	11 15
16	240	16	4 00	46	11 30
17	255	17	4 15	47	11 45
18	270	18	4 30	48	12 0
19	285	19	4 45	49	12 15
20	300	20	5 00	50	12 30
21	315	21	5 15	51	12 45
22	330	22	5 30	52	13 0
23	345	23	5 45	53	13 15
24	360	24	6 00	54	13 30
25		25	6 15	55	13 45
26		26	6 30	56	14 0
27		27	6 45	57	14 15
28		28	7 00	58	14 30
29		29	7 15	59	14 45
30		30	7 30	60	15 0

LET it be required to reduce $15^h : 17' : 22''$ into Degrees, Minutes, *&c.* of the Equinoctial.

By the Table	15^h =	— —	$225° : 0' : 0''$
	$17'$	— —	$4 : 15 : 0$
	$22''$	— —	$5 : 30$
Anſwer		— —	$229 : 20 : 30$

By the Pen, thus; $15^h : 17' : 22''$

60

4) 917 (229° : 20' : 30'' Anſwer
8

11
8

37
36

1
60

4) 82 (2 0
8

02
60

4) 120 (30''

0

AGAIN, Let it be required to reduce $18^h : 38' : 58''$ to Degrees.

By the Table	— —	18^h — —	$270° : 0' : 0''$
		38 Min. —	$9 : 30 : 0$
		58 Sec. —	$14 : 30$
Anſwer	— —	— —	$279 : 44 : 30$

Or,

Or, by the Pen, thus ; — — $18^h : 38' : 58''$

60

4) 1118 ($279° : 44' : 30''$ Anſwer.

8

31

28

38

36

2

60

4) 178 ($44'$

16

18

16

2

60

4) 120 ($30''$

0

LET it be required to reduce $279° : 44' : 30''$ to Time.

$279° : 44' : 30''$

4

60) 1118 : 58 : 0 ($18^h : 38' : 58''$ Anſwer.

60

518

480

38

To reduce Time given for any other Place that is mentioned in the following Table to that of London, *and the contrary.*

THE Epocha, or Radices of the middle Motions of the Sun and Moon in theſe Tables, are accommodated to the laſt Day of the *Julian* Year, under the Meridian of *London*. But, that they may be uſeful in other Places, that are under different Meridians, we have made a Catalogue of moſt of the chief Places in the World, collected from ſome of the beſt Authors; with their Latitudes, and Difference of Longitude, from *London*.

IN uſing this Catalogue, there are *Two Varieties*, viz.

1. WHEN the Time is given at ſome Place mentioned in the Catalogue, to find what Time it is then at *London*.

2. BY having the Time at *London* given, to find the Time at any of thoſe Places.

Note,

Note That A. ſignifies to add, and S. to ſubſtract; that is, ſuppoſe you had the Time of the Middle of a Lunar Eclipſe given for *London*, and would know the Time agreeing thereto for ſome other Place; then look for that Place, and add or ſubſtract the Difference of Meridians to, or from the Time given at *London*, as the Letters A. or S. direct; then the Sum or Difference will be the Time at that other Place.

Note, That the Meridians of thoſe Places, that have A. placed after the Difference of Meridians, are on the Eaſt Side of that of *London*; and thoſe that have S. ſo placed, are Weſt of that of *London*.

If you have the Time at ſome other Place, and would know what is then at *London*, you muſt apply the Difference of Meridians againſt that other Place to the Time given there contrary to the Title, and you will have the Time at *London*.

EXAMPLE 1.

Suppose you would calculate the Sun or Moon's Place by theſe Tables for *Dublin*, in *Ireland*, it being 4 in the Afternoon at that Place; look in the Catalogue for *Dublin*, and you will find the Meridian thereof to differ from that of *London* 27 Minutes. Now, becauſe the Meridian of *Dublin* is Weſt from *London*, the Time at *London* muſt be more than 4 o'Clock; therefore, inſtead of ſubſtracting ſignified by S. you muſt add.

Thus, the Time at *Dublin*	$4^h : 00'$
Difference of Meridians add	0 : 27
The Time at *London*, for which you are to calculate	4 : 27

EXAMPLE 2.

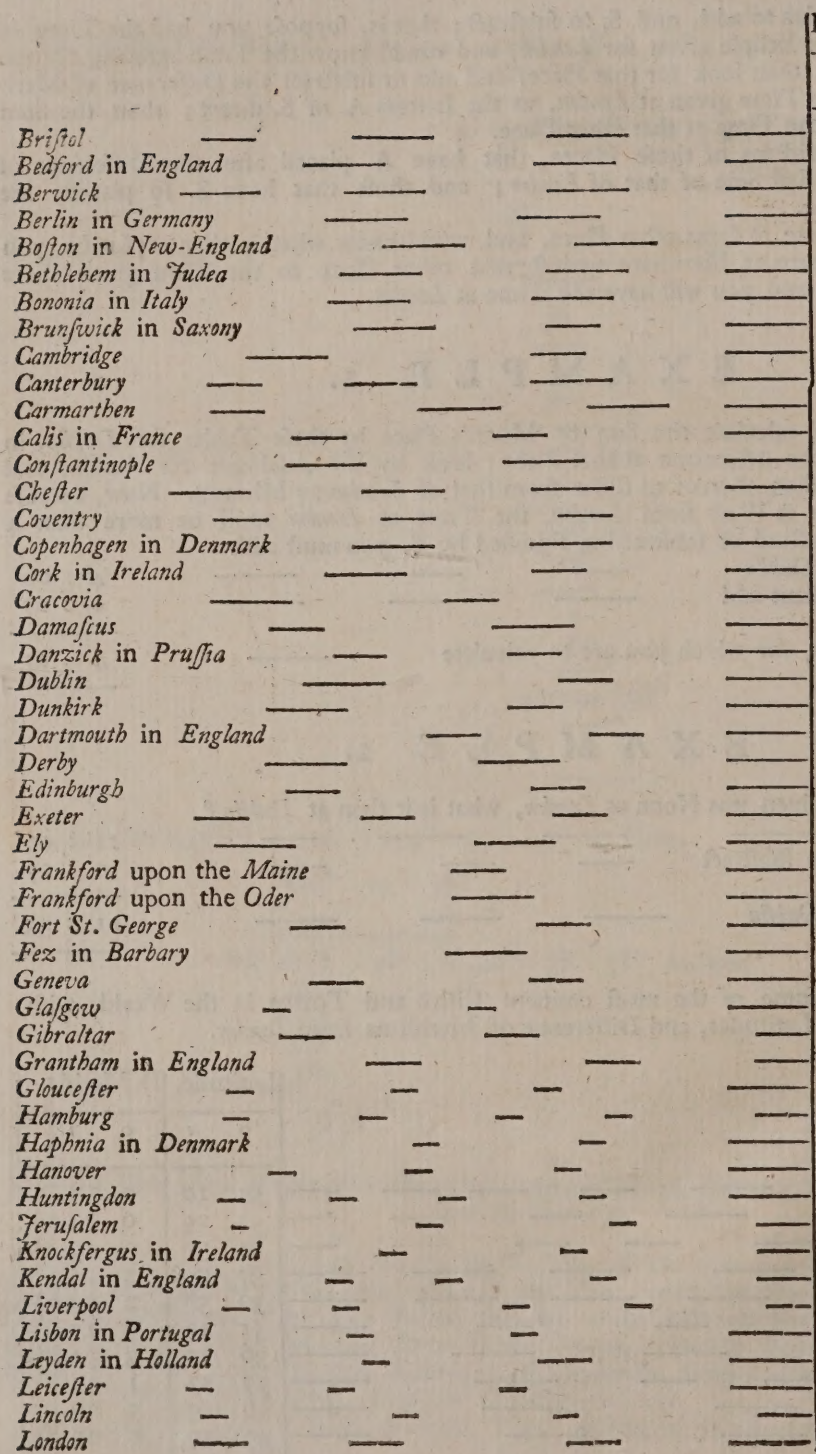
Suppose the Time given was Noon at *London*, what is it then at *Dublin*?

Time at *London*	$12^h : 00'$
Difference of Meridians ſubſtract	0 : 27
Remains the Time at *Dublin*	11 : 33

A Catalogue of ſome of the moſt eminent Cities and Towns in the World, with their Latitudes, and Difference of Meridians from *London*.

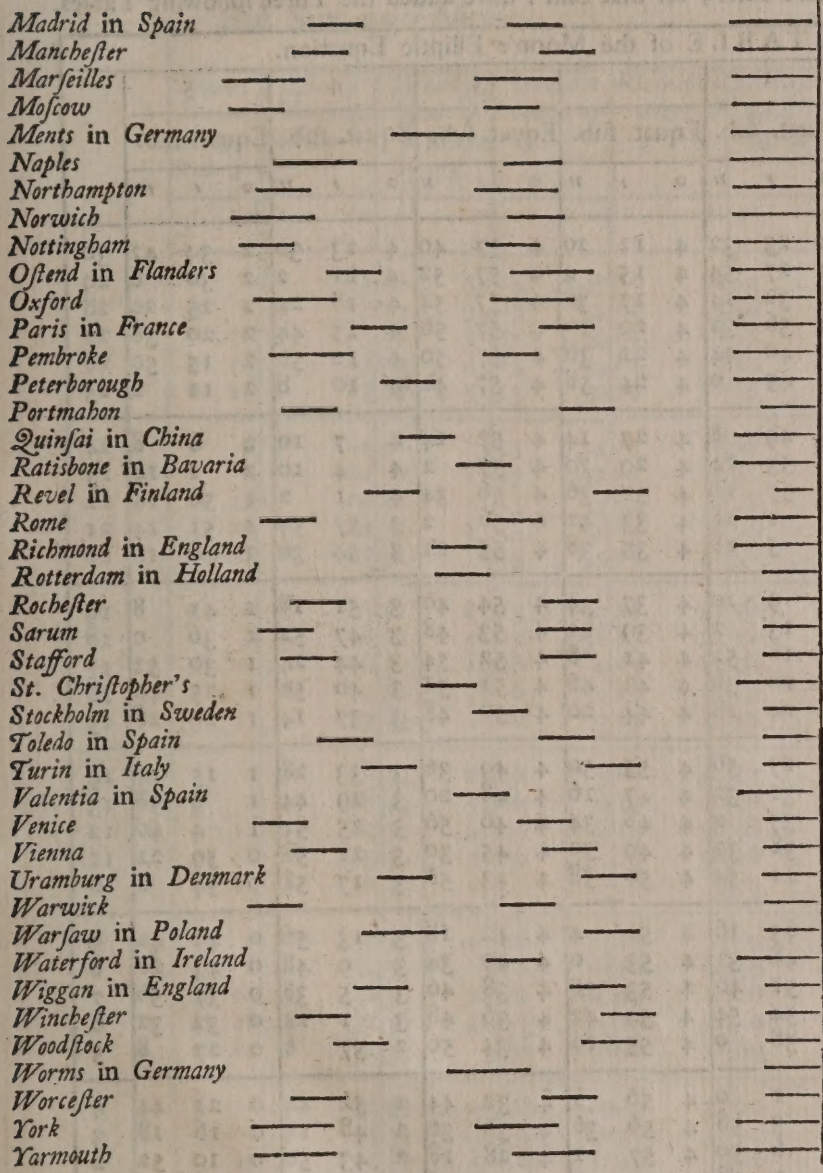
	Latitude.		D. Merid.		
	°	′	°	′	
Aberdeen in *Scotland*	57	10	0	7	S.
Amſterdam	52	29	0	21	A.
Archangel in *Ruſſia*	64	30	2	42	A.
Aleppo	37	20	2	25	A.
Alexandria in *Ægypt*	30	58	2	20	A.
Antwerp	51	12	0	17	A.
Antioch	36	15	3	31	A.
Athens in *Greece*	37	42	1	52	A.
Aracta in *Syria*	36	0	3	20	A.
Ales in *France*	43	40	0	27	A.
Babilon in *Chaldea*	35	00	3	10	A.
Bermudas	32	25	4	14	S.

Briſtol



	Latitude.		D. Merid.		
	°	′	h	′	
Briſtol	51	28	0	12	S.
Bedford in *England*	52	12	0	2	S.
Berwick	55	50	0	6	S.
Berlin in *Germany*	52	33	0	54	A.
Boſton in *New-England*	42	0	4	45	S.
Bethlehem in *Judea*	31	50	2	48	A.
Bononia in *Italy*	43	59	0	45	A.
Brunſwick in *Saxony*	52	16	0	41	A.
Cambridge	52	17	0	01	A.
Canterbury	51	25	0	04	A.
Carmarthen	52	02	0	17	S.
Calis in *France*	50	52	0	10	A.
Conſtantinople	43	0	2	7	A.
Cheſter	53	16	0	10	S.
Coventry	52	26	0	6	S.
Copenhagen in *Denmark*	55	43	0	50	A.
Cork in *Ireland*	51	45	0	30	S.
Cracovia	50	10	1	18	A.
Damaſcus	34	0	3	16	A.
Danzick in *Pruſſia*	54	23	1	14	A.
Dublin	53	11	0	27	S.
Dunkirk	51	2	0	12	A.
Dartmouth in *England*	50	30	0	15	S.
Derby	53	3	0	6	S.
Edinburgh	57	6	0	10	S.
Exeter	50	44	0	14	S.
Ely	52	20	0	1	A.
Frankford upon the *Maine*	50	2	0	35	A.
Frankford upon the *Oder*	52	20	0	58	A.
Fort St. George	13	08	5	24	A.
Fez in *Barbary*	33	10	0	24	S.
Geneva	45	54	0	27	A.
Glaſgow	55	50	0	17	S.
Gibraltar	36	30	0	25	S.
Grantham in *England*	52	58	0	2	S.
Glouceſter	51	58	0	9	S.
Hamburg	53	57	0	41	A.
Haphnia in *Denmark*	55	43	0	52	A.
Hanover	52	35	0	40	A.
Huntingdon	52	24	0	1	S.
Jeruſalem	32	10	3	2	A.
Knockfergus in *Ireland*	54	40	0	29	S.
Kendal in *England*	54	15	0	10	S.
Liverpool	53	22	0	10	S.
Lisbon in *Portugal*	38	45	0	34	S.
Leyden in *Holland*	52	07	0	21	A.
Leiceſter	52	41	0	4	S.
Lincoln	53	15	0	1	S.
London	51	32	0	0	

Madrid



	Latitude.		D.Merid.		
	o	′	h	′	
Madrid in *Spain*	40	45	0	9	S.
Manchester	53	22	0	10	S.
Marseilles	43	20	0	20	A.
Moscow	55	25	2	38	A.
Ments in *Germany*	50	30	0	45	A.
Naples	41	08	1	0	A.
Northampton	52	18	0	3	S.
Norwich	52	46	0	5	A.
Nottingham	53	0	0	4	S.
Ostend in *Flanders*	51	10	0	16	A.
Oxford	51	46	0	5	S.
Paris in *France*	48	51	0	10	A.
Pembroke	51	54	0	19	S.
Peterborough	52	38	0	1	S.
Portmahon	39	45	0	16	A.
Quinsai in *China*	40	0	8	32	A.
Ratisbone in *Bavaria*	49	9	0	51	A.
Revel in *Finland*	59	13	1	36	A.
Rome	41	50	0	52	A.
Richmond in *England*	54	26	0	07	S.
Rotterdam in *Holland*	51	55	0	15	A.
Rochester	51	30	0	2	A.
Sarum	51	12	0	7	S.
Stafford	52	54	0	9	S.
St. Christopher's	17	30	4	6	S.
Stockholm in *Sweden*	58	50	1	3	A.
Toledo in *Spain*	39	54	0	15	S.
Turin in *Italy*	44	50	0	29	A.
Valentia in *Spain*	39	45	0	1	S.
Venice	45	36	0	52	A.
Vienna	38	14	1	1	A.
Uramburg in *Denmark*	55	54½	0	52	A.
Warwick	52	25	0	6	S.
Warsaw in *Poland*	52	14	1	27	A.
Waterford in *Ireland*	52	7	0	31	S.
Wiggan in *England*	53	24	0	11	S.
Winchester	51	3	0	5	S.
Woodstock	51	53	0	6	S.
Worms in *Germany*	50	25	0	31	A.
Worcester	52	17	0	9	S.
York	54	0	0	4	S.
Yarmouth	50	41	0	7	S.

Some Perſons, perhaps, ſeeing that the foregoing Method for finding the Moon's Place in Eclipſes, *&c.* is more tedious than that uſed in *Leadbeater*'s Syſtem, *Wing*, *Street*, *Shakerley*, *&c.* would be willing, in order to calculate an Eclipſe of the Sun or Moon, to do it in the Manner ſuch Authors have done; for that End I have added the Three following Tables.

A TABLE of the Moon's Elliptic Equation.

Signs	0			1			2			3			4			5			
	Equat. ſub.			Equat. ſub.			Equat. ſub.			Equat. ſub.			Equat. ſub.			Equat. ſub.			
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	
0	0	0	0	2	23	32	4	12	20	4	57	40	4	23	30	2	34	42	30
1	0	5	0	2	27	54	4	15	4	4	57	52	4	21	2	2	30	6	29
2	0	9	58	2	32	14	4	17	36	4	57	54	4	18	24	2	25	22	28
3	0	14	56	2	36	32	4	20	10	4	57	56	4	15	44	2	20	38	27
4	0	19	54	2	40	44	4	22	36	4	57	50	4	12	56	2	15	50	26
5	0	24	54	2	45	0	4	24	58	4	57	42	4	10	8	2	11	2	25
6	0	29	52	2	49	8	4	27	14	4	57	24	4	7	10	2	6	8	24
7	0	34	48	2	53	14	4	29	30	4	57	2	4	4	10	2	1	14	23
8	0	39	46	2	57	16	4	31	36	4	56	34	4	1	2	1	56	14	22
9	0	44	44	3	1	18	4	33	42	4	56	2	3	57	54	1	51	14	21
10	0	49	36	3	5	18	4	35	38	4	55	22	3	54	36	1	46	12	20
11	0	54	32	3	9	10	4	37	34	4	54	40	3	51	18	1	41	8	19
12	0	59	22	3	13	2	4	39	22	4	53	48	3	47	52	1	36	0	18
13	1	4	14	3	16	52	4	41	8	4	52	54	3	44	24	1	30	52	17
14	1	9	10	3	20	36	4	42	46	4	51	52	3	40	50	1	25	40	16
15	1	14	0	3	24	18	4	44	20	4	50	48	3	37	14	1	20	28	15
16	1	18	48	3	27	56	4	55	50	4	49	36	3	33	28	1	15	14	14
17	1	23	38	3	31	32	4	47	16	4	48	20	3	29	44	1	9	58	13
18	1	28	24	3	35	2	4	48	34	4	46	56	3	25	52	1	4	40	12
19	1	33	10	3	38	32	4	49	50	4	45	30	3	21	58	0	59	22	11
20	1	37	54	3	41	54	4	50	58	4	43	56	3	17	58	0	54	2	10
21	1	42	32	3	45	16	4	52	4	4	42	18	3	13	56	0	48	42	9
22	1	47	16	3	48	31	4	53	0	4	40	34	3	9	48	0	43	20	8
23	1	51	54	3	51	46	4	53	56	4	38	46	3	5	38	0	37	56	7
24	1	56	32	3	54	54	4	54	42	4	36	48	3	1	24	0	32	32	6
25	2	1	8	3	58	0	4	55	26	4	34	56	2	57	6	0	27	8	5
26	2	5	40	4	1	0	4	56	2	4	32	44	2	52	44	0	21	44	4
27	2	10	12	4	3	58	4	56	36	4	30	36	2	48	18	0	16	18	3
28	2	14	40	4	6	50	4	57	2	4	28	18	2	43	50	0	10	52	2
29	2	19	8	4	9	38	4	57	34	4	25	58	2	39	17	0	5	26	1
30	2	23	32	4	12	20	4	57	40	4	23	30	2	34	42	0	0	0	0
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	
Signs	Equat. add 11			Equat. add 10			Equat. add 9			Equat. add 8			Equat. add 7			Equat. add 6			

A TABLE

A TABLE of the Hourly Motions, Semidiameters, and Horizontal Parallaxes of the Sun and Moon.

Mean Anom. of ☉. and ☽.	Hourly Motion of ☉.	Hourly Mot. of ☽ in the Syzyg.	Semidiameter of ☉.	Semidiameter of ☽.	Horizontal Parallax of ☽.	Mean Anom. of ☉. and ☽.
S : °	′ ″	′ ″	′ ″	′ ″	′ ″	*S* : °
0 0	2 23	29 33	15 49	14 54	54 59	12 0
0 6	2 23	29 34	15 49	14 55	54 59	11 24
0 12	2 23	29 37	15 49	14 56	55 2	11 18
0 18	2 23	29 43	15 49	14 57	55 7	11 12
0 24	2 23	29 51	15 50	14 58	55 13	11 6
1 0	2 24	30 3	15 51	15 1	55 20	11 0
1 6	2 24	30 17	15 52	15 3	55 29	10 24
1 12	2 24	30 31	15 53	15 6	55 40	10 18
1 18	2 24	30 47	15 54	15 9	55 52	10 12
1 24	2 25	31 6	15 55	15 12	56 5	10 6
2 0	2 25	31 27	15 57	15 17	56 20	10 0
2 6	2 26	31 50	15 58	15 22	56 37	9 24
2 12	2 26	32 14	16 0	15 25	56 53	9 18
2 18	2 27	32 39	16 1	15 30	57 12	9 12
2 24	2 27	33 4	16 3	15 36	57 30	9 6
3 0	2 28	33 29	16 5	15 41	57 50	9 0
5 6	2 28	33 55	16 6	15 46	58 10	8 24
3 12	2 29	34 22	16 8	15 52	58 31	8 18
3 18	2 29	34 50	16 10	15 58	58 52	8 12
3 24	2 30	35 17	16 12	16 3	59 11	8 6
4 0	2 30	35 42	16 13	16 9	59 30	8 0
4 6	2 31	36 5	16 15	16 14	59 50	7 24
4 12	2 31	36 26	16 16	16 19	60 9	7 18
4 18	2 32	36 46	16 17	16 24	60 26	7 12
4 24	2 32	37 5	16 18	16 28	60 40	7 6
5 0	2 32	37 23	16 19	16 31	60 53	7 0
5 6	2 33	37 38	16 20	16 34	61 4	6 24
5 12	2 33	37 51	16 21	16 36	61 12	6 18
5 18	2 33	38 0	16 21	16 38	61 19	6 12
5 24	2 33	38 5	16 22	16 39	61 23	6 6
6 0	2 33	38 6	16 22	16 40	61 24	6 0

A TABLE of the Moon's Latitude, with her Reduction from her Orb to the Ecliptic in Syzygias.

Arg. Lat.	Moon's Lat. Signs 0/6 N.A. S.A.			Reduc. Subſtr.		Moon's Lat. Signs 1/7 N.A. S.A.			Reduc. Subſtr.		Moon's Lat. Signs 2/8 N.A. S.A.			Reduc. Subſtr.		Arg. Lat.
	°	′	″	′	″	°	′	″	′	″	°	′	″	′	″	
0	0	0	0	0	0	2	29	51	5	40	4	19	44	5	41	30
1	0	5	14	0	14	2	34	22	5	47	4	22	18	5	34	29
2	0	10	27	0	27	2	38	50	5	53	4	24	49	5	26	28
3	0	15	41	0	41	2	43	15	5	59	4	27	14	5	18	27
4	0	20	54	0	55	2	47	37	6	4	4	29	44	5	10	26
5	0	26	7	1	8	2	51	56	6	9	4	31	50	5	2	25
6	0	31	19	1	22	2	56	11	6	14	4	34	0	5	53	24
7	0	36	31	1	35	3	0	24	6	18	4	36	6	4	43	23
8	0	41	32	1	48	3	4	33	6	21	4	38	6	4	34	22
9	0	46	52	2	1	3	8	39	6	24	4	40	2	4	23	21
10	0	52	2	2	14	3	12	42	6	27	4	41	52	4	13	20
11	0	57	10	2	27	3	16	41	6	29	4	43	37	4	2	19
12	1	2	18	2	40	3	20	36	6	31	4	45	17	3	51	18
13	1	7	24	2	52	3	24	28	6	32	4	46	52	3	40	17
14	1	12	29	3	4	3	28	16	6	33	4	48	21	3	29	16
15	1	17	33	3	16	3	32	0	6	33	4	49	45	3	17	15
16	1	22	36	3	28	3	35	40	6	33	4	51	4	3	5	14
17	1	27	37	3	40	3	39	17	6	32	4	52	18	2	53	13
18	1	32	36	3	51	3	42	49	6	31	4	53	26	2	40	12
19	1	37	34	4	2	3	46	17	6	29	4	54	28	2	28	11
20	1	42	29	4	12	3	49	42	6	27	4	55	26	2	15	10
21	1	47	23	4	23	3	53	2	6	25	4	56	18	2	2	9
22	1	52	16	4	33	3	56	17	6	22	4	57	4	1	49	8
23	1	57	6	4	43	3	59	29	6	18	4	57	45	1	35	7
24	2	1	54	4	52	4	2	36	6	14	4	58	21	1	22	6
25	2	6	39	5	1	4	5	39	6	10	4	58	51	1	8	5
26	2	11	23	5	9	4	8	37	6	5	4	59	16	0	55	4
27	2	16	4	5	18	4	11	30	6	0	4	59	35	0	41	3
28	2	20	42	5	26	4	14	19	5	54	4	59	49	0	27	2
29	2	25	18	5	33	4	17	4	5	47	4	59	57	0	14	1
30	29	51		5	40	4	19	44	5	41	5	0	0	0	0	0
Arg. Lat.	Sig. 11 N.D. S.D.			Add		Sig. 4/10 N.D. S.D.			Add		Signs 3/9 N.D. S.D.			Add		Arg. Lat.

A TABLE

A TABLE of the Angle which the true Motion of the Moon from the Sun makes with the Ecliptic in Conjunctions and Oppositions.

Arg. Lat. Sig. ☌ ☍	The true Hourly Motion of the Moon from the Sun.																		Arg. Lat. Sig. ☊ ☋
	27′		28′		29′		30′		31′		32′		33′		34′		35′		36′		
	°	′	°	′	°	′	°	′	°	′	°	′	°	′	°	′	°	′	°	′	
0	5	47	5	46	5	45	5	44	5	43	5	42	5	41	5	41	5	40	5	39	30
1	5	47	5	46	5	45	5	44	5	43	5	42	5	41	5	41	5	40	5	39	29
2	5	47	5	45	5	44	5	43	5	43	5	42	5	41	5	40	5	40	5	39	28
3	5	46	5	45	5	44	5	43	5	42	5	42	5	41	5	40	5	40	5	39	27
4	5	46	5	45	5	44	5	43	5	42	5	41	5	41	5	40	5	39	5	39	26
5	5	45	5	44	5	43	5	42	5	42	5	41	5	40	5	39	5	39	5	38	25
6	5	45	5	44	5	43	5	42	5	41	5	40	5	39	5	39	5	38	5	38	24
7	5	44	5	43	5	42	5	41	5	40	5	40	5	39	5	38	5	38	5	37	23
8	5	43	5	42	5	41	5	40	5	40	5	39	5	38	5	37	5	37	5	36	22
9	5	42	5	41	5	40	5	39	5	39	5	38	5	37	5	37	5	36	5	35	21
10	5	41	5	40	5	39	5	38	5	38	5	37	5	36	5	36	5	35	5	34	20
11	5	40	5	39	5	38	5	37	5	37	5	36	5	35	5	35	5	34	5	33	19
12	5	39	5	38	5	37	5	36	5	35	5	35	5	34	5	33	5	33	5	32	18
13	5	38	5	37	5	36	5	35	5	34	5	33	5	33	5	32	5	31	5	31	17
14	5	36	5	35	5	34	5	34	5	33	5	32	5	31	5	31	5	30	5	29	16
15	5	35	5	34	5	33	5	32	5	31	5	31	5	30	5	29	5	29	5	28	15

To find the Angle that the Moon's Way makes with the Ecliptic, you must find the Argument of Latitude either in the first or last Column; and, guiding your Eye from thence, until you come under the Hourly Motion of ☽ *à* ☉; then in the common Angle of Meeting you will have the Angle required.

LET the Argument of Latitude be 0. Signs, or 6ˢ : 10°, and the Hourly Motion of ☽ *à* ☉ = 32′, then under 0ˢ, or 6ˢ in the first Column, and against 10 Degree in the same, looking along the Line from thence towards the Right until you come under 32′, there you will find 5° : 37′ the Angle of the Moon's Way.

WHEN

When you would calculate an Eclipſe, and have a Mind to uſe theſe laſt Tables, you may find the Sun's true Place, and mean Anomaly, as you ſee in the Part before going: But to find the Moon's Place in her Orb (which is what you compare with the true Place of the Sun in Conjunctions and Oppoſitions), you muſt,

1. Find her mean Longitude, Apogee, and Node, as before; then from the mean Longitude ſubſtract her Apogee, what remains is her mean Anomaly.

2. Take this to the Table of the Moon's Elliptic Equation, and thence take out the Equation, which being apply'd to her mean Longitude, according to the Title, will give the Moon's Place in her Orb.

To make this plain, we will reſume the Eclipſe of the Moon, that is to be in *December*, 1749.

The mean Time was found to be *December* ——	12ᵈ : 18ʰ : 6′ : 27″
The Interval, according to the Moon's Place, found as above directed, is (ſubſtract.) —— ——	10 : 2 : 47
Remains the true equal Time of the Oppoſition — ——	12 : 8 : 3 : 40
At which Time the Sun's true Place is —— ——	9ˢ : 2 : 15 : 02
And the mean Anomaly —— —— ——	5 : 23 : 58 : 36

For Moon's Place at the ſame Time, work as under.

	Long. ☽.				Apog. ☽.				Node ☽.			
	ſ : ° : ′ : ″				ſ : ° : ′ : ″				ſ : ° : ′ : ″			
1749 —	6	23	53	48	4	11	32	35	9	28	58	44
Dec. 12 —	7	29	1	57	1	8	32	50	0	18	19	21
Hours 8 —		4	23	32			2	14			1	4
Min. 3 —			1	39			0	01			0	0
Sec. 40 —			0	22								
									0	18	20	25
Mean Long. ☽ —— —	2	27	21	18	5	20	7	40	9	10	38	19
Apogee ſubſtract — —	5	20	7	40								
Remains mean Anom. ☽ —	9	7	13	38								
Elliptic Equat. add — —		4	53	44								
The Sum is ☽ in Orb —	3	2	15	02								
North Node ſubſtract — —	9	10	38	19								
Remains the Arg. of Latitude —	5	21	36	43								
True Lat. ☽. N. D. ——			43	42								
Reduction add ——			1	53								
☽ in the Ecliptic ——	3	2	16	55								

The Hourly Motion of ☽ by her mean Anomaly ——	33′ : 01″
Hourly Motion of ☉ by his mean Anomaly ——	2 : 33

Their

Their Difference is the Hourly Motion of ☽ *à* ☉			30′ : 28″	
Time of Reduction (contrary to its firſt Title) ſubſtract			3 : 43	
From the true equal Time of the Oppoſition, *December*		12ᵈ : 8ʰ :	3 : 40	
Remains the true equal Time of the Ecliptic Oppoſition		12 : 7 :	59 : 57	}
But add it (according to its firſt Title) and the Sum is the equal Time of the Middle of the Eclipſe		8 : 7 :	23	}
Equation of Time add			0 : 1	
The Sum is the apparent Time of the Ecliptic Oppoſition		7 : 59 :	58	}
The apparent Time of the Middle of the Eclipſe		8 : 7 :	28	}

The Horizontal Parallax of ☽	57′ : 26″	
Horizontal Parallax of ☉ (always 10″) add	0 : 10	
Their Sum is	57 : 36	
Semidiameter of ☉ ſubſtract	16 : 22	
Remains the Semidiameter of the Earth's Shadow	41 : 14	
Semidiameter of the Moon add	15 : 12	
Sum of the Semidiameters of the Moon and Earth's Shadow	56 : 26	
Latitude of ☽ ſubſtract	43 : 42	
Remains the Parts deficient	12 : 44	
Digits Eclipſed (found by the Rule given before)	5° : 1 : 24	

By the Rule formerly given for the ſame Eclipſe, you will find the Time of half Duration to be	1ʰ : 7′ : 41″	}
Which take from the Middle, or Time of greateſt Obſcuration	8 : 7 : 24	
Remains the apparent Time of the Beginning	6 : 59 : 43	}
And being added to the ſame, gives the apparent Time of the End	9 : 15 : 05	}
The whole Duration	2 : 15 : 22	

THE Manner of delineating this Eclipſe, was ſhewn before; and here obſerve, that whether the Moon's Latitude be North or South deſcending, the Angle of the Moon's Way with the Ecliptic muſt be ſet from either the North or South End of the Ecliptic towards the Left-hand (as in this Caſe). But when the Latitude is aſcending, then it muſt be ſet off from the Axis towards the Right-hand.

THOUGH this Method is more expeditious than the former, yet I would recommend that; becauſe it being generally nearer the Truth, is a ſufficient Reward for the Labour.

CHAP.

CHAP. XVIII.

Of Navigation.

NAVIGATION (that very uſeful Part of the Mathematicks), is an Art that has been highly valued by the Ancients; it being the Beauty and Bulwark of *England*, the Wall and Wealth of *Britain*, and the Bridge that joins it to the Univerſe.

THIS noble Art conſiſts of Three Parts, *viz.*

1. PLAIN
2. MERCATOR's
3. GREAT CIRCLE

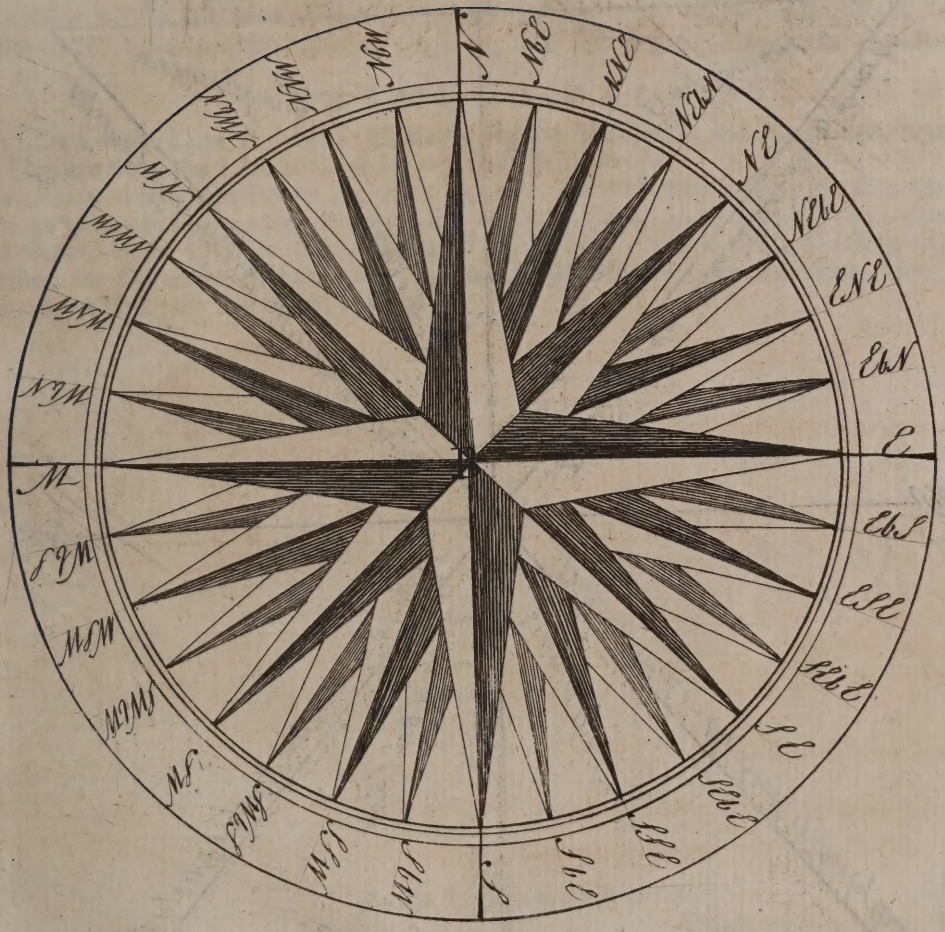
} SAILING.

PLAIN Sailing is founded on the Suppoſition of the Earth, being a Plain or Flat, which, though notoriouſly falſe, yet Places laid down accordingly; and ſo a long Voyage, broke into many ſhort ones, the Voyage may be tolerably performed by it near the ſame Meridian.

PLAIN Sailing is divided into Three Parts.

1. In a Right-lin'd Triangle, relating to a ſingle Courſe.
2. In a Right-angled Triangle, relating to ſeveral Courſes, called, a Traverſe.
3. In an Oblique Triangle.

Note, THAT where-ever the Mariner is at Sea (if upon the North Side of the Equator) he muſt ſuppoſe his Ship to be in the Center of his Compaſs, by which he is to direct his Courſe, and his Face towards the North; then his Right hand will be to the Eaſt, his Left to the Weſt, and his Back to the South: So, when he is protracting or drawing his Work, he muſt ſuppoſe the Top of the Book, or Paper, to be the North Part, then the reſt will be as before.



PLAIN SAILING contains the Six following Cases. Here you must observe the next Figure, in which you have the Names of the Sides and Angles, as they are given or required. Supposing B the Place from whence you sail.

Case

Case I.

GIVEN the Course and Distance, to find the Difference of Latitude and Departure.

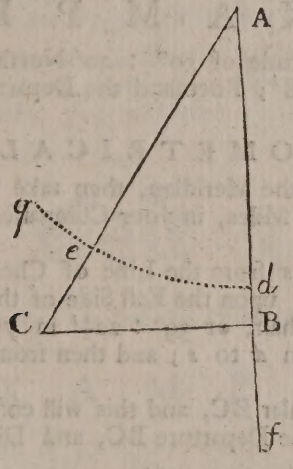
EXAMPLE.

ADMIT a Ship sails from A in the Latitude of 52° : 20′ North, away S. W. by S. 52 Miles to C; I demand the Latitude she is in, and her Departure from the Meridian.

GEOMETRICALLY.

1. DRAW the Line A *f* out at Pleasure, for the Meridian; and, upon the Point A, with 60 Degrees from the Line of Chords, describe the Arch *d q*.

2. FROM *d* lay three Points or 33° : 45′ to *e*, through which draw the Line A C; then take 52° in your Compasses, from a Line of equal Parts, and lay that Extent from A to C, and from C draw C B, at Right Angles to A *f*. The Triangle being thus formed, you may measure the Difference of Latitude A B, and Departure B C, by the same Line of equal Parts by which you laid down A C.



By the LOGARITHMS.

	10.
As Radius — — — —	1.716003
Is to the Distance run A C = 52 — —	9.744739
So is the Sine of the Rumb B A C = 33° : 45′ — —	
	1.460742
To the Departure B C = 28.79 Miles — —	

AGAIN,	10.
As Radius — — — —	1.716003
Is to the Distance run 52 — —	9.919846
So is the Sine of 56° : 15′ (the Complement of 33° : 45′) —	
	1.635849
To the Difference of Latitude A B = 43.24 — —	

From the Latitude given	—	—	—	52° : 20′
Subſtract the Difference of Latitude		—	—	0 : 43.24
Remains the Latitude that the Ship is in		—	—	51 : 36.76

By GUNTER's Scale.

Extend the Compaſſes from 90° on the Line of Sines to 33° : 45′, and the ſame Extent will reach from 52 on the Line of Numbers to 28.79 upon the ſame Line; and the Extent from 90° to 56° : 15′ on the Sines will reach from 52 on the Numbers to 43.24 on the ſame Line.

By the Sliding Rule.

Move the Sliding Piece, ſo that 90° upon the Line of Sines may ſtand againſt 52 on the Line of Numbers; then will 33° : 45′ on the Sines ſtand againſt 28.79 on the Numbers; and, without altering the Sliding Piece, againſt 56° : 15′ on the Sines you have 43.24 upon the Line of Numbers.

Caſe 2.

The Courſe and Difference of Latitude being given, to find the Departure and Diſtance run.

EXAMPLE.

Admit a Ship from the Latitude of 50° : 30′ North, ſails S. E. by S. ½ Eaſterly till ſhe be in the Latitude of 49° : 18′; I demand the Departure and Diſtance ſailed.

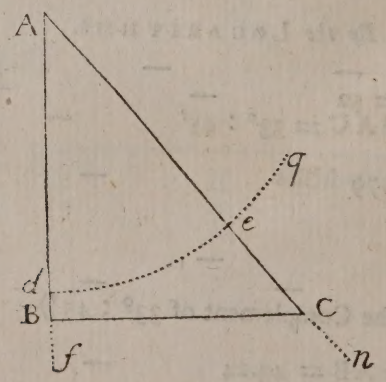
GEOMETRICALLY.

1. Draw A*f* at Pleaſure for the Meridian, then take the Difference of the given Latitudes 1° : 12′ = 72 Minutes, or Miles, in your Compaſſes from a Line of equal Parts, and lay that down from A to B.

2. Take 60° in the Compaſſes from the Line of Chords, and ſetting one Foot in A, with the other draw the Arch *d q*, upon the Eaſt Side of the Meridian.

3. Take three Points and a half, or 39° : 22½′ in your Compaſſes from the Line of Chords, and ſet that Extent from *d* to *e*; and then from A through *e* draw the Line A*n* of any convenient Length.

4 Upon B erect the Perpendicular BC, and this will compleat the Triangle A B C; which being done, you may meaſure the Departure BC, and Diſtance ſailed AC.



By

By the LOGARITHMS.

As the S. *c.* of the Rhumb = BAC = 39° : 22½′ *co. ar.*	0.111667
Is to the Difference of Latitude AB = 72 Miles	1.857332
So is the Sine of the Rhumb 39° : 22½′	9.802358
To the Departure BC = 59.07 Miles	1.771357

AGAIN,

As S. *c.* of the Rhumb. 39° : 22½′ *co. ar.*	0.111667
Is to the Difference of Latitude AB = 72	1.857332
So is Radius	10.
To 93.12 = Miles the Diſtance run = AC	1.968999

By GUNTER's SCALE.

1. EXTEND the Compaſſes from 50° : 37½′ on the Line of Sines to 39° : 22½′, and the ſame Extent will reach from 72 on the Line of Numbers to 59.07 the Departure.
2. EXTEND the Compaſſes from 50° : 37½′ upon the Sines to 90°, the ſame Extent will reach from 72 on the Numbers to 93.12 for the Diſtance run.

By the SLIDING RULE.

SET 50° : 37½′, the Complement of the Rhumb, to 72 on the Numbers, then againſt 39° : 22½′, on the Sines, you will have 59.07 on the Numbers, the Departure required; and againſt Radius, or 90°, on the Sines, you will have 93.12 on the Numbers for the Diſtance ſailed.

Caſe 3.

GIVEN the Courſe and Departure, to find the Diſtance and Difference of Latitude.

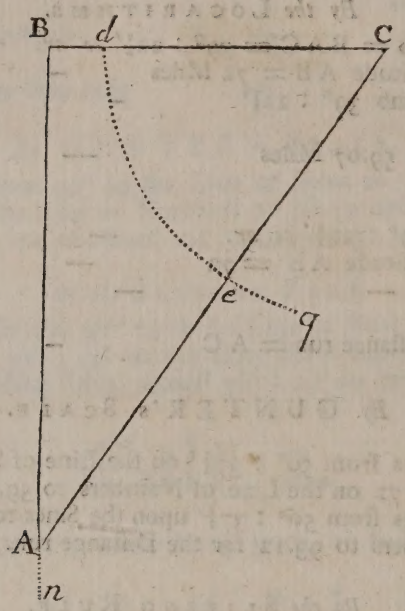
EXAMPLE.

A SHIP ſailed from the Latitude of 47° : 20′ North, upon the N. E. by N. till her Departure from the Meridian was 52.6: What was the Diſtance run, and Difference of Latitude.

GEOMETRICALLY.

1. DRAW the Meridian B*n* of any convenient Length, and from B draw BC at Right Angles to the Meridian B*n*; upon which ſet 52.6 from B to C, for the Departure.
2. TAKE 60° in your Compaſſes from the Line of Chords, and, with one Foot in C, draw the Arch *d q*; upon this Arch from *d* ſet 56° : 15′ (the Complement of the Rhumb) to *e*.
3. FROM C through *e*, draw a Line till it cuts the Meridian in the Point A; and that will compleat the Triangle ABC.

By



By the LOGARITHMS.

As the Sine of the Courſe BAC = 33° : 45′ *co. ar.*	—	0.255261
Is to the Departure BC = 52.6	—	1.720986
So is the S. *c.* of the Courſe	—	9.919846
To the Difference of Latitude AB = 78.73	—	1.896093

THE Latitude that the Ship ſailed from at A	—	47° : 20′
Difference of Latitude add	—	1 : 18.73
The Latitude the Ship was in when at C	—	48 : 38.73

FOR the Diſtance run AC, ſay,

As the Sine of the Courſe 33° : 45′ *co. ar.*	—	0.255261
Is to the Departure 52.6	—	1.720986
So is Radius 90°	—	10.
To the Diſtance run AC = 94.68	—	1.976247

By GUNTER's SCALE.

1. EXTEND the Compaſſes from 33° : 45′ on the Sines to 56° : 15′, the ſame Extent will reach from 52.6 on the Line of Numbers to the Difference of Latitude 78.8.

2. SET one Foot in 33° : 45′ on the Sines, and extend the other to 90°; the ſame will reach from 52.6 to 94.8, the Diſtance run.

By the SLIDING RULE.

Move the Sliding Piece, ſo that 33° : 45′ on the Line of Sines may be againſt 52.6 on the Numbers; then againſt 56° : 15′ you will find 78.73, the Difference of Latitude; and, againſt 90°, you have 94.68 for the Diſtance run.

Caſe

Caſe 4.

THE Diſtance run, and Difference of Latitude given, to find the Courſe and Departure.

EXAMPLE.

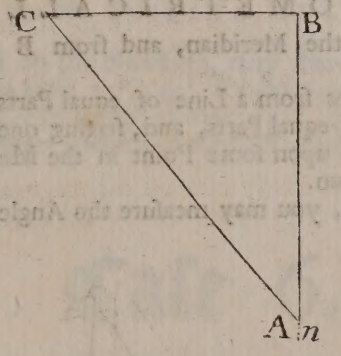
ADMIT a Ship ſail from the Latitude of 46° : 36′ North, on ſome Point of the Compaſs between the North and Weſt 97 Miles, then finds herſelf in the Latitude of 47° : 51′ North, the Courſe and Departure is required

From	—	47° : 51′	
Take	—	46 : 36	
Remains	—	1 : 15	= 75 Miles, the Difference of Latitude.

GEOMETRICALLY.

1. DRAW B*n* to repreſent the Meridian, then from B ſet 75 to A, and draw B C at Right Angles to B A.

2. TAKE 97 in your Compaſſes, and putting one Foot in the Point A, turn the other about till it falls upon the Point C, in the Line B C; then draw the Line A C, and that will compleat the Triangle. You may find the Angle at A by Help of the Line of Chords, and the Departure B C by the Line of equal Parts.



By the LOGARITHMS.

1. As the Diſtance run A C = 97 Miles — —	1.986772
Is to Radius — — —	10.
So is the Difference of Latitude A B = 75 — —	1.875061
To the S. *c.* of the Courſe = 39° : 12′ — —	9.889289
2. For the Departure, ſay,	
As Radius — — —	10.
Is to the Diſtance run 97 — —	1.986772
So is the Sine of the Courſe 33° : 12′ — —	9.800737
To the Departure B C = 61.31 — —	1.787509

By

By GUNTER's SCALE.

EXTEND your Compaſſes from 97 on the Numbers, to 75; the ſame Extent will reach from 90 on the Sines to 50° : 48′, whoſe Complement is 39° : 12′ for the Courſe, that is N. W. by N. and 5° : 27′ more Weſterly, or near N. W.

AGAIN, Extend your Compaſſes from 90 on the Sines, to 39° : 12′; the ſame Extent will reach from 97 on the Numbers to 61.31, the Departure.

By the SLIDING RULE.

SET 97 on the Numbers, to 90 on the Sines; then againſt 75 on the Numbers, you will find 50° : 48′ on the Sines; and. as the Slider is now ſet, againſt 39° : 12′ on the Sines, you have 61.31 on the Line of Numbers.

Caſe 5.

GIVEN the Diſtance and Departure, to find the Courſe and Difference of Latitude.

EXAMPLE.

SUPPOSE a Ship ſails 86 Miles between the South and Weſt, until her Departure is 49 Miles; I demand her Courſe, and Difference of Latitude?

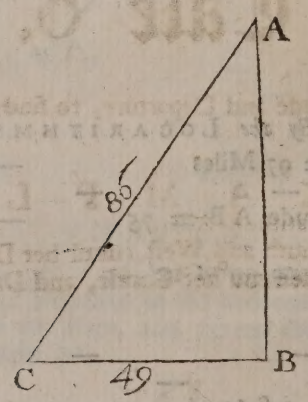
GEOMETRICALLY.

1. DRAW the Line AB for the Meridian, and from B draw the Line BC at Right Angles thereto.

2. TAKE 49 in your Compaſſes from a Line of equal Parts, and ſet that from B to C.

3. TAKE 86 from the Line of equal Parts, and, ſetting one Foot of the Compaſſes in C, turn the other about, until it falls upon ſome Point in the Meridian, as at A; then draw the Line A C for the Diſtance run.

THE Triangle thus compleated, you may meaſure the Angle at A, and the Side AB.



By

By the LOGARITHMS.

1. As the Diſtance run AC = 86 — — —	1.934498
Is to Radius — — —	10.
So is the Departure BC = 49 — — —	1.690196
To the Sine of the Angle of the Courſe 34° : 44′ —	9.755698
That is S. W. by S. and 0° : 59′ more Weſterly.	
2. As Radius — — — —	10.
Is to the Diſtance run 86 — — —	1.934498
So is the S. *c.* of the Courſe 55° : 16′ — —	9.914773
To the Difference of Latitude AB = 70.78 — —	1.849271

By GUNTER's SCALE.

1. EXTEND the Compaſſes from 86 on the Line of Numbers, to 49; and the ſame Extent will reach from 90 on the Sines to 34° : 44′, the Angle of the Courſe.

2. EXTEND the Compaſſes from 90 on the Sines, to 55° : 16′; and the ſame Extent will reach from 86 on the Numbers, to 70.78, for the Difference of Latitude.

By the SLIDING RULE.

MOVE the Slider, ſo that 86 on the Numbers, may ſtand againſt 90 on the Sines; then againſt 49 on the Numbers you have 34° : 44′, for the Angle of the Courſe; and, without altering the Rule, againſt the Complement thereof, *viz.* 55° : 16′ on the Sines, you will find 70.78 on the Line of Numbers; which is the Difference of Latitude required, as above.

Caſe 6.

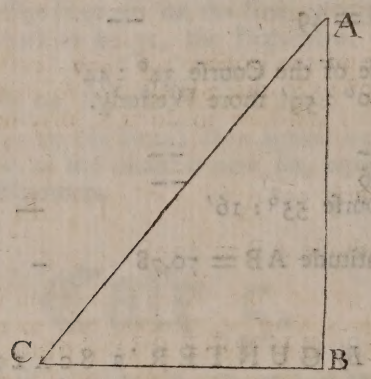
GIVEN the Difference of Latitude and Departure, to find the Courſe and Diſtance.

EXAMPLE.

SUPPOSE a Ship ſails between South and Weſt, until her Departure is 58 Miles, and Difference of Latitude 74 Miles; I demand her Courſe, and Diſtance run.

GEOMETRICALLY.

MAKE AB = 74, and from B draw the Line BC, upon which ſet 58 to C, then draw the Line AC for the Diſtance run; the Length of which you may find by the Line of equal Parts, and the Angle of the Courſe BAC by a Line of Chords.



By the LOGARITHMS.

1. As the Difference of Latitude AB = 74 ——— 1.869232
Is to Radius ——— 10.
So is the Departure BC = 58 ——— 1.763428

To the Tangent of the Angle of the Courſe 38° : 5′ ——— 9.894196

That is S. W. by S. and 4° : 20′ more Weſterly.

2. As the Sine of the Courſe (the Angle at A = 38° : 5′) ——— 9.790149
Is to the Departure BC = 58 ——— 1.763428
So is Radius ——— 10.

To the Diſtance run AC = 94.03 Miles ——— 1.973279

By GUNTER's SCALE.

1. EXTEND the Compaſſes from 74, upon the Line of Numbers, to 58; and the ſame Extent will reach from 45, on the Tangents to 38° : 5′, the Angle of the Courſe.
2. EXTEND the Compaſſes from 38° : 5′, on the Sines, to 90; and the ſame Extent will reach from the Departure 58, on the Numbers, to 94.03, the Diſtance run.

By the SLIDING RULE.

1. SET 74 upon the Numbers, to 45 on the Tangents; then againſt 58 on the Numbers, you will find 38° : 5′ on the Tangents.
2. SET 38° : 5′ on the Sines, to 58 on the Numbers; then againſt 90° on the Sines, you have 94.03 on the Numbers, as before.

To work a Traverse.

A TRAVERSE is the Alteration of a Ship's Course, or Way, by Reason of contrary Winds, or other Interruptions, by Rocks, Sands, *&c.* so that to work a Traverse, is to bring several Courses into one; and how to do that, may be known by the following

EXAMPLE.

SUPPOSE a Ship bound to a certain Port, which bears from where she is N. N. E. 5° : 11′ E. but forced by contrary Winds, Currents, Rocks, *&c.* to make her Way good by sailing first N. E. by N. $\frac{3}{4}$ E. 43 Miles, S. E. by E. $\frac{1}{2}$ E. 20 Miles, then N. E. by N. 44 Miles; N. W. by W. $\frac{3}{4}$ W. 68 Miles, N. by E. 45 Miles, N. E. by E. $\frac{1}{2}$ E. 84 Miles, S. S. W. 60 Miles, N. W. by N. 85 Miles, S. E. by E. $\frac{1}{2}$ E. 33 Miles, N. E. by N. 36. E. N. E $\frac{1}{4}$ E. 45 Miles, and then arrived at the Port desired. The Departure or Difference of Meridians, the Difference of Latitude, and Distance in a direct Course between the Ports, is required.

THE most regular Way to solve this, will be to make a Table, and divide it into Ten Columns: In the First, you may set down the Number of Courses; in the Second, the Courses; in the Third, the Distance run upon each Course; in the Fourth, the Number of Points from the Meridian that you sail upon; in the Fifth, the same in Degrees and Minutes; so that in protracting, you may use either the Line of Rhumbs upon a Scale, or a Line of Chords: In the Sixth, you may set down the Points for laying down the Courses, according to Mr. *Atkinson*, in his *Epitomy of Navigation*. In the other Four, you may set down the Northing, Southing, Easting and Westing, as you shall find them, either Geometrically, Instrumentally, or by Calculation; or, more speedily, by a Traverse Table, or Table of Difference of Latitude and Departure.

The Table.

Number of Courſes.	Courſes.	Diſtance ſailed.	Points from the Merid.	Points for laying down the ſame in Degrees.	Points for laying down without a Meridian.	Diff. Latitude. North.	Diff. Latitude. South.	Departure. East.	Departure. Weſt.
1	N. E. by N. $\frac{3}{4}$ E.	43	$3\frac{3}{4}$	42° 11′ 15″	$3\frac{3}{4}$	31.8	0.	28.9	0.
2	S. E. by E. $\frac{1}{2}$ E.	20	$5\frac{1}{2}$	61 52 30	$9\frac{1}{4}$	0.	9.4	17.6	0.
3	N. E. by N.	44	3	33 45	$8\frac{1}{2}$	36.6	0.	24.4	0.
4	N. W. by W. $\frac{3}{4}$ W.	68	$5\frac{3}{4}$	64 41 15	$7\frac{1}{4}$	29.1	0.	0.	61.5
5	N. by E.	45	1	11 15	$9\frac{1}{4}$	44.1	0.	8.8	0.
6	N. E. by E. $\frac{1}{2}$ E.	84	$5\frac{1}{2}$	61 52 30	$11\frac{1}{2}$	39.6	0.	74.1	0.
7	S. S. W.	60	2	22 30	$3\frac{1}{2}$	0.	55.4	0.	23.
8	N. W. by N.	85	3	33 45	5	70.7	0.	0.	47.2
9	S. E. by E. $\frac{1}{2}$ E.	33	$5\frac{1}{2}$	61 52 30	$2\frac{1}{2}$	0.	15.6	29.1	0.
10	N. E. by N.	36	3	33 45	$8\frac{1}{2}$	29.9	0.	20.	0.
11	E. N. E. $\frac{1}{4}$ E.	45	$6\frac{1}{4}$	70 18 45	$12\frac{3}{4}$	15.2	0.	42.4	0.
	Sum'd up — — —					297.0	80.4	245.3	131.7
	Subſtract — — —					80.4		131.7	
	Diff. Latitude — —					216.6	Depar.	113.6	

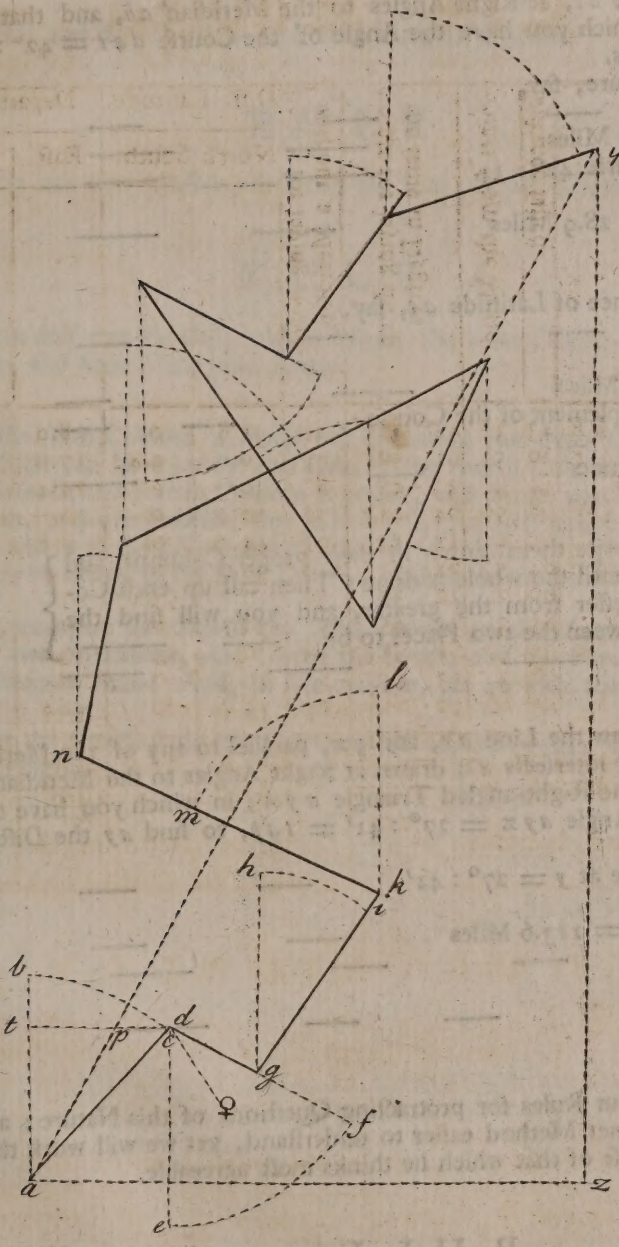
GEOMETRICALLY.

1. DRAW the Meridian *a b*, and taking 60 Degrees in your Compaſſes from a Line of Chords, deſcribe the Arch *b g* upon the Center *a*, and the Right-hand of the Meridian, becauſe the Point ſailed upon is between the North and Eaſt; then from *b* lay off $3\frac{3}{4}$ Points, or 42° : 11′ : 15″ to *c*, and through *c* draw the Line *a d*, upon which from a Line of equal Parts ſet 43 Miles to *d*.

2. DRAW the Meridian *d e* parallel to *a b*; and, upon *d*, as a Center, draw the Arch *e f*, upon the Right hand of the Meridian downward, becauſe the Point is between the South and Eaſt; and upon that ſet $5\frac{1}{2}$ Points, or 61° : 52′ : 30″ from *e* to *f*, and draw the Line *d g*, upon which ſet 20 Miles from *d* to *g*.

3. DRAW the Meridian *g h* parallel to *a b*, or *d e*; and, upon *g*, as a Center, deſcribe the Arch *h i*, and from *h* ſet off three Points, or 33° : 45′ to *i*; and from *g*, through *i*, draw *g k*, upon which ſet 44 Miles to *k*, and draw the Meridian *k l*; and, upon *k*, as a Center, deſcribe the Arch *l m*; then from *l* ſet off $5\frac{3}{4}$ Points, or 64° : 41½′ to *m*, and draw the Line *k m n*, upon which from *k* ſet 68 Miles; and ſo on, until you have compleated the Figure.

By


y
l
n
m
h
k
i
b
t
p
d
c
g
♀
f
a
z
e

By the LOGARITHMS.

FROM *d* draw the Line *dt*, at Right Angles to the Meridian *ab*, and that will complete the Triangle *adt*; in which you have the Angle of the Courſe *dat* = 42° : 11′, and Diſtance run *ad* = 43 Miles.

1. To find the Departure, ſay,

As Radius	10.
Is to the Diſtance run 43 Miles	1.633468
So is the Sine of the Courſe 42° : 11′	9.827049
To the Departure *dt* = 28.9 Miles	1.460517

2. To find the Difference of Latitude *at*, ſay,

As Radius	10.
Is to the Diſtance run 43 Miles	1.633468
So is the Sine of the Complement of the Courſe	9.869818
To the Difference of Latitude	1.503386

HAVING found theſe, write them down in their proper Columns, and ſo proceed with the reſt, until the whole is done: Then caſt up each Column, and ſubſtract the leſſer from the greater, and you will find the Difference of Latitude between the two Places to be 216.6 Miles.

And Departure Eaſtward 113.6 Miles.

Now, if from *a* you draw the Line *ay*, and *yz*, parallel to any of the Meridians already drawn, as *ab*, *gh*, until it interſects *az* drawn at Right Angles to the Meridian *ab*, in the Point *z*, theſe will form the Right-angled Triangle *ayz*; in which you have the Departure *az* = 113.6 Miles, and Angle *ayz* = 27° : 41′ = *tap*, to find *ay* the Diſtance between the two Ports *a* and *y*.

As the Sine of the Angle at *y* = 27° : 41′	9.667065
Is to the Departure *az* = 113.6 Miles	2.055378
So is Radius	10.
To *ay* = 244.5 Miles	2.388313

MR. *Atkinſon* gives us Four Rules for protracting Queſtions of this Nature; and though a Learner may think the former Method eaſier to underſtand, yet we will work the ſame over again, that he may make uſe of that which he thinks moſt agreeable.

RULE 1.

Two Meridians and two Parallels, the leſſer Courſe ſubſtracted from the greater gives the Angle between both Courſes.

RULE

RULE 2.

Two Meridians and one Parallel, add both Courſes together.

RULE 3.

ONE Meridian and two Parallels, ſubſtract the Sum of both Courſes from 16 Points.

RULE 4.

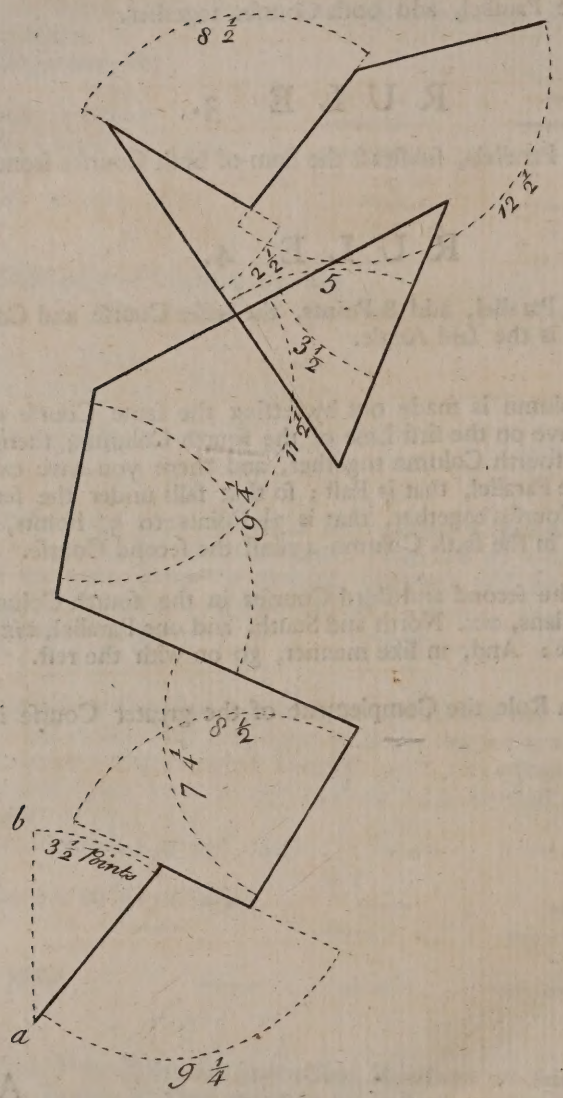
ONE Meridian and one Parallel, add 8 Points, the leſſer Courſe and Complement of the greater together, the Sum is the ſaid Angle.

Note, That the ſixth Column is made out by ſetting the ſame Courſe down on the firſt Line of that which you have on the firſt Line of the fourth Column; then compare the firſt and ſecond Courſes in the fourth Column together, and there you have two Meridians, *viz.* North and South, and one Parallel, that is Eaſt; ſo that falls under the ſecond Rule, which directs you to add both Courſes together, that is $3\frac{3}{4}$ Points to $5\frac{1}{2}$ Points, and the Sum is $9\frac{1}{4}$ Points, which ſet down in the ſixth Column againſt the ſecond Courſe.

THAT done, compare the ſecond and third Courſes in the fourth Column together, and there you have two Meridians, *viz.* North and South, and one Parallel, *viz.* Eaſt. This alſo falls under the ſecond Rule: And, in like manner, go on with the reſt.

Note, That at the fourth Rule the Complement of the greater Courſe is ſo much as it wants of eight Points.

A TABLE


8 1/2
12 1/2
2 1/2
5
3 1/2
11 1/2
9 1/4
8 1/2
7 1/4
b
3 2 Points
a
9 1/4

A TABLE of the Angles which every Rhumb makes with the Meridian.

North.	South.		° : ′ : ″	South.	North.	
		1	2 48 45			
		2	5 37 30			
		3	8 26 15			
N. by E.	S. by E.	1	11 15 00	S. by W.	N. by W.	1
		1	14 3 45			
		2	16 52 30			
		3	19 41 15			
N. N. E.	S. S. E.	2	22 30 0	S. S. W.	N. N. W.	2
		1	25 18 45			
		2	28 7 30			
		3	30 56 15			
N. E. by N.	S. E. by S.	3	33 45 0	S. W. by S.	N.W. byN.	3
		1	36 33 45			
		2	39 22 30			
		3	42 11 15			
N. E.	S. E.	4	45 0 0	S. W.	N. W.	4
		1	47 48 45			
		2	50 37 30			
		3	53 26 15			
N. E. by E.	S. E. by E.	5	56 15 0	S.W. by W.	N.W. byW.	5
		1	59 3 45			
		2	61 52 30			
		3	64 41 15			
E. N. E.	E. S. E.	6	67 30 0	W. S. W.	W. N. W.	6
		1	70 18 45			
		2	73 7 30			
		3	75 56 15			
E. by N.	E. by S.	7	78 45 0	W. by S.	W. by N.	7
		2	81 33 45			
		1	84 22 30			
		3	87 11 15			
East.	East.	8	90 0 0	West.	West.	8

CHAP. XIX.

Oblique Sailing.

Or, The Application of Oblique-angled plain Triangles to plain SAILING.

THIS Part of Navigation admits of great Variety of Problems, but we ſhall confine ourſelves chiefly to thoſe that are propoſed by Mr. *Norwood*, in his *Doctrine of Triangles*; becauſe the Geometrical Conſtruction (which that Author has omitted) will make the Anſwers more plain to a Learner.

Problem I.

SAILING away W. S. W. I ſee a Point of Land, which I ſet, and find to bear from me W. by N. and having ſailed ſix Leagues, or 18 Miles farther, I find it bears from me N. W. by W. I would know how far it was from me at each Obſervation.

GEOMETRICALLY.

1. LET N. S. be the Meridian of the Place, from whence the firſt Obſervation was made.

2. MAKE Choice of any Point therein as at A, and with 60 Deg. in your Compaſſes from the Line of Chords, ſet one Foot in A, and with the other deſcribe the Semicircle *g f e d*, upon which lay ſeven Points, or 78° : 45′ from *g* to *f*, and from A through *f*, draw the Line A C out at Diſcretion.

3. TAKE ſix Points, or 67° : 30′ in your Compaſſes, and lay that from *d* to *e*, then draw the Line A *e* L, upon which ſet 18 Miles from A to B; ſo is B the Place of the ſecond Obſervation.

4. THROUGH B, and parallel to N. S. draw another Meridian Line Q R; and upon B, as a Center, deſcribe the Arch *h i*, and from *i* lay off five Points, or 56° : 15′ to *h*, through which Point draw a Line from B, until it cuts the Line A C in C, ſo the Point of Land is repreſented at C, and the Triangle formed; which being done, you may meaſure the Length of the Lines A C and B C by the ſame Line of equal Parts by which A B was laid down.

By the LOGARITHMS.

1. To *d e* = 6 Points add *f g* = 7 Points, the Sum is 13 Points; take this from 16 Points, there will remain 3 Points = *f e* = *f* A *e* = 33° : 45′.

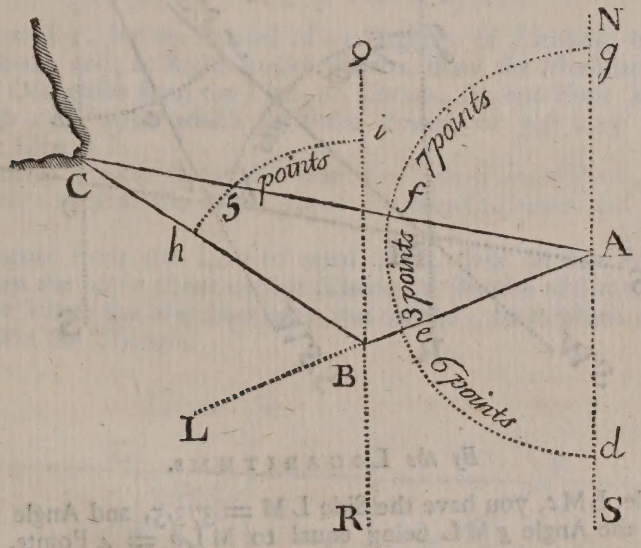
2. THE

2. THE Angle QBA is equal to $eAd = 6$ Points, to which add $CBQ = 5$ Points, the Sum is 11 Points; take this from 16, there will remain the external Angle $CBL = 5$ Points, or $56° : 15'$, from which take the Angle $fAe = 33° : 45'$, and there will remain $BCA = 22° : 30'$.

Now, ſay,	
As the Sine of the Angle $BCA = 22° : 30'$ *co. ar.*	0.4171603
Is to the Side $AB = 18$ Miles	1.2552725
So is the Sine of the Angle $fAe = 33° : 45'$	9.7447390
To BC the Diſtance of the ſaid Point of Land at C, from the Place of the Ship at B, at the ſecond Obſervation 26.13 Miles	1.4171718

AGAIN, to find the Diſtance of the ſaid Point of Land from the Ship at the firſt Obſervation, ſay,

As the Sine of the Angle $BCA = 22° : 30'$ *co. ar.*	0.4171603
Is to the Side $AB = 18$ Miles	1.2552725
So is the Sine of $CBL = 56° : 15'$	9.9198464
To $AC = 49.23$ Miles	1,6922792



Prob. 2.

ADMIT the Courſe from the *Lizard* to St. *Mary*'s be S. W. the Diſtance $373\frac{1}{2} = 373.5$ Leagues, a certain Ship bound from the *Lizard* to St. *Mary*'s, ſteers away S. S. W. and afterwards W. by S. and ſo ſometimes upon one of theſe Points, ſometimes upon the other, till ſhe arrives at St. *Mary*'s. Now, I demand how many Leagues ſhe has ſailed upon one of theſe Courſes, and how many upon the other?

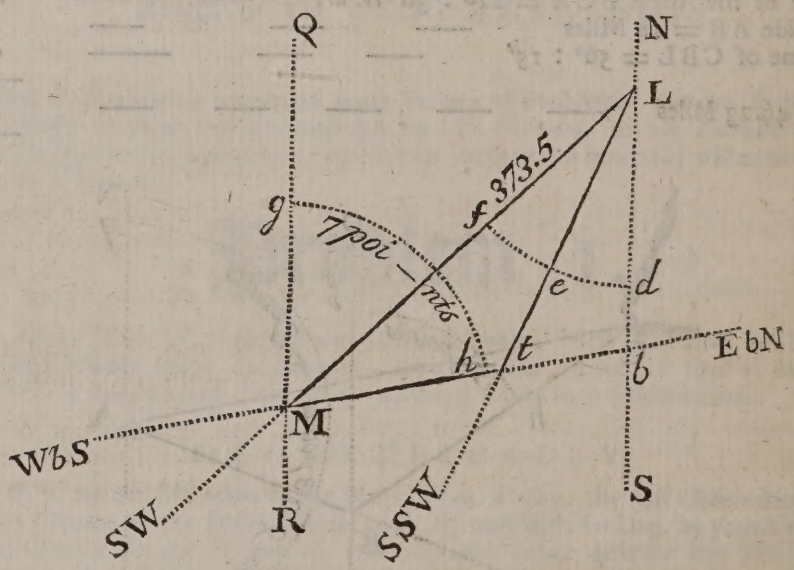
GEOMETRICALLY.

1. DRAW the Meridian N. S. and make Choice of any convenient Point therein, as at L, to repreſent the *Lizard*; then, with 60 Degrees in your Compaſſes, from a Line of Chords, and, with one Foot in L, deſcribe the Arch *df*, upon which ſet off four Points, or 45 Degrees from *d* to *f*, and through *f* draw LM out at Pleaſure.

2. TAKE 373·5 in your Compaſſes, from the Line of equal Parts, and ſet that from L to M, then M ſhall repreſent St. *Mary*'s.

3. THROUGH M, and parallel to N. S. draw the Meridian Q R, then becauſe E. by N. makes the ſame Angle with the Meridian, that W. by S. does, make M a Center, upon which deſcribe the Arch *g h*, and from *g* ſet ſeven Points, or 78° : 45′ to *h*, then draw the Line M*t* out each Way, and if you pleaſe mark it at the Ends with W. by S. and E. by N.

4. TAKE two Points, or 22° : 30′, and ſet it from *d* to *e*, upon the Arch *df*, and from L through *e* draw the Line L*t*, which will cut the Line M*b* at *t*, and compleat the Triangle L M*t*. This being done, you may meaſure the Lines L*t* and M*t*, by a Line of equal Parts.



By the LOGARITHMS.

1. IN the Triangle L M*t*, you have the Side L M = 373·5, and Angle M L*t* = 2 Points, or 22° : 30′; now the Angle *g* M L being equal to M L*b* = 4 Points, if from the Angle *g* M*t* = 7 Points you take *g* M L = 4 Points, there will remain the Angle L M*t* = 3 Points, or 33° : 45′, to which add M L*t* = 22° : 30′, the Sum will be the external Angle L*t b* = 56° : 15′.

2. Now, ſay,

As the Sine of 56° : 15′ *co. ar.*	0.0801536
Is to L M = 373·5	2.5722906
So is the Sine of the Angle M L*t* = 22° : 30′	9.5828397
To M*t* = 172 Leagues	2.2352839

3. As

3. As the Sine of 56° : 15′ *co. ar.*	—	—	—		0.0801536
Is to LM = 373.5	—	—	—		2.5722906
So is the Sine of the Angle LM*t* = 33° : 45′	—	—	—		9.7447390
To L*t* = 249.5	—	—	—		2.3971832

Hence we find that the Ship ſailed S. S. W. — — — 249.5 } Leagues.
and W. by S. — — — 172. }

Prob. 3.

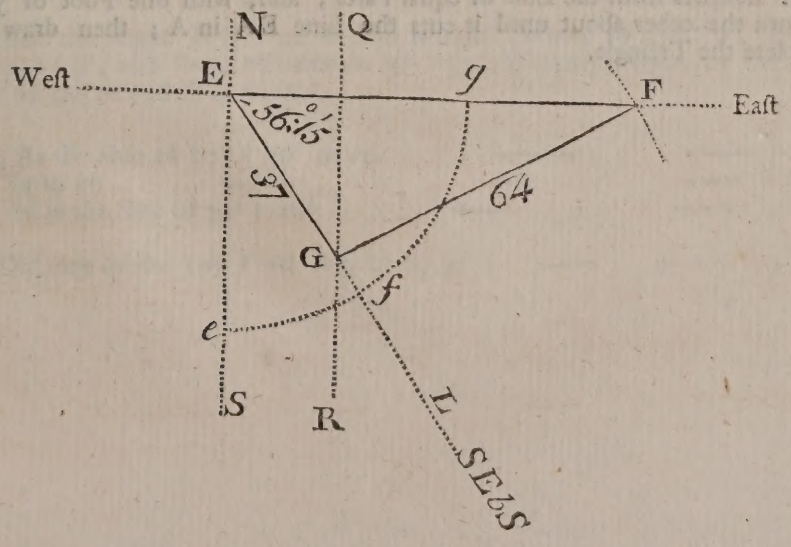
A Merchant-man, being in the Latitude of 43 Degrees, falls into the Hands of Pyrates; who, amongſt other Things, take away his Sea-Compaſs: But when he is gotten clear, he ſails away as directly as he can, and after two Days meets with a Man of War, who alſo had been the Day before in the Latitude of 43°, and had ſailed thence S. E. by S. 37 Leagues; he deſires to find theſe Pyrates, the Merchant-man tells him he left them lying to and fro where they took him, and he had ſailed ſince at leaſt 64 Leagues, between the South and Weſt. What Courſe ſhall the Man of War ſhape to find thoſe Pyrates?

GEOMETRICALLY.

1. DRAW the Line EF, for the Parallel of 43 Degrees of Latitude, in which the Merchant-man was robbed; and, at Right-Angles thereto, draw the Meridian N. S. then taking 60 Degrees in your Compaſſes from the Line of Chords, ſet one Foot in E, and with the other draw the Arch *e f g*, upon which ſet three Points, or 33° : 45′ from *e* to *f*, and through *f* draw the Line E L.

2. TAKE 37 Leagues in your Compaſſes from the Line of equal Parts, and ſet that from E to G; then G ſhall repreſent the Place where the Merchant-man and Man of War did meet.

3. TAKE 64 Leagues from the Line of equal Parts, then ſet one Foot of your Compaſſes in G, and turn the other about until it falls on the Parallel of Latitude at the Point F, and that is the Place where the Merchant-man was robbed; from which draw a Line to G, and that will complete the Triangle.



By

By the LOGARITHMS.

SAY, As the Diſtance run by the Merchant-man G F = 64 Leagues *co. ar.* —	8.1938200
Is to the Sine of 56° : 15′ —— ——	9.9198464
So is the Diſtance run by the Man of War 37 —— ——	1.5682017
To the Sine of the Angle E F G = 28° : 44′ —— ——	9.6818681

The Complement of which is the Angle Q G F = 61° : 16′, that is N. E. by E. 5° : 1′ E. the Courſe that the Man of War muſt ſhape to find the Pyrates; conſequently the Merchant-man had ſailed S. W. by S. 5° : 1′ W.

Prob. 4.

THERE are two Parts lying N. E. and S. W. off one another, a Ship ſails from the Weſtermoſt of the Ports E. S. E. 47 Leagues. Another departing from the Eaſtermoſt Port ſailed 66 Leagues, and then meets with the former. What Courſe hath this ſecond Ship kept, and how far are theſe Ports aſunder?

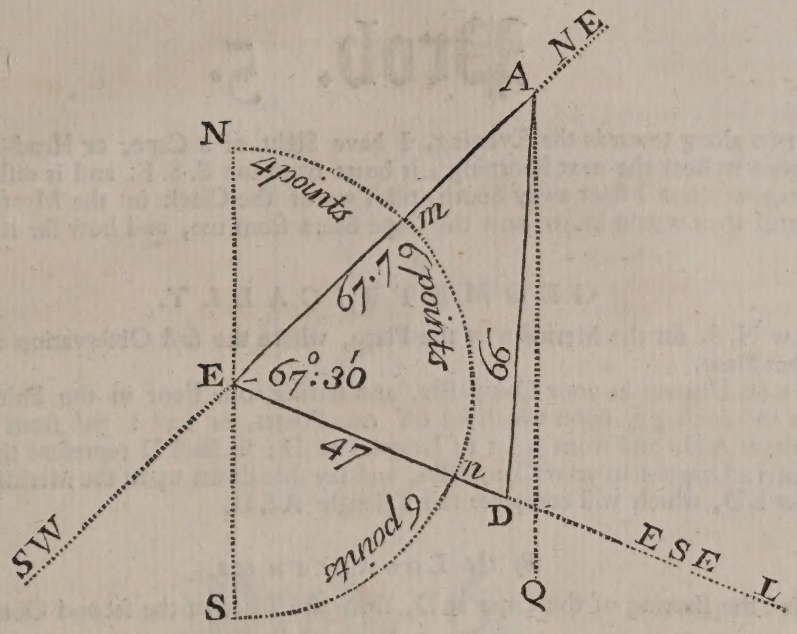
GEOMETRICALLY.

1. DRAW N. S. for the Meridian of the Weſtermoſt Port; and, upon any convenient Point thereof, as at E, draw the Semicircle N *m n* S, then taking four Points, or 45° : 0′, in your Compaſſes, and lay that Extent from N to *m*, and from E through *m* draw the Line E A, out at Pleaſure.

2. TAKE 6 Points, or 67° : 30′ from the Line of Chords, and ſet that off from S to *n*, and from E through *n*, draw E D, upon which ſet 47 Leagues from E to D.

3. TAKE 66 Leagues from the Line of equal Parts; and, with one Foot of your Compaſſes in D, turn the other about until it cuts the Line E A in A; then draw D A, and that will complete the Triangle.

By



By the LOGARITHMS.

As the Side AD = 66 *co. ar.*	8.180456
Is to the Sine of AED = 67° : 30′	9.965615
So is ED = 47	1.672098
To the Sine of the Angle EAD = 41° : 8′	9.818169

To this Angle add the Angle AED = 67 : 30
The Sum is the external Angle ADL = 108° : 38′, the Supplement of which is the Angle ADE = 71° : 22′.

Note, That the Angle NEA = EAQ is four Points, or 45° : 0′, from which take EAD = 41° : 8′, and there will remain 3° : 52′ Westerly, equal to the Angle DAQ, the Course of the second Ship from A.

AGAIN, As the Sine of 67° : 30′ *co. ar.*	0.034385
Is to 66	1.819544
So is the Sine of 71° : 22′	9.976617
To the Distance of the two Ports AE = 67,7	1.830546

Prob. 5.

COASTING along towards the Evening, I have Sight of a Cape, or Head-Land, beyond which I desire to steer the next Morning; it bears from me S. S. E. and is distant by Estimation 11 Leagues; but I steer away South, till Two of the Clock in the Morning, about 12 Leagues, and then would know how the Cape bears from me, and how far it is off?

GEOMETRICALLY.

1. DRAW N. S. for the Meridian of the Place, where the first Observation was made, and let A be that Place.
2. TAKE 60 Degrees in your Compasses, and setting one Foot in the Point A, with the other draw the Arch *g e*, upon which set off two Points, or 22° : 30′ from *g* to *c*, then through *c* draw A D, and from A set 11 Leagues to D; so shall D represent the Cape.
3. TAKE 12 Leagues in your Compasses, and lay that down upon the Meridian from A to E, and draw E D, which will complete the Triangle A E D.

By the LOGARITHMS.

1. To find the Bearing of the Cape at D, from the Place of the second Observation at E, say,

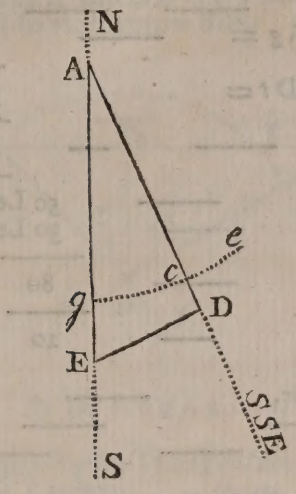
As A E + A D = 23 Leagues *co. ar.*	8.638272
Is to A E — A D = 1 League	0.
So is the Tangent of $\frac{180° - 22° : 30'}{2} = 78° : 45'$	10.701338
To the Tangent of 12° : 20′ (subtract)	9.339610

From —— 78 : 45
Remains —— 66 : 25 = A E D; that is, N. E. by E. 10° : 10′ Easterly.

2. To find the Distance of the Cape E D, say,

As the Sine of the Angle A E D = 66° : 25′ *co. ar.*	0.037877
Is to A D = 11 Leagues	1.041393
So is the Sine of the Angle E A D = 22° : 30′	9.582840
To the Distance in the Morning E D = 4.6	0.662110

That is four Leagues and above an half.



Prob. 6.

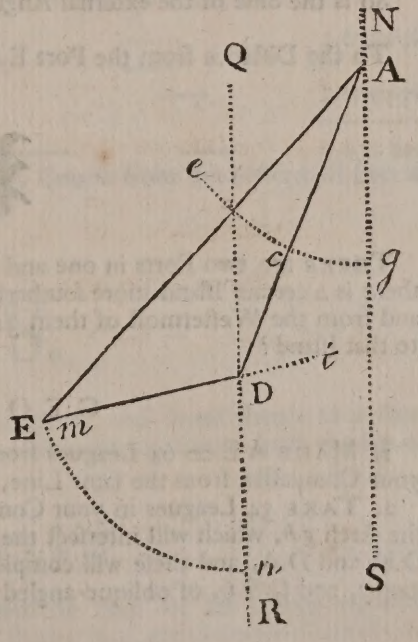
ADMIT I ſail away from a certain Port S. S. W. 50 Leagues, and thence again W. *b*. S. 30 Leagues; upon what Point have I made my Way good, and how far am I come from that Port?

GEOMETRICALLY.

1. DRAW N. S. for the Meridian of the Port from whence the Ship ſet Sail, and let the Point A be the Port, upon which as a Centre deſcribe the Arch *g c e*, and thereon ſet off two Points, or 22° : 30′ from *g* to *c*; and thro' *c* draw A D, upon which ſet 50 Leagues from A to D.

2. THRO' D, and parallel to A S, draw another Meridian Q R, and upon D as a Center deſcribe the Arch *n m*; upon this Arch ſet 7 Points, or 78° : 45′ from *n* to *m*; and thro' *m* draw D E, and thereon ſet 30 Leagues from D to E.

3. From E draw the Line A E, and that will complete the Triangle A D E.



By the LOGARITHMS.

1. THE Angle $m\,D\,n = Q\,D\,t$ is ——— ——— 78° : 45′
From which take $Q\,D\,A = t\,A\,g$ = ——— 22 : 30

Remains the external Angle $A\,D\,t$ = 56 : 15 = EAD + AED

Half of this Remainder is ——— — 28 : 8

AD is ——— ——— 50 Leagues
ED ——— ——— 30 Leagues

Sum ——— ——— 80

Difference —— —— 20

THEN, As 80 Leagues *co. ar.* ——— —— 8.096910
Is to 20 Leagues ——— ——— —— 1.301030
So is the Tangent of 28° : 8′ ——— ——— —— 9.728109

To the Tangent of 7 : 37 (ſubſtract ——— — 9.126049
From ——— 28 : 8

Remains the Angle EAD = 20 : 31
To this add $D\,A\,g$ = 22 : 30

The Sum is $E\,A\,S$ = 43 : 01, that is S. W. *b.* S. 9° : 16′ W. for the Courſe from A to E.
For the Diſtance, ſay
As the Sine of $E\,A\,D$ = 20° : 31′ *co. ar.* ——— ——— 0.455337
Is to ED = 30 Leagues ——— ——— —— 1.477121
So is the Sine of the external Angle $A\,D\,t$ = 56° : 15 ——— —— 9.919846

To the Diſtance from the Port EA = 71.17 Leagues ——— 1.852304

Prob. 7.

THERE are two Ports in one and the ſame Parallel of Latitude, diſtant 64 Leagues; and there is a certain Iſland more ſoutherly, diſtant from the Eaſtermoſt of the Ports 47 Leagues, and from the Weſtermoſt of them 34 Leagues; I demand the Courſe from the eaſtermoſt Port to that Iſland?

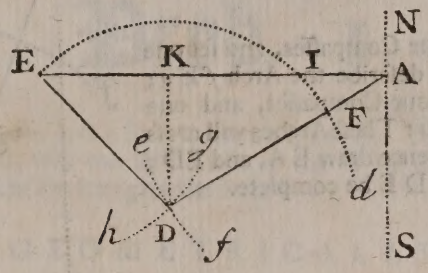
GEOMETRICALLY.

1. MAKE AE = 64 Leagues from a Line of equal Parts; and then taking 47 Leagues in your Compaſſes from the ſame Line, ſet one Foot in A, and with the other draw the Arch *ef*.

2. TAKE 34 Leagues in your Compaſſes, and ſet one Foot in E, and with the other deſcribe the Arch *g h*, which will interſect the former Arch in D, from which Point draw the Lines D E, and D A, and theſe will complete the Triangle E A D (See *Prob.* 22. of Practical Geometry, and *Caſe* 6. of oblique-angled Triangles).

3. SET

3. Set one Foot of your Compaſſes in D, and the other in E, and with that Extent deſcribe the Arch E *d*, which will cut E A in I, and D A in F; then A F will be the Difference of the Sides D A and D E, and A I will be the alternate Baſe.



By the LOGARITHMS.

By *Axiom* the 3d of oblique-angled plain Triangles find the alternate Baſe, by ſaying,

As the true Baſe A E = 64 Leagues,	*co. ar.*	8.193820
Is to the Sum of the Sides D A + D E = 81 Leagues		1.908485
So is the Difference of the Sides D A — D E = 13		1.113943
To the alternate Baſe A I =	16.45 Leagues	1.216248
To which add A E =	64.	
The Sum is	80.45	
Half of this Sum is A K =	40.22	

Now in the right-angled Triangle A K D, you have the Hypothenuſe A D, and A K, to find the Angle A D K.

Say, As A D = 47	1.672098
Is to Radius	10.
So is A K = 40.22	1.604442
To the Sine of A D K = 58° : 51′ = D A S	9.932344

THAT is S. W. *b.* W. 2° : 36′ weſterly; and this is the Courſe from the eaſtermoſt Port at A, to the Iſland at D.

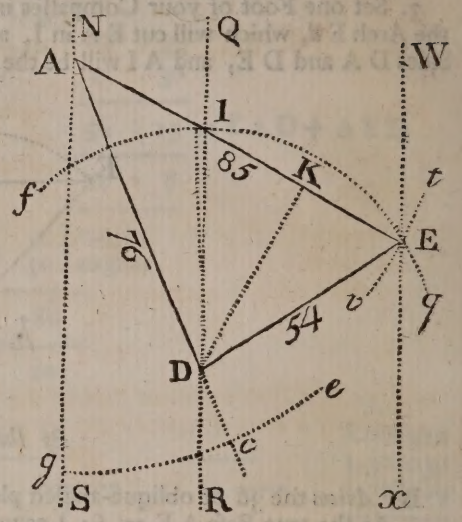
Prob. 8.

A SHIP ſails from one Port to a ſecond S. S. E. 76 Leagues, and from thence to a third 54 Leagues, and from the third to the firſt 85 Leagues; I demand the Courſe from the ſecond Port to the third, and from the third to the firſt?

GEOMETRICALLY.

1. DRAW N. S. and let A be the firſt Port from whence the Ship ſet ſail; then upon the Point A deſcribe the Arch *g e*, upon which ſet 22° : 30′ from *g* to *c*, and draw the Line A *c* —; upon this ſet 76 Leagues from A to D.

2. TAKE 54 Leagues in your Compaſſes, and ſet one Foot in D; with the other deſcribe the Arch *f* E *q*; and with 85 Leagues in your Compaſſes, and one Foot in A draw the Arch *t v*: Theſe Arches will croſs one another in E, from whence draw E A, and E D; and ſo will the Triangle A D E be complete.



By the LOGARITHMS.

ACCORDING to *Axiom* the 3d, and *Caſe* the 6th of oblique-angled plain Triangles, ſay,

1. As the true Baſe A E = 85 *l.* *co. ar.*			8.070581
Is to the Sum of the Sides A D + E D = 130 *l.*			2.113943
So is the Difference of the Sides A D − E D = 22 *l.*			1.342422
To the alternate Baſe	A I = 33.65		1.526946
Add the true Baſe A E =	85.		
The Sum is	118.65	Leagues.	
The half of which is A K =	59.32		
2. As A D = 76 Leagues			1.880813
Is to Radius			10.
So is A K = 59.32			1.773201
To the S. *c.* of A D K =	38° : 42′		9.892388
To this add *g* A *c* =	22 : 30		
The Sum is *g* A E = A E W =	61 : 12, *i. e.* N. W. *b.* W. 4° : 5′ W.		

And this is the Courſe from the third Port at E to the firſt at A.

3. As D E = 54 Leagues, *co. ar.*		8.267606
Is to the Sine of D A E = 38° : 42		9.796049
So is A D = 76 Leagues		1.880813
To the Sine of the Angle A E D = 61° : 38′		9.944468
To which add the Angle A E W = 61 : 12		
The Sum is W E D = 122 : 50		

The Supplement of which is D E *x* = Q D E = 57 : 10. *i. e.* N. E *b.* E. 0° : 55′ E.
The Courſe from the ſecond Port to the third.

The

The next four Problems are uſed in turning to Windward.

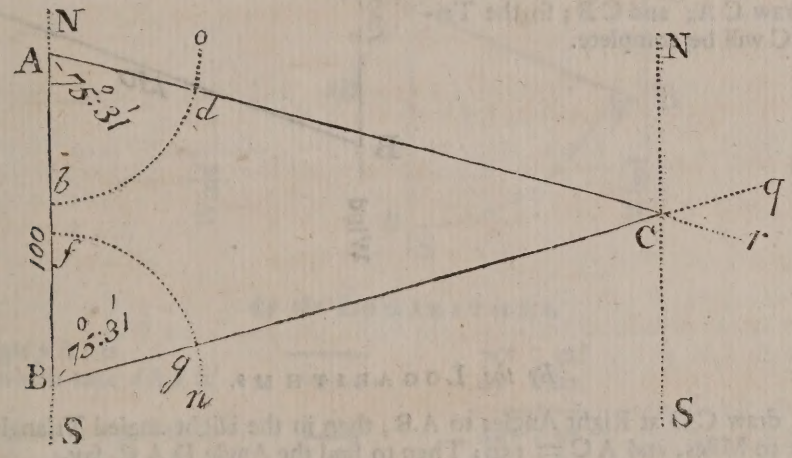
Prob. I.

SUPPOSE the Wind at South, and a Ship is bound from the Port at A, to another at B 100 Miles directly to Windward, who can lie no nearer the Wind than 75° : 31′ from it; I demand the Diſtance upon each Tack to gain the Port?

GEOMETRICALLY.

1. DRAW A B to repreſent Part of the Meridian; and from any Point therein, as A, ſet off 100 Miles by a Line of equal Parts, from A to B.

2. Upon the Points A and B, with 60 Degrees in your Compaſſes from a Line of Chords, deſcribe the Arches 60, and *f n*; upon theſe ſet 75° : 31′ from *b*, and *f* to *d*, and *g*; thro' which Points from A and B draw A *r*, and B *q*, which will interſect in C, and complete the Triangle A B C.



By the LOGARITHMS.

THE Angle B A C is		75° : 31′	
and A B C		75 : 31	
Their Sum is		151 : 02	
Whoſe Supplement is		28 : 58 = A C B	
Now ſay, As the Sine of 28° : 58		*co. ar.*	0.314885
Is to A B = 100 Miles			2.
So is the Sine of the Angle at A = 75° : 31′			9.985974
To the Side B C = 200 Miles *fere*			2.300859 = A C.

HENCE her Diſtance upon each Tack is 200 Miles, or 400 Miles in both.

Prob.

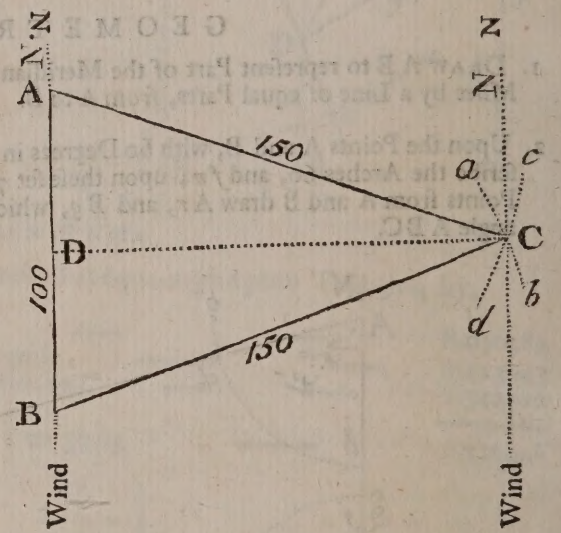
Prob. 2.

LET A, be a Port from whence a Ship ſet ſail to another at B, bearing directly ſouth from A, and diſtant 100 Miles, the Wind at ſouth (ſhe making one or many Boards) doth run 300 Miles before ſhe can reach B; I demand how near the Wind ſhe makes her Way good?

GEOMETRICALLY.

1. Draw the Line A B for the Meridian, and from A ſet off 100 Miles to B.

2. TAKE 150 Miles in your Compaſſes, and ſetting one Foot in A and B, ſeverally with the other deſcribe the Arches *c d*, and *a b*, theſe will interſect one another in C, from whence draw C A, and C B; ſo the Triangle A B C will be complete.



By the LOGARITHMS.

FROM C, draw C D at Right Angles to A B; then in the Right-angled Triangle A D C you have A D = 50 Miles, and A C = 150: Then to find the Angle D A C, ſay,

As the Side A C = 150	*co. ar.*	7.823909
Is to Radius		10.
So is A D = 50		1.698970
To the Sine of the Angle A C D = 19° : 28′		9.522879

Whoſe Complement is 70° : 32′ = D A C; which ſhews that ſhe makes her Way good within 70° : 32′ of the Wind.

Prob. 3.

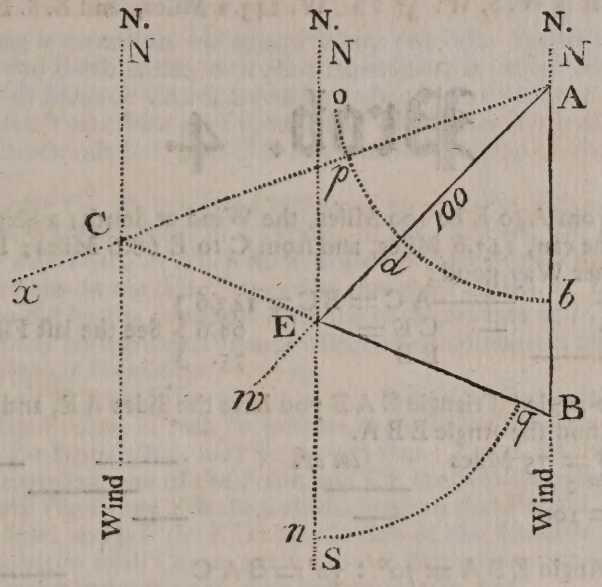
LET the Diſtance from A to E be 100 Miles South-Weſt, the Wind at South; and let the Ship make her Way good within 70° : 32′ of the Wind; I demand the Diſtance A C, and E C, that is, the Ship's Way by dead Reckoning upon one Tack, and upon the other?

GEO-

GEOMETRICALLY.

1. DRAW A B for Part of the Meridian; and upon A, as a Centre describe the Arch *b o*; upon which set 45° : 0′ from *b* to *d*, and 70° : 32′ from *b* to *p*; and thro' *d*, and *p*, draw the Lines A *w*, and A *x*, and upon A *w* set 100 Miles from A to E; then thro' E draw a Meridian N S parallel to A B.

2. With 60° in your Compasses, and one Foot in E, draw the Arch *n q*, upon which from *n* set 70° : 32′ to *q*: Then thro' *q* and E draw the B *q* E C, which will cut the Line A *x* in C, and complete the Oblique-angled Triangle A E C.



By the LOGARITHMS.

THE Angle *p* A *b* is	70°	32′
From which take *d* A *b* =	45	00
There remains *p* A *d* =	25	32
Twice the Angle B A C =	141	04
The Supplement of which is the Angle A C B	38	56
To this add *p* A *d* =	25	32
The Sum is the external Angle A E B =	64	28

Now to find A C, say,

As the Sine of A C B = 38° : 56′	*co. ar.*	0.201753
Is to A E = 100		2.
So is the Sine of A E B = 64° : 28′		9.955367
To A C = 143.6		2.157120

For

For C E, ſay,

As the Sine of A C B = 38° : 56′	*co. ar.*	0.201753
Is to A E = 100 Miles		2.
So is the Sine of *p* A *d* = 25° : 32		9.634514
To C E = 68.6		1.836267

Thus it appears, that to ſail from A to E, which is South-weſt 100 Miles, with the Wind at South, and making her Way good within 70° : 32′ of the Wind, ſhe muſt ſail with her Larboard-tack-aboard near 143.6 Miles, and with her Star-board-tack-aboard near 68.6 Miles, in all 212.2 Miles, that is W. S. W. 3° : 2′ W. 143.6 Miles, and E. S. E. 3° : 2′ Eaſt 68.6 Miles.

Prob. 4.

Let the Diſtance from A to E be 100 Miles, the Wind at South; a Ship ſails from A to C, ſo near the Wind as ſhe can, 143.6 Miles, and from C to E 68.6 Miles; I demand how near the Wind ſhe makes her Way good?

From	A C =	143.6	} See the laſt Figure.
Take	C E =	68.6	
Remains	E B	75.	

Now in the Oblique-angled Triangle E A B you have the Sides A E, and E B, with the Angle E A B = 45° : 0′ to find the Angle E B A.

Say, As the Side E B = 75 Miles	*co. ar.*	8.124939
Is to the Sine of 45° : 0′		9.849485
So is the Side A E = 100		2.
To the Sine of the Angle E B A = 70° : 32′ = B A C		9.974424

Hence we conclude, that ſhe makes her Way good within 70° : 32′ of the Wind; ſo that the Wind being at South, ſhe makes her Way good upon the one Tack W. S. W. 3° 2′ W. and upon the other E. S. E. 3° : 2′ E.

Here perhaps the Reader might expect, that I ſhould have gone on to Problems of parallel Sailing, Sailing in a Current, *&c.* But becauſe many very good Authors, as Mr. *Hodgſon*, *Harris*, *Atkinſon*, *Ward*, *&c.* have handled what is of moſt Uſe in theſe already with Applauſe, and what I have done, or ſhall do in Navigation, being only to excite the inquiſitive Reader to look over ſuch Authors, I ſhall only touch upon *Mercator*'s, and Great Circle Sailing.

CHAP.

CHAP. XX.

Mercator's Sailing.

THAT Plain Sailing is erroneous will appear to any one who knows that it is built upon a Suppoſition that the Earth is flat, or a plain Superficies, as before ſaid, and not ſpherical, as it really is; and as all Maps or Charts drawn according to that Opinion muſt have been falſe. *Ptolomey*, many hundred Years ſince gave ſome Hints of a better Method of projecting them; however this was not much minded, until *Mercator* publiſh'd a Map of that Kind, whence it is called *Mercator*'s Chart.

BUT Mr. *Wright* taking this Matter into Conſideration, was the firſt who compoſed Tables of meridional Parts, as you have them in a Treatiſe of his, intitled, *The Correction of Errors in Navigation*: Printed, *Anno* 1610, in which he hath ſhewed how to enlarge the Degrees of the Meridian towards the Poles in the ſame Proportion that the Parallels of Latitude do decreaſe towards them; this he did by the continual Addition of the Secants of $1'$, $2'$, $3'$, $4'$, $5'$, $6'$, *&c.* But the learned Dr. *Halley* hath given us various Methods of computing the ſame to the utmoſt Exactneſs. See *Philoſophical Tranſactions*, N° 219.

That the Meaſure of a Degree of Longitude in any Parallel of Latitude is leſs than a Degree of Longitude in the Equinoctial, in ſuch Proportion as the Semidiameter of the Parallel is to the Semidiameter of the Equinoctial, may be proved thus: Let A B C be one quarter of any Meridian, and let A repreſent one of the Poles, and C E the Semidiameter of the Equinoctial, B C the Latitude of any Place, and F B the Semidiameter of that Parallel.

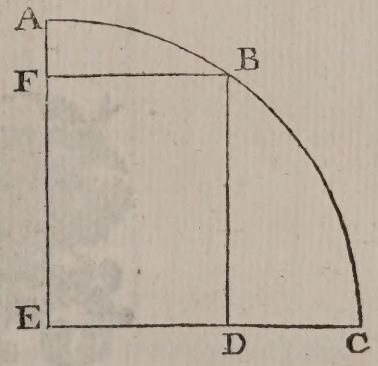
THEN is B D the Sine, and B F (= E D) the Co-ſine of that Latitude.

Now ſince the Peripheries of all Circles are in a direct Proportion to their Diameters, conſequently they are in a direct Proportion to their Semidiameters, and

As the Periphery of one Circle
Is to the Periphery of another
So is any Part of the firſt Circles Periphery
To the like Part of the ſecond Circle's Periphery

Hence we may ſay,

As E C the Radius of the Equinoctial
Is to F B the Co-ſine of any Latitude
So is any Difference of Longitude in Miles
To the Number of Miles in that Parallel which are contain'd between the ſame two Meridians or Difference of Longitude.



EXAMPLE.

SUPPOSE it was required to find how many Miles in the Latitude of 54 Degrees will be equal to one Degree of Longitude, *viz.* 60 Miles in the Equinoctial.

It will be as Radius	10.
Is to the Co-ſine of the Latitude	9.769219
So is the Difference of Longitude 60 Miles	1.778151
To the Number of Miles required 35.26	1.547370

ACCORDING to this Analogy, the following Table was calculated, by which having any Latitude given, you may find how many Miles muſt be taken to make one Degree of Longitude in that Latitude.

So againſt the Latitude of 38 Degrees you have 47.28 Miles, and againſt 70° is 20.52.

BUT if the given Latitude conſiſts of Degrees and Minutes, you muſt find the Difference of Miles and Parts, or Parts between the given Degrees, and thoſe next above, then ſay,

As 60 is to that Difference, ſo are the Minutes given to a proportional Part of the Difference which ſubſtract from the Miles and Parts that you found againſt the given Degrees of Latitude, and what remains will be the Miles and Parts for the Degrees and Minutes given.

EXAMPLE.

I demand the Miles anſwering to a Degree of Longitude in the Latitude of 34° : 28′.

AGAINST 34 Degrees you have 49.74, and againſt 35 you have 49.15 their Difference is 59.

As 60′ : 59 :: 28 : 27, ſubſtract this .27 from 49.74, and there will remain 49.47 for the Miles and decimal Parts required.

THE Diſtance ſailed in any Parallel of Latitude being given to find the Difference of Longitude.

EXAMPLE. Suppoſe a Ship in the Latitude of 54° ſails directly Eaſt or Weſt, in that Parallel 1642 Miles, and it be required how much ſhe has alter'd her Longitude.

Say, As S. *c.* 54°	9.769219
Is to Radius	10.
So is the meridional Diſtance 1642	3.215373
To the Difference of Longitude 2793	3.446154

A TABLE

A TABLE, ſhewing how many Miles anſwer to a Degree of Longitude, at every Degree of Latitude.

Latitude. D.	Miles.	Latitude. D.	Miles.	Latitude. D.	Miles.
1	59.99	31	51.43	61	29.09
2	59.96	32	50.88	62	28.17
3	59.92	33	50.32	63	27.24
4	59.86	34	49.74	64	36.30
5	59.77	35	49.15	65	25.36
6	59.67	36	48.54	66	24.41
7	59.56	37	47.92	67	23.45
8	59.42	38	47.28	68	22.48
9	59.28	39	46.62	69	21.50
10	59.08	40	45.95	70	20.52
11	58.89	41	45.28	71	19.54
12	58.68	42	44.59	72	18.55
13	58.46	43	43.88	73	17.54
14	58.22	44	43.16	74	16.53
15	57.95	45	42.43	75	15.52
16	57.67	46	41.68	76	14.51
17	57.37	47	40.92	77	13.50
18	57.06	48	40.15	78	12.48
19	56.73	49	39.36	79	11.45
20	56.38	50	38.57	80	10.42
21	56.01	51	37.76	81	9.38
22	55.63	52	36.94	82	8.35
23	55.23	53	36.11	83	7.32
24	54.81	54	35.26	84	6.28
25	54.38	55	34.41	85	5.23
26	53.93	56	33.55	86	4.18
27	53.46	57	32.68	87	3.14
28	52.97	58	31.79	88	2.09
29	52.47	59	30.90	89	1.05
30	51.96	60	30.00	90	0.00

THE Diſtance ſailed in any Parallel of Latitude being given to find the Difference of Longitude by the Table.

EXAMPLE. Suppoſe a Ship in the Latitude of 54° ſails directly Eaſt or Weſt, in that Parallel 1642 Miles, and it be required how much ſhe has altered her Longitude.

AGAINST the given Latitude in the Table you have 35.26, then it will be as 35.26 : 60 :: 1642 : 2793, the Difference of Longitude required.

THERE is a Line mark'd M.L. (for Middle Latitude) fitted to a Line of Chords, commonly placed upon *Gunter*'s Scale ; theſe ſerve to the ſame Purpoſe as the Table ; for if you extend your Compaſſes from the Point in the Braſs Pin, at the Beginning of the Line of Chords, to 54°, that Extent will reach from the Point at 60 upon the Line M. L. to 35.26, as by the Table.

A Table of Meridional Parts.

M	0	1	2	3	4	5	6	7	8	9	M
0	0	60	120	180.1	240.2	300.4	360.7	421.1	481.6	542.2	0
10	10	70	130	190.1	250.2	310.4	370.7	431.1	491.7	552.4	10
20	20	80	140	200.1	260.3	320.5	380.8	441.1	501.8	562.5	20
30	30	90	150	210.1	270.3	330.5	390.8	451.3	511.9	572.6	30
40	40	100	160	220.1	280.3	340.6	400.9	461.4	522.0	582.8	40
50	50	110	170	230.1	290.3	350.6	411.0	171.5	532.1	592.9	50
M	10	11	12	13	14	15	16	17	18	19	M
0	603.1	664.1	725.3	786.8	848.5	910.5	972.8	1035.3	1098.2	1161.5	0
10	613.2	674.3	735.6	797.1	858.9	920.8	983.1	1045.8	1108.7	1172.1	10
20	623.4	684.5	745.8	807.3	869.2	931.2	993.6	1056.2	1119.2	1182.7	20
30	633.5	694.7	756.1	817.6	879.6	941.5	1004.0	1066.7	1129.7	1193.2	30
40	643.7	704.9	766.3	827.9	889.9	951.9	1014.4	1077.2	1140.3	1203.9	40
50	653.9	715.1	776.6	838.2	900.2	962.3	1024.8	1087.7	1150.9	1214.5	50
M	20	21	22	23	24	25	26	27	28	29	M
0	1225.1	1289.2	1353.7	1418.7	1484.1	1550.0	1616.5	1683.6	1751.2	1819.5	0
10	1235.8	1299.9	1364.4	1429.5	1495.0	1561.0	1627.6	1694.8	1762.5	1830.9	10
20	1246.4	1310.6	1375.3	1440.4	1506.0	1572.1	1638.8	1706.0	1773.9	1842.4	20
30	1257.1	1321.4	1386.1	1451.3	1517.0	1583.2	1649.9	1717.3	1785.2	1853.8	30
40	1261.8	1332.1	1396.9	1462.2	1528.0	1594.3	1661.1	1728.6	1796.6	1865.3	40
50	1278.5	1342.9	1407.8	1473.1	1539.0	1605.4	1672.3	1739.9	1808.0	1876.8	50
M	30	31	32	33	34	35	36	37	38	39	M
0	1888.4	1958.1	2028.4	2099.6	2171.5	2244.3	2318.0	2390.7	2468.3	2544.9	0
10	1899.9	1969.7	2040.2	2111.5	2183.6	2256.5	2330.4	2405.2	2481.0	2557.8	10
20	1911.5	1981.4	2052.0	2123.4	2195.7	2268.8	2342.8	2417.8	2493.7	2570.7	20
30	1923.1	1993.1	2063.9	2135.4	2207.8	2281.0	2355.2	2430.3	2506.5	2583.7	30
40	1934.7	2004.9	2075.7	2147.4	2219.9	2293.3	2367.7	2443.0	2519.3	2596.7	40
50	1946.4	2016.6	2087.7	2159.4	2232.1	2305.7	2381.0	2455.6	2532.1	2609.7	50
M	40	41	42	43	44	45	46	47	48	49	M
0	2622.7	2701.6	2781.7	2863.1	2945.9	3030.0	3115.6	3202.8	3291.6	3382.1	0
10	2635.8	2714.9	2795.1	2876.8	2959.8	3044.1	3120.0	3217.4	3306.5	3397.4	10
20	2648.9	2728.2	2808.7	2890.5	2973.7	3058.3	3144.5	3232.2	3321.5	3412.7	20
30	2662.0	2741.5	2822.3	2904.3	2987.7	3072.6	3159.0	3246.9	3336.6	3428.1	30
40	2674.1	2754.9	2835.8	2918.1	3001.8	3086.9	3173.5	3261.8	3351.7	3443.5	40
50	2688.4	2768.3	2849.5	2932.0	3015.8	3101.2	3188.0	3276.6	3366.9	3459.0	50

A Table of Meridional Parts.

M	50	51	52	53	54	55	56	57	58	59	M
0	3474.5	3568.8	3665.2	3763.8	3864.7	3968.0	4073.9	4182.7	4294.3	4409.2	0
10	3490.1	3584.8	3681.5	3780.4	3881.7	3983.5	4091.9	4201.1	4313.2	4428.6	10
20	3505.7	3600.7	3697.8	3797.2	3898.8	4003.0	4109.9	4219.5	4332.2	4448.2	20
30	3521.4	3616.8	3714.2	3813.9	3916.0	4020.6	4127.9	4238.1	4351.3	4467.8	30
40	3537.2	3632.9	3730.7	3830.8	3933.3	4038.3	4146.0	4256.8	4370.5	4487.6	40
50	3553.0	3649.0	3747.2	3847.7	3950.6	4056.1	4164.3	4275.5	4389.8	4507.5	50
M	60	61	62	63	64	65	66	67	68	69	M
0	4527.4	4649.3	4775.0	4905.0	5039.5	5178.8	5323.6	5474.0	5630.9	5794.6	0
10	4547.5	4669.9	4796.4	4927.1	5062.3	5202.6	5348.2	5499.7	5657.6	5822.6	10
20	4567.6	4690.7	4817.8	4949.3	5085.3	5226.5	5373.0	5525.6	5684.6	5850.8	20
30	4587.8	4711.6	4839.4	4971.6	5108.5	5250.5	5398.0	5551.6	5711.8	5879.3	30
40	4608.2	4732.6	4861.2	4994.1	5131.8	5274.7	5423.2	5577.8	5739.2	5907.9	40
50	4628.7	4753.8	4883.0	5016.7	5155.3	5299.0	5448.5	5604.3	5766.8	5936.8	50
M	70	71	72	73	74	75	76	77	78	79	M
0	5966.0	6145.7	6334.9	6534.4	6745.7	6970.3	7210.0	7467.2	7744.6	8045.7	0
10	5995.3	6176.6	6367.4	6568.8	6782.2	7009.2	7251.6	7511.9	7792.9	8098.5	10
20	6024.9	6207.6	6400.2	6603.4	6819.0	7048.4	7293.6	7557.2	7842.0	8152.1	20
30	6054.7	6239.0	6433.2	6638.5	6856.2	7088.1	7336.2	7603.1	7891.8	8206.5	30
40	6084.8	6270.7	6466.7	6673.9	6893.8	7128.3	7379.4	7649.6	7942.4	8261.8	40
50	6115.1	6302.6	6500.4	6709.6	6931.8	7169.0	7423.0	7696.8	7993.7	8318.1	50
M	80	81	82	83	84	85	86	87	88	89	M
0	8375.3	8739.1	9145.6	9605.9	10137.0	10764.7	11532.6	12522.3	13916.6	16299.8	0
10	8433.3	8803.6	9218.1	9689.0	10234.0	10881.4	11679.1	12718.8	14215.8	16926.5	10
20	8492.3	8869.3	9292.3	9774.0	10333.8	11002.2	11832.0	12927.4	14543.5	17693.6	20
30	8552.3	8936.2	9368.1	9861.3	10436.6	11127.4	11992.0	13149.3	14905.8	18682.5	30
40	8613.4	9004.6	9445.6	9950.8	10542.6	11257.2	12159.9	13386.6	15310.7	20075.2	40
50	8675.7	9074.4	9524.8	10042.6	10651.9	11392.2	12336.3	13641.4	15769.8	22458.0	50

By the Help of this Table (which is calculated to every 10 Minutes of a Degree of Latitude from the Equinoctial to the Poles) having the Latitudes of two Places given, you may find the meridional Miles or Minutes between them; in order to this, you muſt conſider whether the Places be one under the Equinoctial, and the other on the North or South Side thereof; or the one on one Side of the Equinoctial, and the other on the other Side; or whether they both lie on the ſame Side; for according to theſe Poſitions, there are Three Caſes.

Case 1.

If one of the propoſed Places be under the Equinoctial, and the other have either North or South Latitude, the meridional Parts found againſt the Latitude, will be the meridional Difference of Latitude, or Latitude enlarged.

EXAMPLE.

Suppose it be required to find the meridional Parts between the Equinoctial and the Latitude of 50° : 30′ in the Column under 50° and againſt 30′ ſtand 3521.4, which are the meridional Parts required.

Case 2.

If the two Places propos'd are in contrary Hemiſpheres, *viz.* if one of them have North, and the other South Latitude, then the Sum of the meridional Parts found againſt each Latitude, will be the meridional Minutes required.

EXAMPLE.

Let it be required to find the meridional Parts between the Latitude of 25° : 10′ South and 51° : 30 North.

Under 25° and againſt 10′ is	1561.0
Under 51° and againſt 30′ is	3616.8
The meridional Parts between them are	5177.8

Case 3.

If the two Places propoſed are both in one Hemiſphere, *viz.* either both North, or both South, then the Difference of the meridional Parts found againſt each Latitude, will be the meridional Parts required.

EXAMPLE.

Let it be required to find the meridional Parts between the Latitude of 25° : 40′ and the Latitude of 51° : 50′ both North.

Under 51°, and againſt 50′ is	3649.0
And under 25°, and againſt 40′ is	1594.3
The meridional Parts between the Latitudes propoſed are	2054.7

When the given Latitude conſiſts of any Number of Degrees contain'd in this Table, and Minutes between 10′ and 20′, 30′ and 40, *&c.* you may find the meridional Parts, by taking the Parts proportional.

Suppose

SUPPOSE it be required to find the meridional Parts anſwering to 36° : 44′.

Under 36°, and againſt 40′ is	2367.7
Under 36°, and againſt 50′ is	2381.0
Their Difference is Parts	13.3
Then, As 10 : 13.3 :: 4′ : to	5.32
Add 5.32 to 2367.7, and the Sum is the meridional Parts required	2373.02

THERE are two Lines upon *Gunter*'s Scale, one is mark'd at the Beginning with *Mer.* for Meridian, and the other with *E. P.* for equal Parts, theſe are to ſupply the Want of a Table of meridional Parts, thus, Let it be required to find the meridional Parts anſwering to the Latitude of 25 Degrees.

EXTEND your Compaſſes from the Braſs Point at the Beginning of the meridian Line to 25 Degrees upon the ſame Line, then ſet one Foot upon 20 in the Line of equal Parts, and the other will reach to about 25.83, which being multiplied by 60 will produce 1549.8 for the meridional Parts required.

AGAIN, Let it be required to find the meridional Parts for the Latitude of 58 : 30′.

EXTEND your Compaſſes from the Beginning of the meridional Line to 58 : 30′, then placing one Foot in 70 on the Line of equal Parts, the other will be on about 72.52, that is 72 Degrees, and $\frac{52}{100}$ of a Degree; multiply this by 60, and the Product will be 4351.2, for the meridional Parts required.

THERE are Rules (deduced from Dr. *Halley*'s Works) to ſupply the Want of a Table of meridional Parts by a Table of Logarithmic Tangents, As,

1.) To find the meridional Parts to any given Latitude.

RULE.

To the given Latitude add 90°, then find the Tangent of half that Sum, from which ſubſtract Radius, then divide the Remainder by 1263.3 and the Quotient will be the true meridional Parts contained between the Equinoctial and that Latitude.

EXAMPLE 1.

LET it be required to find the meridional Parts contained between the Equinoctial and the Latitude of 36° : 40′.

The given Latitude 36° : 40′ + 90° = 126° : 40′, half of that is 63° : 20′ } The Tangent of which is	10.2991070
From which take Radius	10.
There remains	0.2991070

Which divided by 1263.3 gives in the Quotient 2367.7 the meridional Parts to the Latitude of 36° : 40′.

EXAMPLE 2.

LET it be required to find the meridional Parts for the Latitude of 88° : 50.

Here 88° : 50′ + 90 = 178° : 50′ the half of which is 89° : 25′, and the Tangent thereof is	11.9921908
From which take the Radius	10.
Remains	1.9921908

This

This divided by 1263.3, the Quotient is 15769.7, the meridional Parts anſwering to 88° : 50′, *&c.*

2.) To find the meridional Parts contained between the Latitudes of any two Places that are on the ſame Side of the Equinoctial.

RULE.

To each of the given Latitudes add 90°, then find the Tangents of each of their half Sums (as before the Difference of theſe two Tangents being divided by 1263.3 the Quotient will be the true meridional Parts contain'd between thoſe two Latitudes.

EXAMPLE.

LET it be required to find the meridional Parts between the Latitude of 46° : 10, and 16° : 30′, both North or South.

Here 46° : 10′ + 90° = 136° : 10, its half is 68° : 5′, the Tangent is —— 10.3954118
And 16 : 30′ + 90 = 136 : 30′, its half is 53 : 15 the Tangent is —— 10.1268332
Difference of the Tangents is —— —— —— 2685786
This divided by 1263.3 the Quotient is 2126, for the meridional Parts required.

3.) To find the meridional Parts contained between the Latitudes of two Places, one being on the North Side of the Equinoctial, and the other on the South.

RULE.

To each of the given Latitudes add 90°, and find the Tangent to each of their half Sums, then add theſe two Tangents together, and their Sum made leſs by twice the Radius, the Remainder divided by 1263.3, the Quotient is the true meridional Parts.

EXAMPLE.

SUPPOSE it be required to find the meridional Parts between the Latitude of 22° : 0′ North, and the Latitude of 12° : 40′ South.

Latitude North 22° : 0′ + 90° = 112° : 0′ half is 56° : its Tangent —— 10.1710126
Latitude South 12° : 40′ + 90 = 102 : 40 half is 51 : 20 its Tangent —— 10.0968034

Sum of the Tangents is —— —— —— 20.2678160
From which take twice Radius —— —— —— 20.0000000

The Remainder is —— —— —— —— 2678160

THIS being divided by 1263.3, the Quotient will be 2120, the meridional Parts contain'd between the two Latitudes.

To do the ſame by the meridian Line and Line of equal Parts upon *Gunter*'s Scale.

RULE.

EXTEND your Compaſſes from the beginning of the meridian Line to 12° : 40′, then lay that Extent from 22° : 0′ towards the Left-hand, and the Foot towards the Left-hand will reach to a certain Point upon the meridian Line, where fix it, then the Diſtance between that Point and the beginning of the Line, apply'd to the Line of equal Parts, will reach to about

about 35.3, which multiply'd by 60 gives 2119.8 near the ſame as above, and ſo of any other.

Caſe I.

THE Latitudes of two Places, and their Difference of Longitude, being given to find the Courſe, and direct Diſtance between them.

EXAMPLE.

ADMIT a Ship ſails from a certain Port in the Latitude of 48° : 30′ North to another, whoſe Latitude is 30° : 10′ North, their Difference of Longitude being 54° : 0′ Weſterly, what will be their direct Courſe and Diſtance?

Latitudes { 48° : 30′ / 30 : 10 } meridional Parts { 3336.6 / 1899.9 } ſubſtract

Difference 18 : 20 — 1436.7 meridional Diſtance.
60

Miles — 1100

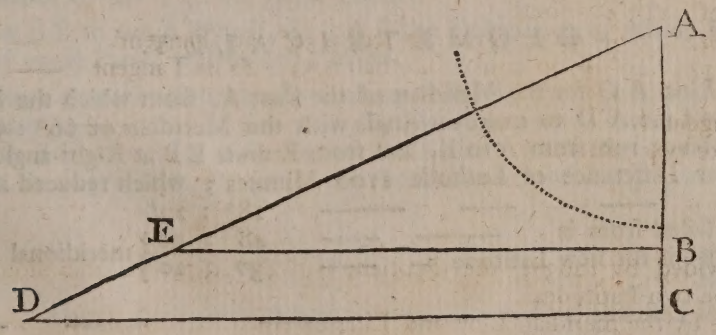
Difference of Longitude 54° × 60′ = 3240 Difference of Longitude in Minutes.

GEOMETRICALLY.

1. LET A repreſent the Port from whence the Ship ſail'd, from which draw A C = 1436.7 (from a Scale of equal Parts.)

2. UPON C erect a Perpendicular, and from a Scale of equal Parts ſet 3240 from C to D, then draw A D and the Triangle will be compleat.

3. FROM the ſame Scale of equal Parts, ſet off 1100 Miles from A to B, and from B draw B E parallel to C D.



IN this Figure A repreſents the Port from whence the Ship ſet Sail, and E the Port to which ſhe ſailed; A the Angle of the Ship's Courſe, A B the true Difference of Latitude, A C is the meridional Difference of Latitude, C D is the true Difference of Longitude in Miles, B E the Departure, and A E the true Diſtance of the two Ports.

By the LOGARITHMS.

IN the Triangle A C D, we have A C and C D given to find the Angle at A, or the Courſe.

As A C = 1436.7	3.157366
Is to Radius	10.
So is C D = 3240.	3.510545
To the Tangent of the Angle of the Courſe 66° : 5′	10.353179
That is S. W. *b.* W. + 9° : 50′ W.	

AGAIN, in the Triangle A B E there is given A B and the Angle at A = 66° : 5′ to find the Diſtance A E.

As the S. *c.* of 66° : 5′	9.607892
Is to A B = 1100, the proper Difference of Latitude	3.041393
So is Radius	10.
To A E the Diſtance of the Ports 2714 Miles	3.433501

Caſe 2.

GIVEN the Courſe and Diſtance, with one Latitude to find the other Latitude, and Difference of Longitude.

EXAMPLE.

SUPPOSE a Ship ſails from a Port in the Latitude of 48° : 30′ North, away S. W. *b.* W. + 9° : 50′ W. = 66° : 5′ South weſterly, 2714 Miles, what Latitude will ſhe be in, and what will the Difference of Longitude be?

GEOMETRICALLY.

1) Draw the Line A C for the Meridian of the Port A, from which the Ship ſail'd, and from A draw the Line A D to make an Angle with the Meridian of 66° : 5′; upon which ſet 2714 the Diſtance run from A to E, and from E draw E B at Right-angles to A C, then is A B the proper Difference of Latitude 1100 Minutes; which reduced into Degrees, is 18° : 20′

The Latitude ſailed from is	48 : 30	meridional Parts 3336.6
Their Difference is the new Latitude	30 : 10	1899.9
The meridional Diſtance is		1436.7

2) TAKE 1436.7 in your Compaſſes from a Line of equal Parts, and ſet that off from A to C, then from C draw C D parallel to B E, and that will compleat the Figure, then take C D in your Compaſſes to the Line of equal Parts, and you'll find it to be 3240, for the Difference of Longitude; that is 54° : 0′.

By

By the LOGARITHMS.

In the Right-angled Triangle A B E is given the Angle of the Course at A = 66° : 5′ and Distance run A E = 2714 Miles to find the Difference of Latitude A B.

As Radius	10.
Is to the Distance run A E = 2714	3.433609
So is the S. *c.* of the Course = 66° : 5′	9.607892
To the Difference of Latitude 100′ = 18° : 20′	3.041501

By the Difference of Latitude, and the given Latitude 48° : 30′ the new Latitude will be 30° : 10′ and the meridional Parts between these two Latitudes will be A C = 1436.7 as before.

Then in the Right-angled Triangle A C D, you have A C = 1436.7 and the Angle of the Course 66° : 5′ to find the Difference of Longitude C D.

As Radius	10.
Is to the meridional Distance A C = 1436.7	3.157366
So is the Tangent of the Course 66 : 5′	10.353179
To C D = 3240 the Difference of Longitude	3.510545

Which divided by 60′ gives 54° : 0′ for the Difference of Longitude in Degrees towards the West. (See the last Figure.)

Case 3.

GIVEN the Latitude of two Places, and the Course between to find their Distance and Difference of Longitude.

EXAMPLE.

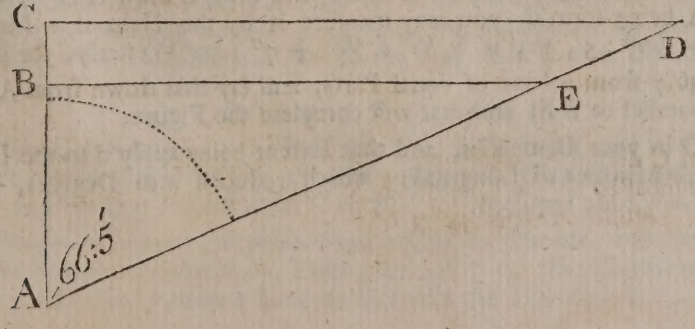
ADMIT a Ship sails from the Latitude of 30° : 10′ North, upon *a* N. E. *b.* E. Course 9° : 50′ more Easterly, till she comes into the Latitude of 48° : 30′, what is her Distance sailed, and how much hath she altered her Longitude?

GEOMETRICALLY.

1) DRAW C A for the Meridian of the Place at A (from whence the Ship did sail) upon which from a Line of equal Parts set off the Difference of Latitudes 18° : 20′ = 1100 Miles from A to B, and from A draw A D upon the East Side of the Meridian, making an Angle therewith of 66° : 5′ the given Course.

2) FROM B draw B E to cut A D in E, so is A E the Distance run, which being measured by the Line of equal Parts, will be 2714 Miles.

3) TAKE in your Compasses from the Line of equal Parts, the meridional Parts between the two Latitudes 1436.7, and set that upon the Meridian from A to C, and from C draw C D parallel to B E, this Line will complete the Figure, and being measured by the Line of equal Parts, is 3240 Miles, or 54° : 0′, the Difference of Longitude required.



By the LOGARITHMS.

1) In the Triangle A B E you have A B = 1100 Miles, and the Angle at A = 66° : 5′, to find A E, ſay,

As the S. *c.* of 66° : 5′	9.607892
Is to 1100	3.041393
So is Radius	10.
To the Diſtance run A E = 2714 Miles	3.433501

AGAIN in the Triangle A C D you have the Side A C = 1436.7 and Angle of the Courſe A = 66° : 5′ to find the Difference of Longitude C D ſay,

As the S. *c.* of 66° : 5′ — — *co. ar.*	9.392108
Is to A C = 1436.7	3.157366
So is the Sine of 66° : 5′	9.961011
To the Difference of Longitude C D = 3240	3.510485

Which being divided by 60 give 54° : 0′.

Caſe 4.

THE Latitude of two Places, and Diſtance, being given to find the Courſe, and the Difference of Longitude.

ADMIT a Ship ſails from the Latitude of 48° : 30′ North, between the South and the Weſt 2714 Miles, and then is found to be in the Latiude of 30° : 10′ North, 'tis requir'd to find what Courſe ſhe hath ſailed upon, and how many Degrees ſhe hath altered her Longitude?

Latitudes { 48° : 30′ / 30 : 10 } meridional Parts — — { 3336.6 / 1899.9 }

Difference 18 : 20 Difference —— — 1436.7 = A C

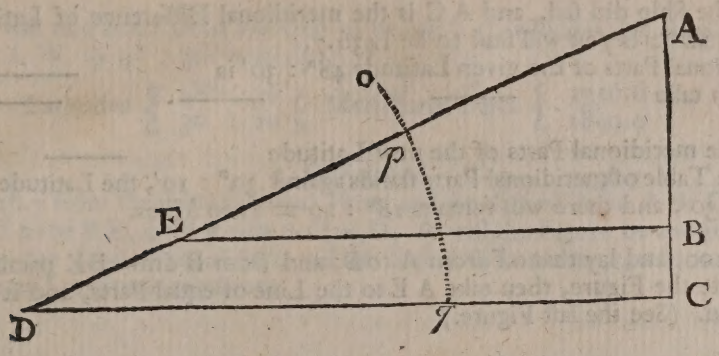
GEOMETRICALLY.

1) DRAW A C for the Meridian of the Place at A, then take 1100 in your Compaſſes from a Line of equal Parts, and lay that down from A to B, and from B draw the Line B E at Right-angles to A C.

2) TAKE 2714 from a Line of equal Parts, and ſetting one Foot of your Compaſſes in A, turn the other about until it cuts B E in E, and thro' E draw A D: Now the Angle of the Courſe at A being form'd, you may meaſure it by the Help of a Line of Chords, and find it to be 66° : 5′, that is S. W. *b.* W. + 9° : 50′ Weſterly, for the Ship's Courſe.

3) TAKE 1436.7 from a Line of equal Parts, and lay that down from A to C, and from C draw C D parallel to B E, and that will complete the Figure.

4) TAKE C D in your Compaſſes, and that Extent being apply'd to the Line of equal Parts, will give 3240 Minutes of Longitude; which reduced into Degrees, is 54° : 0′ the Difference of Longitude required.



By the LOGARITHMS.

1) IN the Right-angled Triangle A B E, is given A B = 1100, and A E = 2714, to find the Angle of the Courſe A, ſay,

As 2714 — — — — — —	3.433609
Is to Radius — — — — — —	10.
So is 1100 — — — — — —	3.041393
To the S. *c.* of the Courſe 66° : 5′ — — —	9.607784

2) IN the Right-angled Triangle A C D, you have A C = 1436.7, and Angle of the Courſe A = 66° : 5′, to find C D, ſay,

As Radius — — — — — —	10.
Is to A C = 1436.7 — — — —	3.157366
So is the Tangent of 66° : 5′ — — — —	10.353119
To C D = 3240 Minutes = 54° : 0′ the Difference of Longitude required—	3.510485

Caſe 5.

GIVEN one Latitude, the Courſe, and Difference of Longitude to find the other Latitude and Diſtance run.

EXAMPLE.

ADMIT a Ship ſails from a certain Port at A, in the Latitude of 48° : 30′ North, S. W. *b.* W. + 9° : 50′ weſterly = 66° : 5′, until the Difference of Longitude be 54° : 0′ = 3240 Minutes, what Latitude will ſhe be in, and what is the Diſtance run?

GEOMETRICALLY.

1) DRAW A C for the Meridian of the Port the Ship ſail'd from, and upon any Point therein as at C, erect a perpendicular C D, then take 3240 in your Compaſſes from a Line of equal Parts, and lay that Extent from C to D.

2) TAKE 60 Degrees in your Compaſſes from a Line of Chords, and ſetting one Foot in D, with the other deſcribe the Arch *q o*, from *q* ſet 23° : 55′ (the Complement of the Courſe) to *p*, then from D thro' *p* draw a Line until it cuts the Meridian in A, ſo is A the Port from

from which the Ship did ſail, and A C is the meridional Difference of Latitude, which by the Line of equal Parts you will find to be 1436.7.

The meridional Parts of the given Latitude 48° : 30′ is	3336.6
From which take	1436.7
Remains the meridional Parts of the new Latitude	1899.9

Which in the Table of meridional Parts ſtands againſt 30° : 10′, the Latitude required, which take from 48° : 30′, and there will remain 18° : 20 = 1100 Miles.

3.) TAKE 1100, and lay that off from A to B, and from B draw B E parallel to C D, and that will complete the Figure, then take A E to the Line of equal Parts, and it will give 2714, the Diſtance Run. (See the laſt Figure.)

By the LOGARITHMS.

1. IN the Triangle A C D, you have the Angle of the Courſe A = 66° : 5′, and Difference of Longitude C D = 3240, to find A C, ſay,

As the Sine of the Courſe 66° : 5′	*co. ar.*	0.038989
Is to D C = 3240		3.510545
So is the S. *c.* of the Courſe		9.607892
To A C = 1436.7		3.157426

By this, and the meridional Parts of the given Latitude, you will find the new Latitude to be 30° : 10′, and the Difference of Latitudes to be 1100 Miles, as above.

IN the Triangle A B E, you have the Angle of the Courſe A = 66° : 5′, and Difference of Latitude A B = 1100 Miles, to find A E, ſay,

As S. *c.* of 66° : 5′	9.607892
Is to 1100	3.041393
So is Radius	10.
To the Diſtance Run A E = 2714	3.433501

Caſe 6.

GIVEN both Latitudes, and Departure, to find the Courſe, the Diſtance ſail'd, and Difference of Longitude.

EXAMPLE.

SUPPOSE a Ship ſailed from a certain Port at A, in the Latitude of 48° : 30′ North between the South and the Weſt, until ſhe come into the Latitude of 30° : 10 North, and her Departure be 2481 Miles, I demand her Courſe, the Diſtance ſailed, and Difference of Longitude.

GEOMETRICALLY.

1.) DRAW A C for the Meridian, then the Difference of Latitude being 18° : 20′ = 1100 Miles, take this in your Compaſſes from the Line of equal Parts, and lay it down from A to B, and upon B erect the Perpendicular B E, upon which ſet 2481 from B to E; then thro' E draw A Q out at Pleaſure, ſo will A E be the Diſtance ſail'd which being meaſured by the Line of equal Parts is 2714 Miles.

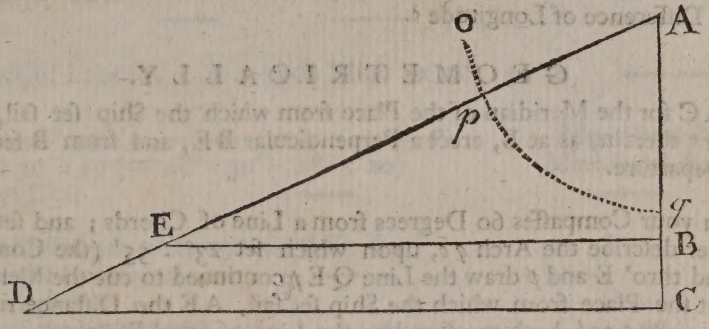
2.) TAKE 60 Degrees in your Compaſſes, out of the Line of Chords, and ſetting one Foot in A, with the other deſcribe the Arch *q q*, then take *q p* in your Compaſſes to the Line of Chords,

Chords, and you will find that Part of the Arch to be 66° : 5′, which is the Angle of the Course from A, or S. W. *b.* W. + 9° : 50′ westerly.

Latitudes { 48° : 30′ / 30 : 10 } Meridional Parts { 3336.6 / 1899.9 }

Difference —— 1436.7

3.) TAKE 1436.7 from the Line of equal Parts, and set that off from A to C, and from C draw C D parallell to be B E, until it cuts A Q in D, so will the Figure be complete; and C D being measured by the Line of equal Parts will be found to contain 3240 Minutes, or 54° : 0′ for the Difference of Longitude required.



By the LOGARITHMS.

1.) IN the Triangle A B E, you have the Difference of Latitude A B = 1100 Miles, and Departure B E = 2481 Miles, to find the Angle of the Course A, say,

As 1100 —— —— —— ——	3.041393
Is to Radius —— —— —— ——	10.
So is 2481 —— —— —— ——	3.394672
To the Tangent of the Course 66° : 5′, or S. W. *b.* W. + 9° : 50′ westerly ——	10.353276

2.) FOR the Distance run A E, say,

As the S. *c.* 66° : 5′ —— —— ——	9.607892
Is to 1100 —— —— —— ——	3.041393
So is Radius —— —— ——	10.
To the Distance Run 2714 —— ——	3.433501

3.) THE meridional Parts between the two given Latitudes was found to be 1436.7. Now in the Triangle A C D, you have the Angle of the Course D A C = 66° : 5′, and Side A C = 1436.7, to find C D, say, As Radius —— —— 10.

Is to 1436.7 —— —— —— ——	3.157366
So is the Tangent of the Course 66° : 5′ —— ——	10.353179
To C D = 3240 Minutes, or 54° : 0′, the Longitude required ——	3.510545

Tale

Caſe 7.

GIVEN one Latitude, the Courſe, and Departure, to find the other Latitude, the Diſtance run, and Difference of Longitude.

EXAMPLE.

SUPPOSE a Ship ſailed from the Latitude of 48° : 30′, S. E. *b.* E. + 90° : 50′ eaſterly, until ſhe had departed from the Meridian 2481 Miles; what Latitude is ſhe then in; what is the Diſtance run, and Difference of Longitude?

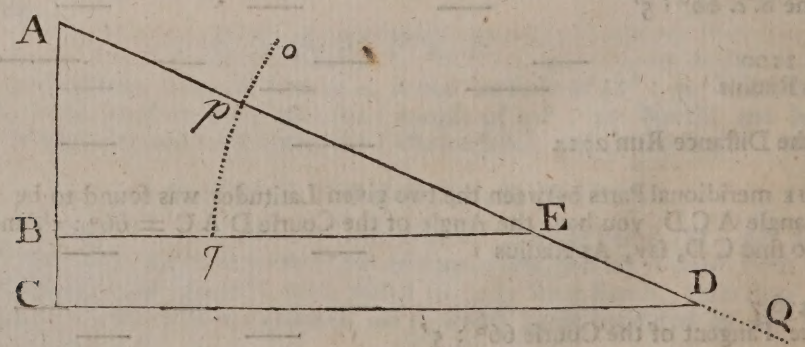
GEOMETRICALLY.

1.) DRAW A C for the Meridian of the Place from which the Ship ſet ſail, then upon any convenient Point therein, as at B, erect a Perpendicular B E, and from B ſet off 2481, to E, ſo is B E the Departure.

2.) TAKE in your Compaſſes 60 Degrees from a Line of Chords; and ſetting one Foot in E, with the other deſcribe the Arch *q o*, upon which ſet 23° : 55′ (the Complement of the Courſe) to *p*, and thro' E and *p* draw the Line Q E *p* continued to cut the Meridian in A; then A ſhall repreſent the Place from which the Ship ſet ſail, A E the Diſtance run = 2714, and A B = 1100′ = 18° : 20′, both meaſured by the Line of equal Parts.

The Latitude the Ship was in at A = 48° : 30′ meridional Parts	—	3336.6
Difference of Latitude ſubſtract — 18 : 20		
Remains the new Latitude — 30 : 10 meridional Parts	—	1899.9
Difference — — — —	—	1436.7

3.) TAKE this Difference 1436.7 in your Compaſſes from the Line of equal Parts, and ſet that Extent from A to C, and from C draw C D parallel to B E, and the Figure will be complete; and C D being meaſured by the iLne of equal Parts will be 3243 Minutes = 54° : 0′ for the Difference of Longitude.



By

By the LOGARITHMS.

1.) In the Triangle A B E is given the Angle at A = 66° : 5′, and Departure B E = 2481, to find the Diſtance run, ſay,

As the Sine of the Courſe 66° : 5′	9.961011
Is to the Departure 2481 Miles	3.394627
So is Radius	10.
To the Diſtance Run 2714 Miles	3.433616

2.) FOR the Difference of Latitude, ſay,

As the Tangent of the Courſe 66° : 5′	10.353119
Is to the Departure 2481	3.394627
So is Radius	10.
To the Difference of Latitude A B = 1100 Miles = 18° : 20′	3.041508

GIVEN Latitude 48° : 30′	Meridional Parts — 3336.6
New Latitude 30 : 10 (= 48° : 30′ − 18° : 20)	Meridional Parts — 1899.9
Difference is A C =	1436.7

3.) To find the Difference of Longitude C D, ſay,

As the S. *c.* of the Courſe, *co. ar.*	0.392108
Is to A C = 1436.7	3.157366
So is the Sine of the Courſe 66° : 5′	9.961011
To C D = 3240′ = 54° : 0′, the Longitude required	3.510485

Great Circle Sailing.

BEFORE we give Examples in this Part of Sailing, it will be of Uſe to know how

To project the Sphere upon the Plane of the Meridian.

THE Projection of the Sphere upon the Plane of the Meridian may be ſaid to be of two Sorts, *viz.* One particular, becauſe to a particular Latitude and Time, and made uſe of in Aſtronomy. The other univerſal, and uſed in Geography, and Navigation; and the Manner of Drawing ſuch a Projection is as follows;

1. DRAW the Line A C, of what Length you pleaſe, for the Equator, and divide it into two equal Parts in the Point B, ſo will A B and B C be the Diameters of two Hemiſpheres. (See *Fig.* 1ſt.

2. DIVIDE A B, and B C into two equal Parts in the Points *b*, and *d*, and theſe are the Centers of the Hemiſpheres, upon which the Radius A *b*, or B *d* deſcribe the primitive Circles ANBS, and BNCI for the firſt Meridian.

3. DRAW N. S. thro' the Points *b*, and *d* at Right-angles with the Equator, and thoſe Lines will cut the Meridian in N. the North Pole, and S. the South Pole.

4. THE Circles or Parallels of Latitude, 10, 20 30, 40, *&c.* being all parallel to the Equator, or Right-circles A B and B C, you may draw them as directed at *Prob.* 9. *Chap.* 11. Thus,

Suppoſe you would draw the Parallel of 30 Degrees from the Equator A B: Firſt, from the Line of Chords take 30 Degrees, and ſet that Extent from A, B, and C, towards N and S. Then ſet the Tangent of 60 Degrees (the Complement of 30°) from 30 to *x* and *y*; or the Secant of 60 Degrees from the Centres *b* and *d*, to *x* and *y*: So will the Points *x*, and *y* be the Centres of two Parallels of Latitude, one North, and the other South, from the Equator. Then ſet one Foot of your Compaſſes in *x* or *y*, and extend the other to 30°; that Extent is the Radius of the Parallel, at which deſcribe it, in like manner proceed to draw the reſt. *Note*, That *x* is ſuppoſed to be as much above *b*, as *y* is below it.

To draw the MERIDIANS, *or* CIRCLES *of Longitude.*

1. BECAUSE theſe are uſually drawn thro' every 10th Degree of the Equator, they will make Angles with the Meridian, or Primitive Circle at the Poles N and S of 10, 20, 30, 40, 50, 60, 70, and 80 Degrees: Therefore by Caſe 2d of Prob. 2d, Chap. 11th, ſet the Tangents of 10, 20, 30, 40, &c. from the Centre *b*, towards B, or beyond it upon Occaſion, or the Secants of the ſame Angles from N or S the ſame Way, and you will have the Centres of the Meridians that are to interſect the Equator between A and *b*.

2. With one Foot of your Compaſſes in any of theſe Centres, and the other extended to N or S deſcribe the Meridians, &c. In like manner you may find the Centers of all the Meridians between B and *b*, by ſetting the Tangents 10, 20, 30, 40, &c. Degrees from *b*, towards A, or beyond, or the Secants of the ſame Angles from N or S (as before) the ſame Way, and draw them at the Diſtance of thoſe Centres from N or S.

To draw the Ecliptic A ♋ B and B ♑ C, which makes an Angle with the Primitive Circle of 66° : 31′ = S B ♑.

TAKE the Tangent of 66° : 31′, and lay it from *b* to ♎ and from *d* to *o*; or the Secant of the ſame Angle, laid from A or B, will reach to the ſame Points; and theſe are the Centres of the North and South Part of the Ecliptic: Upon them deſcribe the Arch A ♋ B for the North Part, and B ♑ C for the South Part. *Note*, That *o* is ſuppoſed to be as much above *d*, as ♎ is below *b*.

To divide the ECLIPTIC *into Signs*, &c.

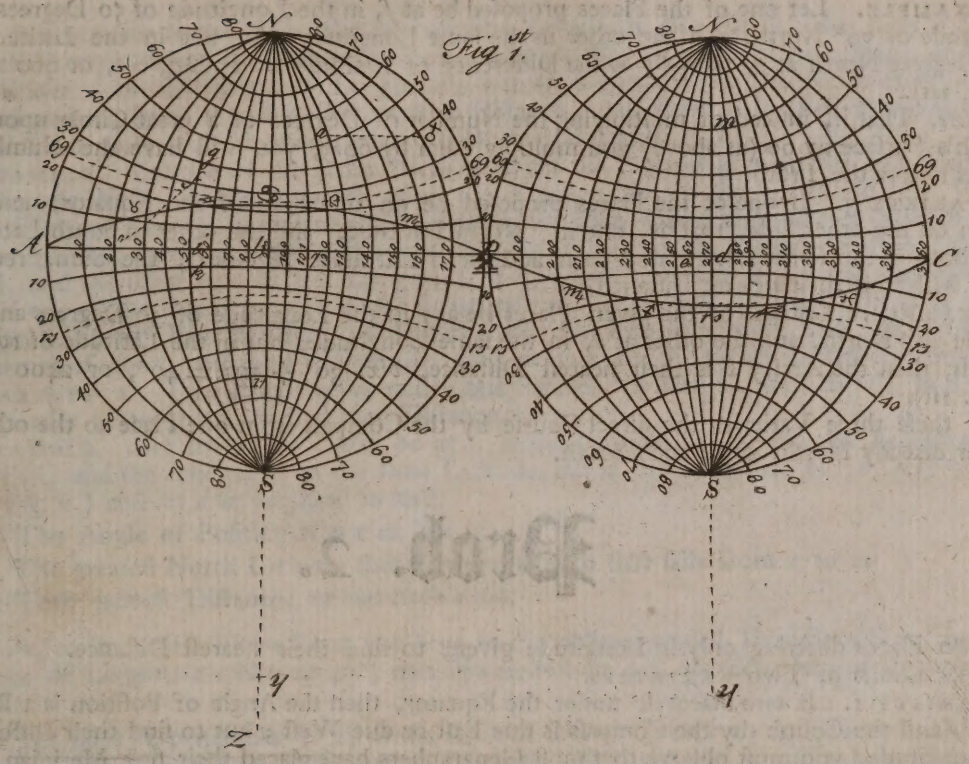
1. By Caſe the 3d of Prob. 1ſt, Chap. 11. find the Pole of the North Part at *n*, and of the South Part at *m*.

2. LAY a Ruler upon *n*, and 30 Degrees and 60° of the Primitive Circle in the Quadrants A N and B N; and it will cut the North Part of the Ecliptic in ♉, ♊, ♌ and ♍.

AGAIN, a Ruler laid upon *m*, the Pole of the South Part of the Ecliptic, and 30 and 60 Degrees of the Primitive, in the Quadrants S B and S C, will cut that Part of the Ecliptic in ♏, ♐, ♒ and ♓. And thus you may divide the Ecliptic into Degrees, &c.

The *Tropics* of *Cancer* and *Capricorn* are drawn the ſame Way as were the Parallels of Latitude, but at the Diſtance of 23° : 29′ from the Equator.

Great



Great Circle Sailing being the trueſt Way of directing a Ship from one Place to another, when there is room to put it in Practice by keeping to the Arch of a great Circle that paſſes over both Places, ſo by having the Latitudes and Longitudes of any two Places given, then how to find their neareſt Diſtance, *&c.* will be eaſily known by what follows.

Prob. I.

If the two Places propoſed lie under the ſame Meridian, then the Arch of their Difference of Latitude is the Arch of their neareſt Diſtance.

This admits of three Varieties.

Variety 1. If one Place is under the Equator, and the other in the Northern or Southern Hemiſphere, then the Latitude of the Place, that is in the Northern or Southern Hemiſphere, is the Difference of Latitude; which being reduced into Miles, gives their neareſt Diſtance.

Example. Let one of the given Places be upon the Equator at *i*, in the Longitude of 50 Degrees, and the other at *f*, in the ſame Longitude, or under the ſame Meridian, but in the Latitude of 30 Degrees North, here the Arch of neareſt Diſtance is $if = 30$ Degrees, or 1800 nautical Miles (See Fig. 1ſt).

Variety 2. If the Places propoſed be on the ſame Side of the Equator, or both in the Northern, or both in the Southern Hemiſphere, and under the ſame Meridian, then ſubtract the leſſer Latitude out of the greater; and what remains will be the Difference of Latitude; which, being reduced into Miles, will be their neareſt Diſtance.

EXAMPLE. Let one of the Places propoſed be at *f*, in the Longitude of 50 Degrees, and Latitude of 30° North, and the other in the ſame Longitude alſo, but in the Latitude of 45 Degrees North at *g*, then *f g* is the Difference of Latitude = 15 Degrees, or 900 Miles (Fig. 1ſt).

Note, That if, inſtead of multiplying the Number of Degrees of a great Circle upon the Earth's Surface by 60 (as above) you multiply them by 69.5, you will have the Number of Miles nearer the Truth.

VARIETY 3. If one of the Places propoſed be on one Side of the Equator, and the other on the other Side, that is, one in North Latitude, and the other in South Latitude, and both in the ſame Longitude, then add both Latitudes together; the Sum, reduced into Miles, is their neareſt Diſtance.

EXAMPLE. Let one of the given Places be at *f* in the Longitude of 50 Degrees and Latitude 30° North, and the other at *k*, in the ſame Longitude, but in the Latitude of 10° : 0′ South, then the Arch *f k* is their neareſt Diſtance, *viz.* 30° + 10° = 40°, or 2400 Miles (Fig. 1ſt).

IN theſe three Varieties the direct Courſe by the Compaſs from one Place to the other is either directly North, or directly South.

Prob. 2.

Two Places differing only in Longitude given, to find their neareſt Diſtance.

This admits of TWO VARIETIES.

VARIETY 1. If two Places lie under the Equator, then the Angle of Poſition is a Right-angle, and the Courſe by the Compaſs is due Eaſt or due Weſt; but to find their Difference of Longitude, you muſt obſerve that moſt Geographers have placed their firſt Meridian over *Pico Teneriffa*, ſome over *Faro*, and thence count the Longitude round the Equator Eaſtward to 360 Degrees, which terminates at the Meridian at which they began; and in this manner Fig. 1ſt. is numbered upon the Equator with 10, 20, 30, 40, 50, *&c.* in the Hemiſphere A N B S, and ſo to 360 at the Point C in the Hemiſphere B N C S; which Point C is ſuppoſed to be the ſame with the Point A: And this is called the Old Way of counting Longitude.

OTHERS place their firſt Meridian at the Metropolis of their own Nation, and count the Longitude from that Meridian upon the Equator both Ways, that is, Eaſtward and Weſtward to 180 Degrees, ending at the oppoſite Part of the ſame Meridian, as it is numbered at the next Figure following: And this is call'd the New Way.

RULE.

1. ACCORDING to the old Way of counting Longitude (ſee Fig. 1ſt) ſubſtract the leſſer Longtitude from the greater, the Remainder is the Difference required, but if the Remainder be more than 180 Degrees, take it out of 360°, and what remains will be their neareſt Diſtance.

EXAMPLE. Suppoſe one of the given Places to be upon the Equator at *i*, in the Longitude of 50 Degrees, and the other upon the Equator at *q*, in the Longitude of 110 Degrees; then from 110° take 50°, and there will remain 60 Degrees equal to the Arch *i q*, which reduced is 3600 Miles.

AGAIN, let one of the Places be that at *i*, in the Longitude of 50 Degrees, and the other at *e*, in the Longitude of 250 Degrees; then from 250° take 50°, and there will remain 200°, which, being more than 180°, muſt be taken out of 360°, and there will remain 160 Degrees, which reduced is 9600 Miles.

2. Ac-

2. ACCORDING to the New Way of counting Longitude (see Fig. 2.) the Rule is thus: If both Longitudes be East, or both West, subtract the lesser from the greater, and the Remainder is the Difference of Longitude; but when one is East, and the other West, add them; and the Sum, if under 180°, is their Distance; but when it is more than 180° subtract it from 360°, and the Remainder will be their Distance.

EXAMPLE. Let one of the given Places be under the Equator at *q*, in the Longitude of 70 Degrees West, and the other at *i* upon the Equator also, in the Longitude 130° West; then from 130 take 70, and there will remain 60° = 3600 Miles, their nearest Distance.

AGAIN, Let one of the given Places be at *q* (as before) in the Longitude of 70 Degrees West, and the other at *e* in the Longitude of 70° East, the Sum is 140° = 8400 Miles; but if one of the Places be at *i*, in the Longitude of 130° West, and the other at *e*, in the Longitude of 70° East, the Sum of these is 200°; but this being more than 180°, must be taken out of 360°, and there will remain 160° = 9600 for the nearest Distance required.

VARIETY 2. Two Places being under one Parallel, or both in one Latitude, and their Longitudes being given, to find their Distance, *&c.*

EXAMPLE. Let one of the Places be at *o*, in the Latitude of 50° N. and Longitude of 30° East, and the other at *v*, in the same Latitude, but in the Longitude of 70 Degrees East (see Fig. 2.) and let it be required to find,

1. The Angle of Position N *v z* = N *o z*.
2. The greatest North Latitude that a Ship must take that sails from *o* to *v*.
3. Their nearest Distance, or the Arch *o z v*.

Now, to find these Three Parts, you have in the oblique-angled Triangle *o* N *v*, the Difference of Longitude *o* N *v* = 40°, and N *o* = N *v* = 40°, the Complement of the Latitude.

1. Let fall a Perpendicular N *z*, to the Arch of a great Circle *o v*; and that will divide the oblique Triangle into two Right-angled Triangles *o* N *z* and *v* N *z*: This being done, you have in the Right-angled Triangle N *v z* the Angle *v* N *z* = 20° : 0′ and Hypothenuse N *v* = 40°, to find the Angle of the Position N *v z* thus,

S. *c.* N *v* = 40° + R	19.884254
—— T. *c.* of *v* N *z* = 20° : 0′	10.438934
Is equal to T. *c.* of N *v z* = 74° : 25′	2.445320

Which is E. N. E. + 6° : 55′ easterly from *v* to *o*, or W. N. W. + 6° : 55′ westerly from *o* to *v*.

2. To find N *z*, the Complement of the greatest North Latitude, it will be

S. *c.* of *z* N *v* = 20° : 0′ + R	19.972986
—— T. *c.* of N *v* = 40° : 0′	10.076186
Is equal to the Tangent of N *z* = 38° : 15′	9.896800

The Complement of which is 51° : 45′: And this is the greatest North Latitude that a Ship bound from *o* to *v* ought to make.

3. To find the nearest Distance of the Places *o* and *v* in the Arch of a great Circle *o z v*.

IN the Right-angled Triangle N *v z* you have N *v*, and the Angle *v* N *z* as before: Then to find *v z*, it will be

As Radius	10.
Is to the Sine of N *v* = 40° : 0′	9.808067
So is the Sine of *v* N *z* = 20° : 0′	9.534052
To the Sine of *v z* = 12° : 42′	9.342119

Which, being doubled, is 25° : 24′ the Distance required, equal to 1524 Miles.

Note,

Note, That the Length of the Parallel from o to v may be found thus:

As Radius	10.
Is to the S. *c.* of the Latitude given, *viz.* 50° : 0′	9.808067
So is the Difference of Longitude 40° = 2400 Miles	3.380211
To the Length of the Arch $o\,v$ = 1543 Miles	3.188278

Prob. 3.

Two Places differing both in Latitude and Longitude, having their Latitudes and Longitudes given, to find

1. The Arch of their neareſt Diſtance.
2. The Angles of Poſition.

This Problem admits of Three Varieties, *viz.*

1. When one of the Places is under the Equator, and the other in the Northern, or Southern Hemiſphere.

2. When both the Places are in the ſame Hemiſphere.

3. When the Places propos'd are in different Hemiſpheres; that is, one in the Northern Hemiſphere, and the other in the Southern.

Variety 1. Let one of the given Places be under the Equator at ♀ in the Longitude of 20 Degrees (ſee Fig. 1.) and the other at f, in the Latitude of 30 Degrees North, and in the Longitude of 50 Degrees; draw the Arch of a great Circle ♀ f, over the given Places, and it will complete the Right-angled Triangle ♀ $f\,i$.

In which you have ♀ i = 30° the Difference of Longitude, and $f\,i$, the Latitude of the Place at f = 30° to find the Angles of Poſition ♀ $f\,i$, and N ♀ f, thus,

The Sine of $f\,i$ = 30° + R is	19.698970
— The Tangent of ♀ i = 30°	9.761439
Remains the *t c* of the Angle ♀ $f\,i$ = i ♀ f = 49° : 6′	9.937531

Here the Angle of Poſition ♀ $f\,i$ is 49° : 6′; which, according to the Compaſs, is S. W. + 4° : 6′ weſterly; and the Complement thereof is 40° : 54′ for the Angle N ♀ f, or the Angle of Poſition from ♀ to f, or N. E. *b.* N. + 7° : 9′ eaſterly.

For the Diſtance ♀ f	S. *c.* $f\,i$ = 30°	9.937531
	+ S. *c.* ♀ i = 30	9.937531
The Sum want. Radius is the S. *c.* of ♀ f = 41° : 25′		9.875062

This reduced into Miles is 2485, the neareſt Diſtance of the two given Places.

Variety 2. Let one of the given Places be at a, in the Latitude of 50 Degrees N. and Longitude 130 Degrees, and the other at ♂ in the Latitude of 40 Degrees North, and Longitude 170° : 0′ to find the Angles of Poſition N a ♂ and N ♂ a, and alſo their neareſt Diſtance a ♂. (See Fig. 1).

Draw the Arch of a great Circle a ♂ over the given Places, and it will complete the oblique angled Triangle N a ♂, of which you have the Sides N a = 40° and N ♂ = 50° (being the Complements of the given Latitudes) and alſo the included Angle a N ♂ = 40° the Difference of Longitude, to find.

The Angles N a ♂ and N ♂ a, and this you may do by obſerving a Rule at Caſe the 4th of oblique angled ſpherical Triangles, according to which a third Example is given for ſolving that Caſe.

N ♂ = 50°
N *a* = 40
Sum —— 90 half is —— —— 45° : 00′
Difference 10 half is —— —— 5 : 00
The Angle *a* N ♂ = 40° half is —— —— 20.00

1. As S. *c.* of 45° : 0′ *co. ar.* —— —— —— —— 0.150515
Is to the S. *c.* of 5° : 0′ —— —— —— —— 9.998344
So is the T. *c.* of 20° —— —— —— —— 10.438934

To the Tangent of 75° : 31′ = half the Sum of the Angles ſought —— 10.587793

2. As the Sine of half the Sides 45° : 0′ *co. ar.* —— 0.150515
Is to the Sine of half their Diff. 5° : 0′ —— —— 8.940296
So is the Tangent of half the contain'd Angle 20° : 0′ —— —— 10.438934

To the Tangent of —— —— —— 18° : 42′ — 9.529745
Half the Sum of the Angles required —— 75 : 31

The Sum is the greater of the Angles, *viz.* N *a* ♂ = —— 94 : 13
And their Difference is the Leſſer, *viz.* N ♂ *a* = —— 56 : 49
The Angle N *a* ♂ being 94° : 13′, its Supplement is 85° : 47′ = S *a* ♂, or E. *b.* S. + 7° : 2′ eaſterly, the Courſe from *a* to ♂.
And the Angle of Poſition N ♂ *a*, is 56° : 49′, or N. W. *b.* W. + 0° : 34′ weſterly, the Courſe from ♂ to *a*.
To find the Diſtance *a* ♂ you may ſay,
As the Sine of the N ♂ *a* = 56° : 49′ *co. ar.* —— 0.077314
Is to the Sine of N *a* = 40° : 0′ —— —— 9.808067
So is the Sine of the Angle *a* N ♂ = 40° : 00′ —— —— 9.808067

To the Sine of *a* ♂ = 29° : 35′ —— —— 9.693448
This reduced to Minutes or Miles is 1775.

VARIETY 3. Let one of the given Places be at *a*, in the Latitude of 40° : 0′ North, and Longitude 30° : 0′ Eaſt, and the other in the Latitude of 30° : 0′ South, and Longitude 80 Degrees Eaſt at *b*, then the Lifference of their Longitudes is 50° : 0′ equal to the Angle *a* N *b*. (See *Fig.* 2.)

FIRST find the Angles N *a b*, and N *b a*, according to the Work of the laſt Variety. Thus,
The Side —— N *b* = 90° + 30° = 120° : 0′
The Side —— N *a* = — 50 : 0

Their Sum is —— —— 170 : 0 half is = 85° : 00′
And Difference —— —— 70 : 0 half is = 35 : 00
Included Angle *a* N *b* is —— 50 : 0 half is = 25 : 00
Now ſay, As the S. *c.* of 85° : 0′ —— *co. ar.* —— —— 1.059704
Is to the S. *c.* of 35° : 0′ —— —— —— 9.913364
So is the T. *c.* of 25° : 0′ —— —— —— 10.331327

To the Tangent of 87° : 20′, half the Sum of the Angles ſought —— 11.304395

AGAIN

AGAIN, As the Sine of 85° : 00′	*co. ar.*		0.001656
Is to the Sine of 35° : 0′			9.758591
So is the T. *c.* of 25° : 0′			10.331327
To the Tangent of half the Difference of the Angles ſought 51° : 0′			10.091574
Half the Sums of the Angles required is	87 : 10		
Their Sum is the Angle N *a b* =	138 : 10		
And their Difference is the N *b a* =	36 : 10		

The Supplement of 138° : 10′ is 41° : 50 = S *a b*, or S. E. *b*. S. + 8° : 5′ eaſterly.
The Angle N *b a* is 36° : 10′, or N. W. *b*. N. + 2° : 25′

To find the Diſtance of the two Places, or the Arch *a b*, ſay,

As the Sine of the Angle N *b a* = 36° : 10′	*co. ar.*		0.229048
Is to the Sine of the Side N *a* = 50′ : 0′			9.884254
So is the Sine of the Angle *a* N *b* = 50° : 0′			9.884254
To the Sine of *a b*, the Diſtance required = 83° : 55′			9.997556

This reduced to Miles is 5035.

THE Diſtance between the two Places *a* and *b*, being found, it may be farther required, to know the Latitudes, and what Courſes a Ship muſt ſhape to keep to the Arch *a b*, at every 10 Degrees of Longitude from *a* to *b*.

HAVING found the Angle N *a b* = 138° : 10′, its Supplement will be the Angle N *a n*, this being found, draw N *n* perpendicular to the Great Circle *n a b x*, produced, and it will complete the Right-angled Triangle N *n a*, in which you have the Angle N *a n* = 41° : 50′, and the Hypothenuſe N *a* = 50° : 0′ (the Complement of the Latitude of the Place at *a*.) by theſe you will find.

1. The vertical Angle *n* N *a* = 60° : 5′
2. The Side N *n* —— 30 : 43
3. The Side *n a* —— 41 : 36

To the vertical Angle add 10°, 20°, 30°, and 40°, then you will have the

Angles { *n* N *c* = 70° : 5′, *n* N *d* = 80 : 5, *n* N *o* = 90 : 5, *n* N *t* = 100 : 5 } of the Triangles { *n* N *c*, *n* N *d*, *n* N *o*, *n* N *t* }

By Help of theſe Angles, and the Side N *n* = 30° : 43′, common to all the Triangles, you will find

		Miles
1. The Side —— *n c* = 54° : 39′	and *n c* − *n a* = *a c* = 13° : 3′ =	783
2. The Side —— *n d* = 71 : 06	conſequently *c d* = 16 : 27 =	987
3. The Side —— *n o* = 90 : 10	*d o* = 19 : 04 =	1144
4. The Side —— *n t* = 109 : 12	*o t* = 19 : 02 =	1142
and —— *n b* = 125 : 36, then	*t b* = 16 : 24 =	984

In the ſame Triangles you have the Angles as above, and alſo the common Side N *n* = 30:43′ by theſe you will find

The Angles { N *c n* = S *c x* = 36° : 4′, N *d n* = S *d x* = 32 : 8, N *o n* = S *o x* = 30 : 43, N *t n* = S *t x* = 32 : 11 } or { S. E. *b*. S. + 2° : 19′, S. S. E. + 9 : 38, S. S. E. + 8 : 13, S. S. E. + 9 : 41 } Eaſterly.

Laſtly,

Laſtly, by the ſame *Data* you will find the

Sides	N c = 60° : 10′	Their Complements are	29° : 50′ = Lat. at c.	North.
	N d = 73 : 50		16 : 10 = Lat. at d.	
	N o = 90 : 8 — 90° =	——	0 : 8 = Lat. at o	South
	N t = 106 : 25 — 90 =	——	16 : 25 = Lat. at t	

If the ſame Things had been required at every 5th Degree of Longitude, or leſs, the Method would have been the ſame.

Fig. 2

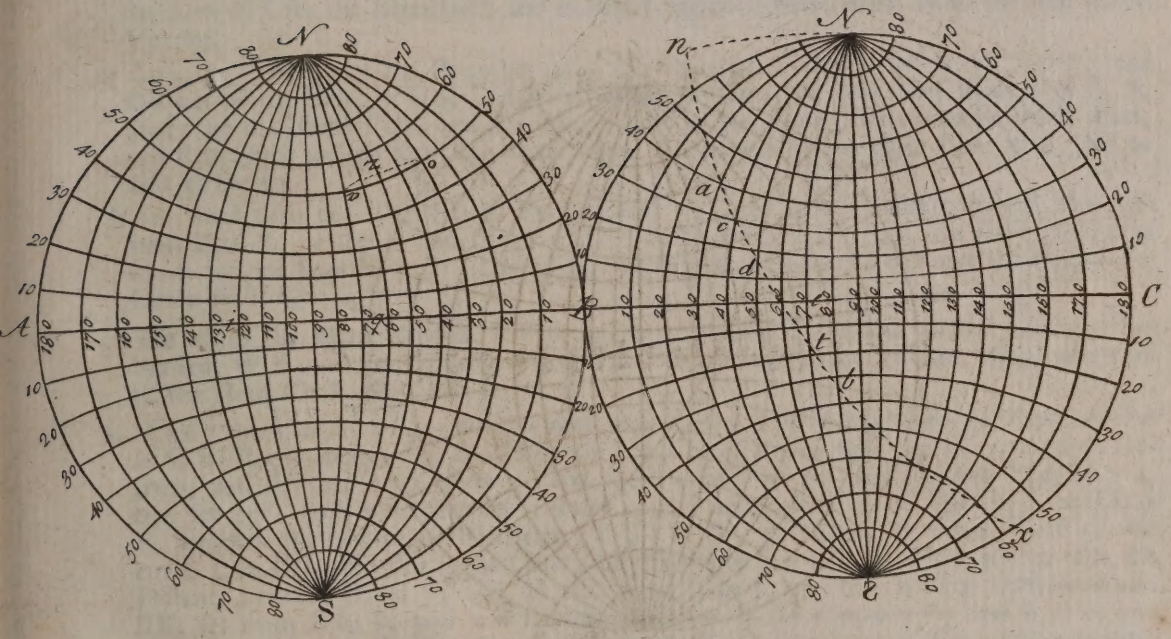
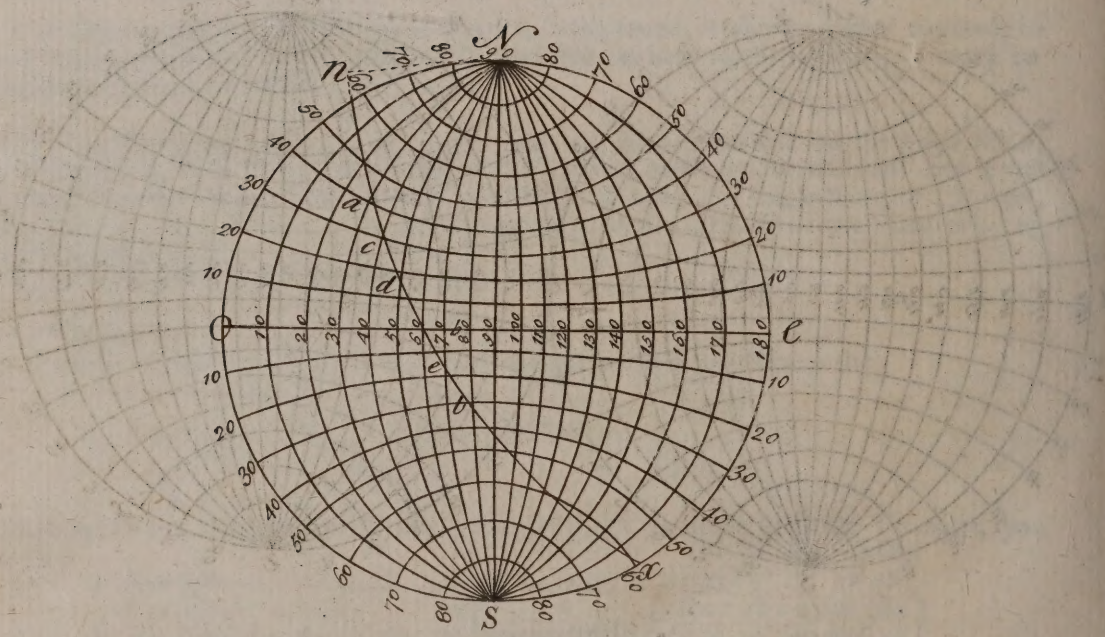


Fig.3



Dialling.

THE great Use of this curious Branch of the Mathematicks has induced many great Men to write upon it, and so well, that it may be thought great Presumption to write upon the same Subject after them; for which Reason I could hardly persuade myself to meddle with it; but as it lay so fair in my Way, and to make the Book as useful to the Reader as I could, I have ventured to touch upon some of the most useful Parts, and collect from some of the best Authors what I thought would be most agreeable to a Learner, both in the calculative and mechanical Parts, that so such as do not understand Trigonometry, may however make a Dial upon any Direct or Declining Plane.

To project the Sphere upon the Plane of the Horizon, suitable to any Latitude.

1. TAKE 60 Degrees in your Compasses from the Line of Chords, and placing one Foot in Z, with the other describe the Circle W N E S, and thro' the Center at Z draw the Line N S for the Meridian; and at Right-angles thereto, draw W E for the prime Vertical.

2. LET the Latitude of the Place be 51° : 32′, then its Complement is 38° : 28″; take this out of the Line of Half Tangents, and set it upon the Meridian from Z to P, so shall P be the North Pole, and the Point where all the Hour Circles intersect each other, and that which passeth thro' the three Points W P E shall be the Hour Line of 6, and to find its Centre.

YOU must take the Tangent of the Latitude 51° : 32′ and set that upon the Meridian from Z to B, or the Secant of 51° : 32′ laid from P, will reach the same Point, in which set one Foot of your Compasses, and extend the other to P, and then draw the Circle W P E.

3. **To draw the Hour Circles.** First draw the Line L L thro' B, and parallel to the prime Vertical W E out from B, both Ways at Discretion; upon this Line the Centers of all the Hour Circles will fall.

4. TAKE the Tangent of 45° and lay that Extent from P to M, and thro' M draw the Line M H at Right-angles with the Meridian; this being done, take the Tangent of 15 Degrees, and set that from M to 5, and the Tangent of 30, and set that from M to 4, *&c.* for 45°, 60°, 75°, then lay a Ruler upon P and 5, and it will cut the Line LL in V, which is the Centre of the Circle 5 P 5. Again, a Ruler laid upon P and 4, will cut L L in IIII, and this is the Centre of the Circle 4 P 4, *&c.* Now if you take the Distance between B and V, and set that from B to 7, and the Distance between B and IIII, set from B to 8, you will have the Centres of the Circles 7 P 7 and 8 P 8, and and so for the rest. Or if you make P B the Radius of the Line of Tangents upon your Sector, and thence take 15° : 30°, *&c.* and set these upon the Line L L from B, you will have the Points V and IIII as before.

5. **To draw the Equinoctial** W *æ* E, which is 90° from the Pole. Lay a Ruler upon E and P, and it will cut the Horizon at *o*, from *o* set of 90° to *p*, then a Ruler laid upon E and *p* will cut the Meridian at *æ*, the Point thro' which the Equinoctial must pass, then from *p* set 23° : 29′ to *v* and *w*. A Ruler laid upon E, and these Points will cut the Meridian in ♋ and ♑, the Points thro' which the Tropicks must pass. Or, rather thus: From the Line of half Tangents, take the Latitude of the Place 51° : 32″, and that being set upon the Meridian from Z, will reach to *æ*, the Point thro' which the Equinoctial must pass as before, then for its Centre, set the Tangent of 38° : 28′ (the Complement of the Latitude) from Z to *c*, and upon *c*, with the Radius *c æ*, draw the Equinoctical W *æ* E.

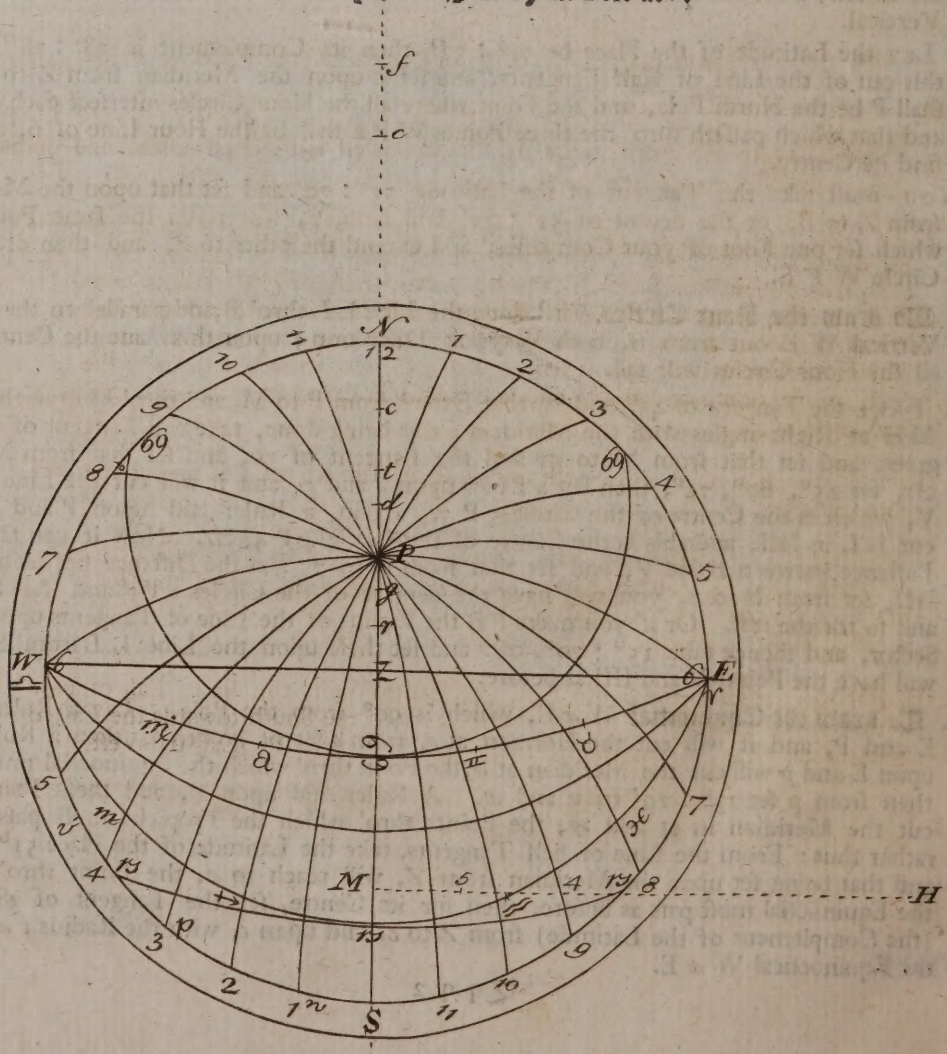
6. **To draw the Tropick of** *Cancer* **and** *Capricorn.*

ADD the Latitude of the Place 51° : 32′ to the Sum's greateſt Declination 23° : 29′ and the Sum is 75° : 1′, and the Difference 28 : 3′ ; then take 75° : 1′ from the Line of half Tangents, and ſet that extent upon the Meridian from Z to ♑, and the half Tangent of 28° : 3′ from Z to ♋, ſo will you have the Points thro' which the two Tropicks muſt paſs, as before ; and for their Centres, you muſt take the Tangent of 58° : 20′, and ſet that upon the Meridian from *z* to *e* ; upon which, with the Radius *e* ♑ deſcribe the Tropic of ♑. Again, take the Tangent of 27° : 47′, and ſet that from Z to *d*, for the Center of the Tropic of ♋, upon which draw it.

7. **To draw the Ecliptic.** Take the Tangent of 61° : 57′, and ſet it from Z to *f*, and the Secant of 61° : 57′ will give its Radius, with which the Circle W ♋ E, repreſenting the Northern half of the Ecliptic, may be deſcribed.

THE Southern half of the Ecliptic being further from the Zenith, than the Equinoctical, then 51° : 32′ + 23° : 29′ = 75° : 1′, take the half Tangent of 75° : 1′, and ſet that from Z towards S, and it will reach to ♑, and the Secant of the Complement of 75° : 1′ = 14° : 59′, ſet upon the Meridian from ♑, will fall upon *g*, which is the Centre ; upon which, with the Radius *g* ♑, draw W ♑ E.

Note, That you may divide W ♋ E into Signs and Degrees, by the Help of its Pole at *r*, and the Southern Part of the Ecliptic W ♑ E by its Pole at *r*.



CHAP. I.

ALL great Circles of the Sphere being projected upon a Plane, howſoever ſituated, become ſtraight Lines; from whence it follows, that the Hour Lines in every Dial being great Circles of the Sphere drawn upon any regular Flat, will become ſtraight Lines.

THE whole Art of Dialling doth conſiſt in finding out thoſe Lines, and their Diſtance from each other, which will vary according to the Situation of the Plane on which they are projected.

THAT Part of the Stile or Gnomon that ſhews the Hours muſt lie parallel to the Axis of the World, by which Means the End of the Stile points to one of the Poles of the World.

Dials, (according to *Clavius*) are diſtinguiſhed into the following Kinds, and are ſituated in one or other of theſe three Poſitions in reſpect of the Horizon of the Place in which the Dial is made; for all Planes are either

Parallel ——— }
Perpendicular, or— } to the Horizon of the Place.
Oblique —— —— }

1. PARALLEL to the Horizon are ſuch as are called Horizontal Dials, and commonly ſet upon a Pedeſtal in a Garden, &c.

2. PERPENDICULAR to the Horizon, are ſuch as are placed upon an upright Wall or Building, and theſe are of two ſorts, either direct or declining, if the Plane or Wall doth face directly to the true North, South, Eaſt or Weſt Points of the Horizon, the Dial made upon ſuch a Plane is called an Erect Direct North, South Eaſt or Weſt Plane.

BUT if the Wall or Plane do not directly reſpect one of theſe four cardinal Points of the Horizon, then it lieth open to two of them, for it muſt lie open to the South and the Eaſt, or the South and the Weſt, and then it is called an Erect South Plane, declining Eaſt or Weſt.

BUT if it lieth open to the North and Eaſt or Weſt, it is called an Erect North Plane, declining Eaſt or Weſt.

3. OBLIQUE (or reclining from the Zenith or inclining to the Horizon) theſe alſo are of two ſorts, that is, Direct Reclining or Declining and Reclining, for if the Reclining Plane do lie open to the North, South, Eaſt or Weſt Points of the Horizon, then it is called a Direct North, South, Eaſt or Weſt Recliner.

BUT if it lie open to the South Eaſt, or South Weſt, the North Eaſt or North Weſt, it is then called a Reclining Plane, declining from the South towards the Eaſt or Weſt, or a Reclining Plane, declining from the North towards the Eaſt or Weſt.

THOSE may be called Recliners whoſe upper Faces behold the Zenith of the Place, and the under Faces of them may be called Incliners.

CHAP.

CHAP. II.

TO know which Pole, whether the North or South, is to be elevated over any Dial Plane, in any Position, remember that the Stile of every Dial represents the Axis of the World, and therefore its Ends must point directly to the two Poles of the World.

GENERAL RULES.

1. UPON the horizontal Plane, in North Latitude, the North Pole; in South Latitude, the South Pole.

2. UPON erect Planes, whether direct or declining, if the Plane lie open to the South, the South Pole is elevated; but if it behold the North, the North Pole is elevated.

3. UPON direct East and West Planes reclining (how far soever) the North Pole is elevated; and upon the East and West Incliners, opposite to them, the South Pole.

4. OVER all North Reclining Planes, whether direct or declining, the North Pole is elevated, and over the Inclining Planes, opposite to them, the South Pole is elevated.

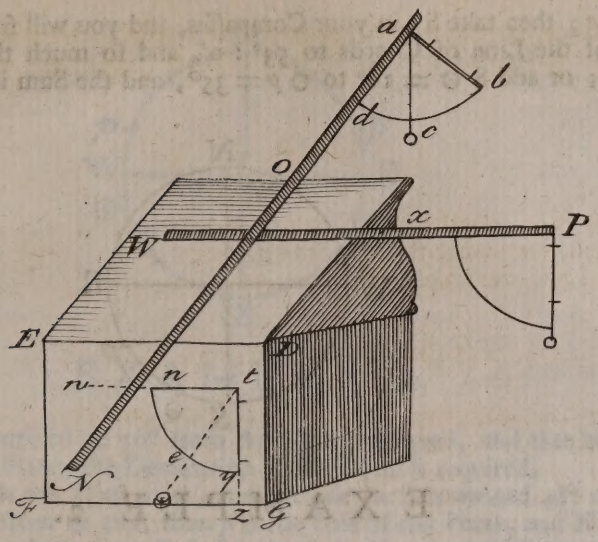
5. OVER all South Reclining Planes, whether direct or declining (if the Plane pass between the Zenith and Pole) the Axis of the Stile must have respect to the South Pole; and on the Inclining Planes, opposite to them, the North Pole must be elevated.

BUT if the Plane pass between the Pole and the Horizon, the North Pole, and on the Incliners opposite to them, the South Pole must be elevated.

CHAP. III.

To find the Inclination or Reclination of a Plane.

THE Inclination of a Plane is the Angle that it makes with the Horizon. When you have drawn the horizontal Line W X by the Help of a Level, or straight Ruler W P, and a Quadrant, and at Right-angles thereto draw the Line N O, upon which lay a Ruler N *a*, to the under Side of this apply a Quadrant, so that the Thread and Plummet may hang freely by the Side thereof; and when it resteth, Note what Degree of the Limb the Thread falls upon; for that's the Quantity of the Plane's Reclination, and here represented by the Arch *c d*.



CHAP. IV.

To find the Declination of a Plane.

THE beſt Inſtrument that I know of, for the Purpoſe, is *Collins*'s Quadrant; becauſe, by that you may find the Time of the Day to a Minute, and the Sun's Azimuth within a ſmall Matter of Truth.

THEN having ſuch an Inſtrument, find thereby the horizontal Line *w t* (ſee the laſt Figure) and to that apply one Side of the Quadrant, ſo that the Limb *n e y* may be towards the Sun, and the Inſtrument itſelf held parallel to the Horizon, as near as you can, then the Pole of the Plane is in the Line *t z*; and to know how many Degrees the Sun wants of coming to it, or is paſt, take a Thread and Plummet, and move them near the Limb until the Shadow of the Thread falls upon the Centre of the Quadrant, which it will do when it comes to *e*, then the Degrees contained between *e* and *y* is what the Sun is paſt the Pole of the Plane (in this laſt Caſe).

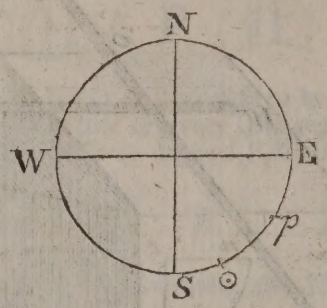
EXAMPLE 1.

1.) LET E F D G (See the laſt Fig.) be the Wall of a Building, upon which a Dial is to be made; then having done as above directed, and found the Arch *e y* to be ſuppoſe 35 Degrees, the Sun being ſo much paſt the Pole of the Plane, and his Azimuth at the ſame time, ſuppoſe 18 Degrees from the South Eaſtward, then

2. TAKE 60 Degrees in your Compaſſes, from a Line of Chords, and with that Extent deſcribe the Circle W N E S, and quarter it with the Meridian N S and prime Vertical W E, then from S ſet off the Sun's Azimuth 18° from S to ⊙ (towards the Eaſt); and becauſe the Sun is paſt the Pole of the Plane 35°, ſet 35° from ⊙ towards the Eaſt to *p*, ſo is *p* the

Pole

Pole of the Plane; then take S p in your Compaſſes, and you will find that Extent reach from the Beginning of the Line of Chords to 53° : 0′, and ſo much the Wall declines from the South Eaſtward; or add S ⊙ = 18° to ⊙ p = 35°, and the Sum is 53° : 0′ as before.

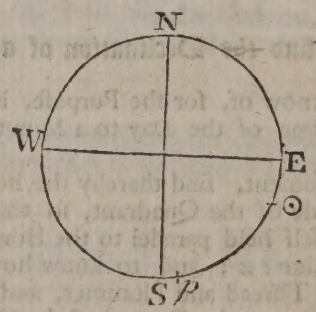


EXAMPLE 2.

Suppose, that by the former Method, I found the Sun to want of the Pole of the Plane 62°, and his Azimuth to be 70° from the South Eaſtward.

1. Describe the Circle W N E S and quarter it as before, then take the Sun's Azimuth 70°, and ſet it from S to ⊙ Eaſtward, becauſe 'tis Eaſtward.

2. Because the Sun wanted 62° of the Pole of the Plane, I ſet 62 from ⊙ to p, ſo is p the Pole of the Plane; then take S p in your Compaſſes to the Line of Chords, and that will give 8 Degrees for the Plane's Declination Eaſtward, or from 70° take 62, and the Remainder will be 8 Degrees as before

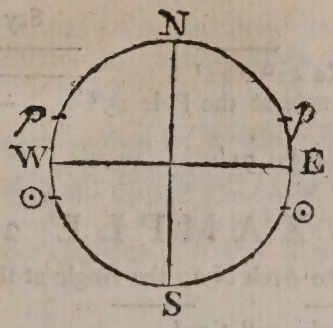


EXAMPLE 3.

Admit the Sun's Azimuth to be 70° from the South Eaſtward, as before, and that he is paſt the Pole of a Plane 46 Degrees, what's the Declination of this Plane?

The Horizon being drawn, &c. as above, take the Sun's Azimuth 70°, and lay that Extent from S to ⊙, then take 46° in your Compaſſes; and becauſe the Sun is paſt the Pole of the Plane ſo much, ſet that back to p, ſo is p the Pole of the Plane; this being done, take N p in your Compaſſes, and lay that Extent from the Beginning of the Line of Chords, it will give 64°, which is the Plane's Declination from the North Eaſtward, or to S ⊙ = 70° add ⊙ p = 46°, the Sum is 116°; the Supplement of which is 64° = N p.

E X-



EXAMPLE 4.

ADMIT the Sun's Azimuth to be 70° from the South weſtward, and that he wanted 46° of coming to the Pole of the Plane, the Declination of the Plane is required.

SET 70° from S. towards the Weſt to ⊙, and becauſe the Sun wanted 46° of coming to the Pole of the Plane, ſet 46° from ⊙ to *p*, then *p* is the Pole of the Plane, and N *p* its Declination from the North weſtward, which you will find to be 64°, (See the laſt Figure.)

CHAP V.

To draw an horizontal Dial.

DIALS by ſome are denominated from that Part of the Sphere where their Poles lie, and others choſe to denominate them from the Circles of the Sphere, to which the Plane is parallel, ſo that it is moſt commonly called an horizontal Dial, becauſe parallel to the Horizon.

THERE is but one Arch required to make an horizontal Dial, and that is the Latitude of the Place where it is to be ſet up, and in the fundamental *Diagram* is repreſented by N P, which, if the Dial is to be made for *London* is 51° : 32′.

THE primitive Circle W N E S repreſents the Plane, N P S, the Meridian Z, the Zenith P, the Pole of the World the Circles 1 P 1, 2 P 2, 3 P 3, *&c.* are Hour-Circles, from the Meridian their Diſtance upon the Horiſon being N 1, N 2, N 3, N 4, N 5, N 6.

Now to find theſe Arches, you have ſix Triangles, *viz.* N P 1, N P 2, N P 3, N P 4, N P 5, N P 6, all right angled at N, having one Side N P common to all, the Angles at the Pole encreaſing from the Meridian by 15 Degrees, if you calculate only for whole Hours, if for half Hours, then by 7° : 30′; but if for Quarters, then by 3° : 45′.

EXAMPLE 1.

SUPPOSE it was required to find the Arch N 1, or N 11, then in the Triangle N P 1, you have N P = 51° : 32′, and the Angle at the Pole N P 1 = 15° (See the ſecond Column of the following Table.)

	Say,		
As Radius			10.
Is to the Sine of the Latitude 51° : 32′			9.893745
So is the Tangent of the Angle at the Pole 15°			9.428052
To the Tangent of N 1 = 11° : 51′			9.321797

EXAMPLE 2.

LET it be required to find the Arch N 2, the Angle at the Pole being now NP 2 = 30°

Then as Radius			10.
Is to the Sine of the Latitude 51° : 32′			9.893745
So is the Tangent of 30°			9.761439
To the Tangent of the Arch N 2 = 24° : 19′			9.655184

AND in this manner you may find the reſt of the Hour Arches, and ſet them in the third Column of the Table againſt the Hours to which they belong.

Noon.	Angles at the Pole. D : 1	Angles at the Plane. D : 1	Hours.
	0 : 0	0 : 0	
NP 1 = NP 11	15 : 0	11 : 51	1 : 11
NP 2 = NP 10	30 : 0	24 : 19	2 : 10
NP 3 = NP 9	45 : 0	38 : 3	3 : 9
NP 4 = NP 8	60 : 0	53 : 36	4 : 8
NP 5 = NP 7	75 : 0	71 : 6	5 : 7
NP 6 = NP 6	90 : 0	90 : 0	6 : 6

To draw the Dial.

1. DRAW the Line VI VI for the Hour of Six, and from the Middle thereof A, draw A B at Right-angles thereto, and this ſhall be the Hour-line of 12.

2. TAKE 60° from the Line of Chords, and with one Foot of your Compaſſes in A, as a Center, with the other draw the Semicircle *q o r*.

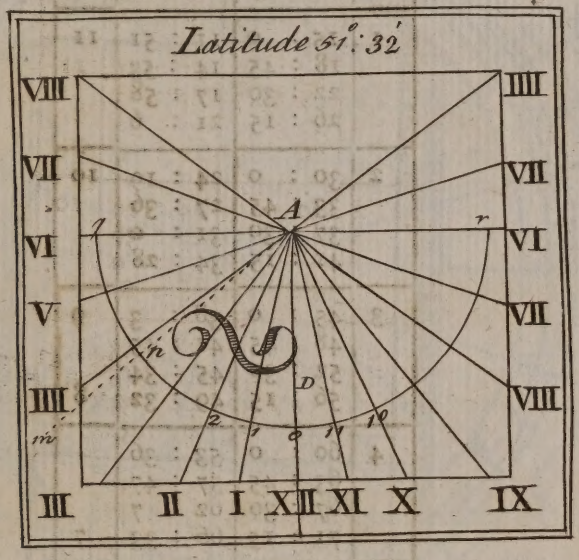
3. TAKE from the ſame Line of Chords the ſeveral Arches that you have in the third Column of the Table, and ſet them on both Sides of the Hour of 12, and thro' thoſe Points in the Semicircle draw the Hour-lines out as far as the Plane will admit.

THUS take 11° : 51′ in your Compaſſes from the Line of Chords, and ſet that from *o* to 1 and 11.

AGAIN, take 24° : 19′, and ſet that off from *o* to 2 and 10, and do the like for 3 and 9, 4 and 8, &c. then from the Centre A, and thro' the Points thus found, draw the Hour Lines

Lines I and XI, II and X, *&c.* but when you draw the Hour Lines for 8 and 7 in the Morning, or 4 and 5 in the Afternoon, continue them thro' the Centre, as you ſee in the Figure, and ſet the Figures to them.

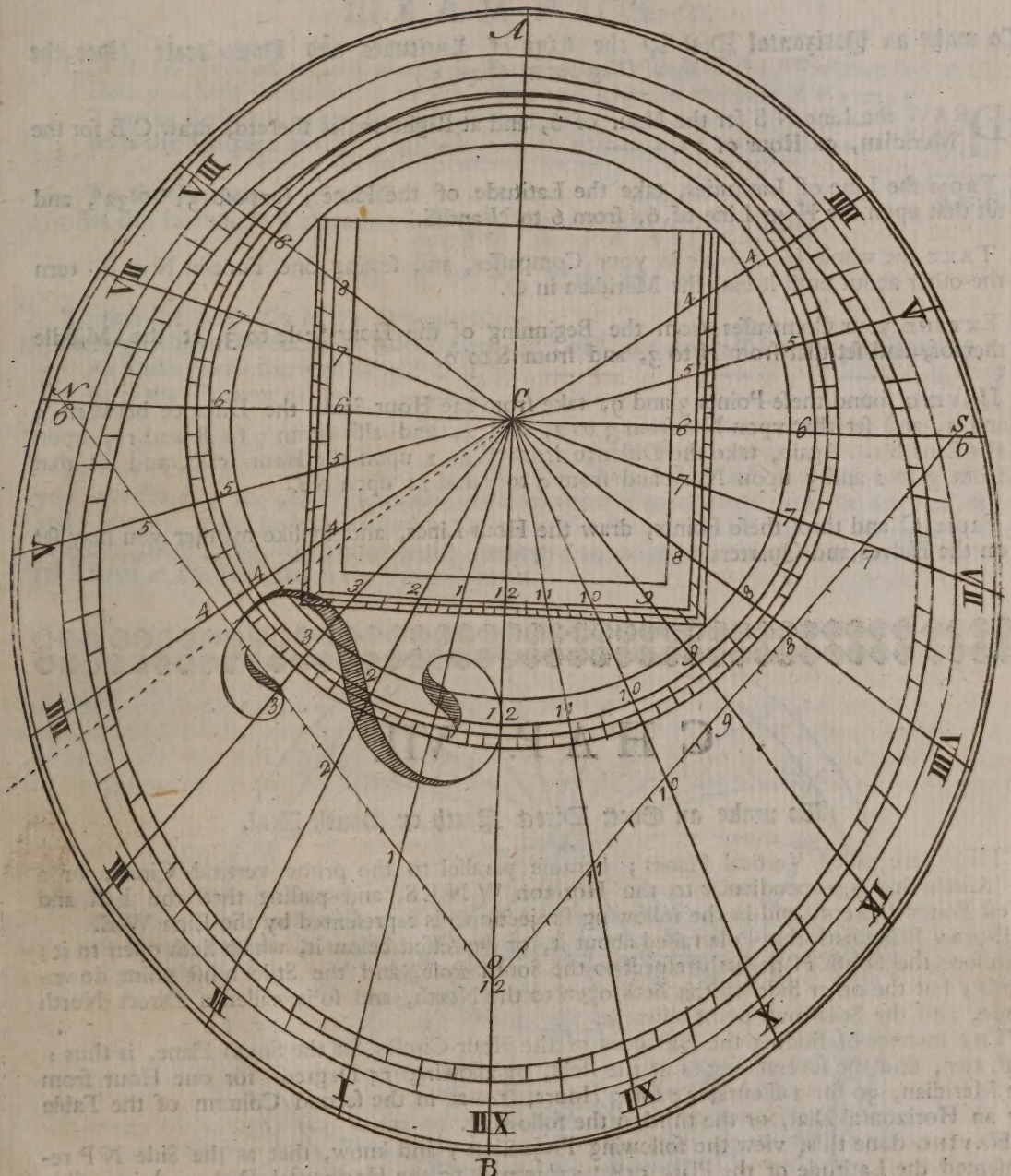
4. HAVING drawn all the Hour-lines, then for the Cock, Gnomon, or Stile, take the Latitude of the Place 51° : 32′ in your Compaſſes, and ſet that upon the Arch *q o r* from *o* to *n*, then from A thro' *n* draw the prick'd Line A *m*, this ſhall repreſent the Axis of the World ; and to be ſupported by ſomething like what you ſee in the Fig. *&c. y s* will be the Stile to be ſet upon the Meridian A XII at Right-angles to the Plane, and ſo the Dial will be finiſhed.



IF you would draw the Dial for Hours and Parts, as for Quarters or leſs, then divide 15 Degrees into ſuch Parts of an Hour as you would have, if for Hours and Quarters, make ſuch a Table as follows ; in the ſecond Column of which, you have the Angles at the Pole to every Quarter of an Hour ; and in the third Column, you have the Arches for the Times given ; and theſe are found by the former Analogy.

Latitude 51° : 32′			
Ho.	An. Po.	An. Pl.	Ho.
	D : 1	D : 1	
12	0 : 0	0 : 0	12
	3 : 45	2 : 57	
	7 : 30	5 : 53	
	11 : 15	8 : 51	
1	15 : 0	11 : 51	11
	18 : 45	14 : 53	
	22 : 30	17 : 58	
	26 : 15	21 : 6	
2	30 : 0	24 : 19	10
	33 : 45	27 : 36	
	37 : 30	31 : 0	
	41 : 15	34 : 28	
3	45 : 0	38 : 3	9
	48 : 45	41 : 45	
	52 : 30	45 : 34	
	56 : 15	49 : 32	8
4	60 : 0	53 : 36	
	63 : 45	57 : 47	
	67 : 30	62 : 7	
	71 : 15	66 : 33	7
5	75 : 0	71 : 6	
	78 : 45	75 : 44	
	82 : 30	80 : 27	
	86 : 15	85 : 13	
6	90 : 0	90 : 0	6

As for the Form of this, or any other Dial, that is commonly left to the Fancy of the Maker, whether in a Parallogram, a Square, a Circle, an Oval, &c. for that will not alter the Angles in whatever Shape the Dial is made; as may be seen by the following Figure.


A
B
C
N
S
I
II
III
IIII
V
VI
VII
VIII
IX
X
XI
XII
IIX
IIIX
IIIIX
VIIIX
VIIIIX
VIIIIX

CHAP. VI.

To make an Horizontal Dial by the Line of Latitudes and Hour-Scale (See the *Diagram* at *Chap.* 5.)

1. DRAW the Line N S for the Hour of 6, and at Right-angles thereto, draw C B for the Meridian, or Hour of 12.

2. FROM the Line of Latitudes, take the Latitude of the Place; suppose 51° : 32′, and set that upon the Hour Line of 6, from 6 to N and S.

3. TAKE the whole Hour-Scale in your Compasses, and setting one Foot in N or S, turn the other about until it cuts the Meridian in O.

4. EXTEND your Compasses from the Beginning of the Hour-Scale to 3, at the Middle thereof, and set that from N to 3, and from S to 9.

5. HAVING found these Points 3 and 9, take from the Hour-Scale the Distance between 3 and 2, and set that upon N *o* from 3 to 2, and 4, and also from 9 to 8 and 10, upon the Line S *o*. Again, take the Distance from 3 to 1 upon the Hour-Scale, and set that from 3 to 1 and 5 upon N *o*, and from 9 to 7 and 11 upon S *o*.

6. FROM C and thro' these Points, draw the Hour Lines, and in like manner you may set on the Halves and Quarters.

CHAP. VII.

To make an Erect Direct North or South Dial.

THESE are called Vertical Planes; because parallel to the prime vertical Circle, or a Circle that is perpendicular to the Horizon W N E S, and passing thro' the East and West Points thereof, and in the following Projection, is represented by the Line W E.

EVERY Plane hath that Pole raised about it, or depressed below it, which lieth open to it; therefore the South Plane hath respect to the South Pole, and the Stile must point downwards; but the other Side of this lieth open to the North, and so is called a Direct North Plane, and the Stile must point upwards.

THE manner of finding the Distances of the Hour-Circles for the South Plane, is thus:

FIRST, find the several Angles at the Pole, by allowing 15 Degrees for one Hour from the Meridian, 30 for 2 Hours, 45 for 3 Hours, *&c.* as in the second Column of the Table for an Horizontal Dial, or the third in the following.

HAVING done this, view the following Projection; and know, that as the Side N P represented the Latitude of the Place 51° : 32′ in making the Horizontal Dial, and was a Side common to all the Triangles N P 1, N P 2, *&c.* so P z, represents the Complement of the Latitude 38° : 28′, and is a Side common to all the Triangles Z P 1, Z P 2, *&c.* And to find

find the ſeveral Hour Arches of the Plane Z 1, Z 2, Z 3, *&c.* in order to fill up the 4th Column ſay,—As Radius

Is to the Sine of the Complement of the Latitude,
So are the Tangents of the Numbers in the 3d Column
To the Tangents of the Hour Arches in the 4th Column.

EXAMPLE 1.

LET it be required to find the Arch Z 1, in the Triangle Z P 1, Right-angled at Z.
Here you have the Side P Z = 38° : 28′, and Angle at the Pole Z P 1 = 15°.

Then as Radius is to the Sine of 38° : 28′ —— —— ——	9·793832
So is the Tangent of 15° : —— —— ——	9·428052
To the Tangent of 9° : 28′ —— —— ——	9·221884

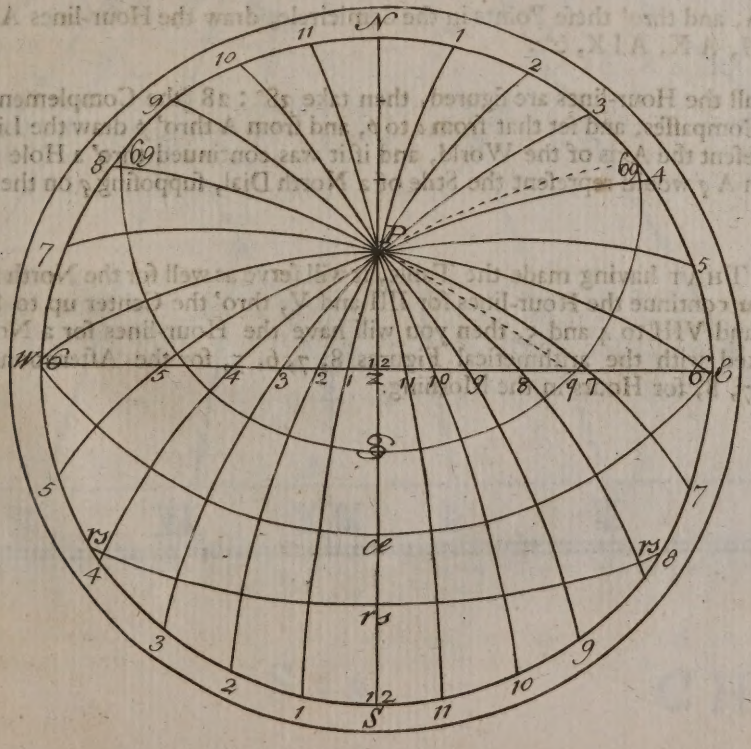
Set this in the 4th Column againſt 15°, or the Hours of 1 and 11.

EXAMPLE 2.

To find the Arch Z 2 in the Triangle Z P 2.
Here you have the common Side P Z = 38° : 28′, and Angle Z P 2 = 30°, then ſay,

As Radius is to the Sine of 38° : 28′ —— —— ——	9·793832
So is the Tangent of 30° —— —— ——	9·761439
To the Tangent of 19° : 45′ —— —— ——	9·555271

Set this in the 4th Column againſt 30°, or the Hours of 2 and 10, and ſo go on until you have filled up the 4th Column for whole Hours: If you would have it for half Hours and Quarters, make ſuch a Column as the 2d in the ſecond Table, for an Horizontal Dial, and then uſe the two firſt Terms in the Analogy, as above, and the Third muſt be the ſeveral Angles at the Pole, from the third Column.



	Ho.	Ang. Pole D : ′	Ho. Arches. D : o	Ho.
	12	o : o	o : o	12
Z P 1 = Z P 11	1	15 : o	9 : 28	11
Z P 2 = Z P 10	2	30 : o	19 : 45	10
Z P 3 = Z P 9	3	45 : o	31 : 53	9
Z P 4 = Z P 3	4	60 : o	47 : 8	8
Z P 5 = Z P 7	5	75 : o	66 : 42	7
Z P 6 = Z P 6	6	90 : o	94 : o	6

To draw the Dial.

1. DRAW the Line 6 A 6 for the Hour of 6, and from the Middle thereof at A draw A B at Right-angles to it.

2. TAKE 60 Degrees in your Compaſſes from a Line of Chords, and ſetting one Foot in A as a Center, with the other draw the Semicircle *w o r*.

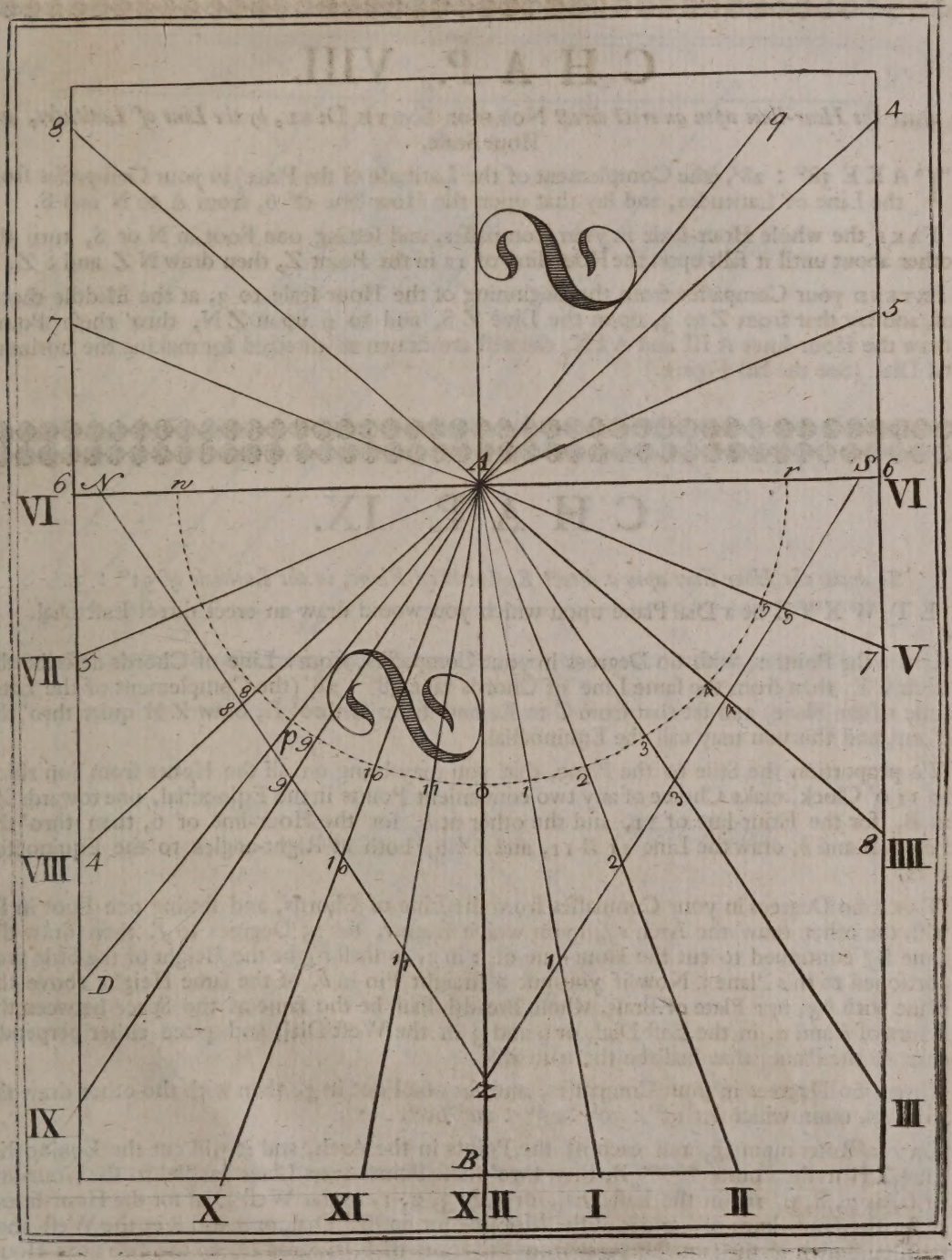
3. TAKE 9° : 28′ from the Line of Chords, and ſet that from O to 1 and 11.

4. TAKE 19° : 45 in your Compaſſes, and ſet it from O to 2 and 10, and in like manner, ſet 31′ 53′ from O to 3, and 9, *&c.*

5. FROM A, and thro' theſe Points in the Semicircle, draw the Hour-lines A I, A II, AIII, *&c.* and A XI, A X, A IX, *&c.*

6. WHEN all the Hour-lines are figured, then take 38° : 28 (the Complement of the Latitude) in your Compaſſes, and ſet that from *o* to *p*, and from A thro' *p* draw the Line A D, and this ſhall repreſent the Axis of the World, and if it was continued thro' a Hole in the Plane *up*, to *q*, then A *q* would repreſent the Stile of a North Dial, ſuppoſing *q* on the other Side of the Plane.

N. B. THAT having made the Table, it will ſerve as well for the North Dial as the South; for if you continue the Hour-lines for IIII and V, thro' the Center up to 8 and 7, and thoſe for VII and VIII to 4 and 5, then you will have the Hour-lines for a North Dial, ſuch as are marked with the arithmetical Figures 8, 7, 6, 5, for the Afternoon Hours, and with 4, 5, 6, 7, 8, for Hours in the Morning.


8
9
4
7
5
A
6
N
n
r
S
6
VI
VI
VII
5
8
9
p
9
10
11
0
1
2
3
4
5
5
7
V
VIII
4
10
11
1
2
3
8
IIII
D
IX
Z
III
B
X
XI
XII
I
II

CHAP. VIII.

To draw the Hour-lines upon an erect direct NORH or SOUTH DIAL, *by the Line of Latitudes, and* Hour Scale.

1. TAKE 38° : 28′, (the Complement of the Latitude of the Place) in your Compaſſes from the Line of Latitudes, and lay that upon the Hour-line of 6, from A to N and S.

2. TAKE the whole Hour-ſcale in your Compaſſes, and ſetting one Foot in N or S, turn the other about until it falls upon the Hour-line of 12 in the Point Z, then draw N Z and S Z.

3. EXTEND your Compaſſes from the Beginning of the Hour-ſcale to 3, at the Middle thereof, and lay that from Z to 3, upon the Line Z S, and to 9 upon Z N, thro' theſe Points draw the Hour-lines A III and A IX, the reſt are drawn as directed for making the horizontal Dial. (See the laſt Figure.)

CHAP. IX.

To draw the Hour-lines upon a direct Eaſt or Weſt Plane, in the Latitude of 51° : 32′.

LET W X Y Z be a Dial Plane upon which you would draw an erect direct Eaſt Dial.

1. UPON the Point z, with 60 Degrees in your Compaſſes, from a Line of Chords deſcribe the Arch C E, then from the ſame Line of Chords take 38° : 28′ (the Complement of the Latitude of the Place) and ſet that from C to E, and from Z thro' E, draw Z H quite thro' the Plane, and this you may call the Equinoctial.

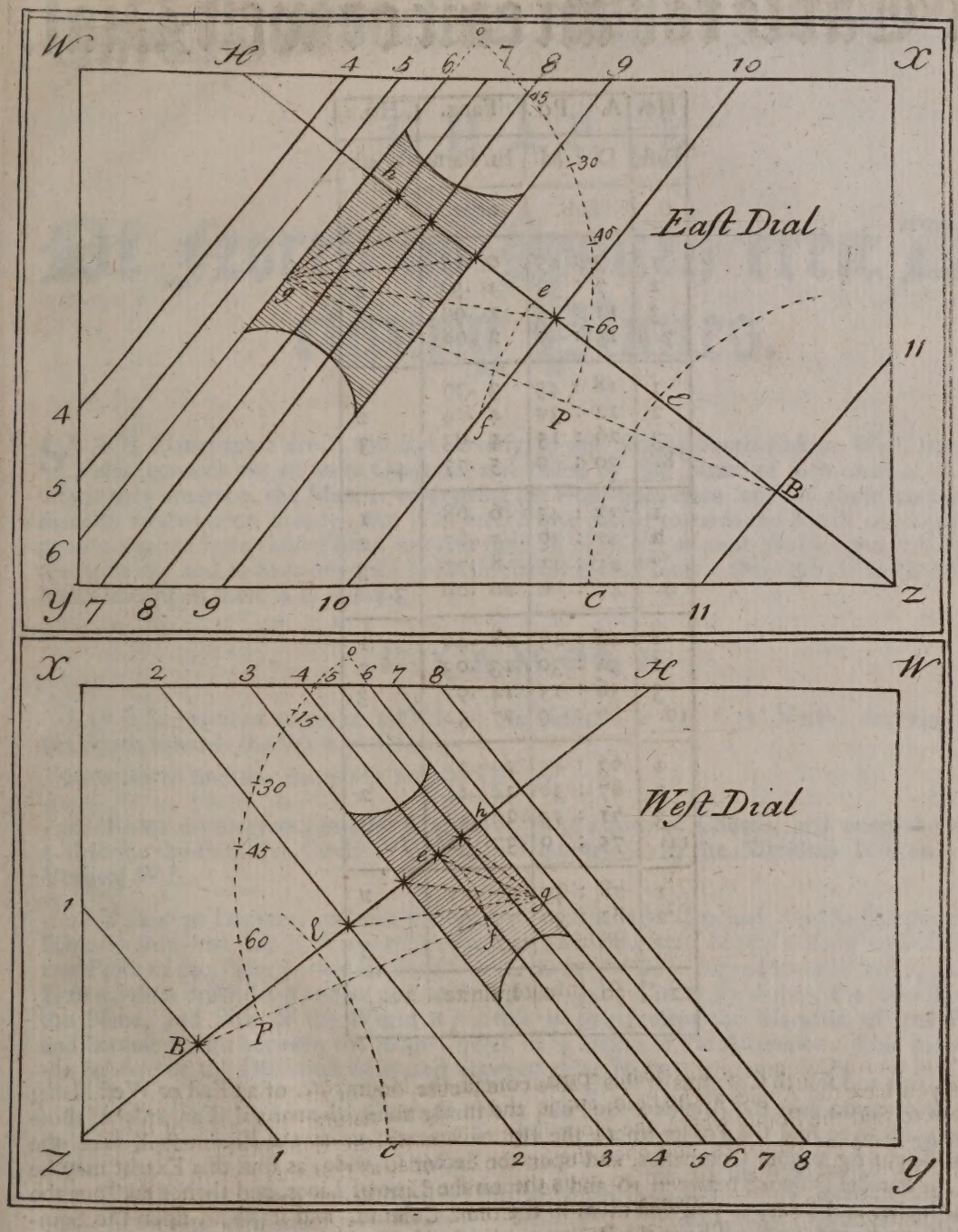
2. To proportion the Stile to the Plane, that you may bring on all the Hours from Sun riſing to 11 o' Clock, make Choice of any two convenient Points in the Eqinoctial, one towards Z, as B, for the Hour-line of 11, and the other at *b*, for the Hour-line of 6, then thro' the Points B and *b*, draw the Line 11 B 11, and 6 *b* 6, both at Right-angles to the Equinoctial B H.

3. TAKE 60 Degrees in your Compaſſes from the Line of Chords, and ſetting one Foot in B, with the other draw the Arch *e f*, upon which from *e*, ſet 15 Degrees to *f*, then draw the Line B *f* continued to cut the Hour-line of 6 in *g*, ſo ſhall *b g* be the Height of the Stile proportioned to this Plane: Now if you put a ſtraight Pin in *b*, of the ſame Height above the Plane with *b g*, or a Plate of Braſs, whoſe Breadth ſhall be the ſame as the Space between the Hours of 6 and 9, in the Eaſt Dial, or 6 and 3 in the Weſt Dial, and place either perpendicular to the Plane, that ſhall be the true Stile.

4. TAKE 60 Degrees in your Compaſſes, and ſet one Foot in *g*, then with the other draw the Arch *o p*, upon which ſet 15° : 30° : 45° : and 60°.

5. LAY a Ruler upon *g*, and each of the Points in the Arch, and it will cut the Equinoctial Line Z H in the Points **** B, then thro' theſe Points draw Lines parallel to the Hour-line of 6, as 7, 8, 9, 10, in the Eaſt Dial, or 5, 4, 3, 2, 1, in the Weſt Dial for the Hour-lines; as for the Hour-lines of 4 and 5 in the Morning for an Eaſt Dial, or 7 and 8 in the Weſt, they muſt be drawn at the ſame Diſtance from the Hour of 6, towards H, as the two next Hour-lines from 6 towards B is from the Hour-line of 6.

WHAT is here ſaid about making the Eaſt Dial, will ſerve for drawing the Hour-lines upon the Weſt Dial, only obſerve to ſet proper Figures to each, as in the Examples, the Directions ſerving for either.


East Dial
West Dial
W
H
X
Y
Z
C
P
B
h
e
f
g
l
0
15
30
45
60
4
5
6
7
8
9
10
11
1
2
3

A Table for an east or west Dial.

Ho.	A Po.	Tang.	Ho.
East.	D : M	In. Parts.	West.
6.	Sub.	Stile.	6.
1	3 : 45	0 .65	1
2	7 : 30	1 .32	2
3	11 : 15	1 .99	3
7.	15 : 0	2 .68	5.
1	18 : 45	3 .39	1
2	22 : 30	4 .14	2
3	26 : 15	4 .93	3
8.	30 : 0	5 .77	4.
1	33 : 45	6 .68	1
2	37 : 30	7 .67	2
3	41 : 15	8 .77	3
9.	45 : 0	10 .00	3.
1	48 : 45	12 .40	1
2	52 : 30	13 .03	2
3	56 : 15	14 .97	3
10.	60 : 0	17 .32	2.
1	63 : 45	20 .28	1
2	67 : 30	24 .14	2
3	71 : 15	29 .46	3
11.	75 : 0	37 .32	1.
1	78 : 45	50 .27	1
2	82 : 30	75 .96	2
3	86 : 15	152 .57	3
12.	90 : 0	Infinite	12.

THE first and fourth Columns of this Table contain the Hours, *&c.* of an East or West Plane; in the second you have the Angles at the Pole, and in the third, the natural Tangents of those Angles. Now to find the Points where the Hour-lines will cross the Equinoctial, take the Stile's Height *h g* in your Compasses, and open the Sector so wide, as that this Extent may be made the Parallel Distance between 10 and 10 upon the Line of Lines, and thence take out the several Inches and Parts, as you find them in the third Column, and set them upon the Equinoctial from *h*, and you will have the Points required, thro' which you may draw the Hour-lines as before.

CHAP.

CHAP. X.

Of North or South erect Declining Planes.

SUCH Planes as are erect, and face directly to the North, South-East or West, have their Poles towards one of those Quarters, and called by the Name of such cardinal Points to which they are open, the Manner of drawing the Hour-lines upon any of those has been sufficiently treated upon already; but if an erect Plane facing towards the North or South happens to decline from those Points towards the East or West (as most Walls upon which Dials are made do) and so have not their Poles in any of those Points; then the Way to draw the Hour-lines upon those is as follows.

EXAMPLE.

LET it be required to make a Dial in the Latitude of 51° : 32′ North, declining from the South towards the West 30 Degrees.

IN order to find the Requisites for this Plane,

1. TAKE 60 Degrees, in your Compasses, from a Line of Chords, and upon the Center z describe the Primitive Circle W N E S, and quarter it by the Meridian N S and prime Vertical W E.

2. TAKE 30 Degrees, and set that from S towards W (because the Declination of the Plane is from the South Westward) to p, and the same from N to d; so p and d are the two Poles of the Plane; then for the Plane itself, set the same Extent from W to w, and from E to x, then draw $d\,z\,p$ and $w\,z\,x$, and also the great Circle $p\,y\,d$ thro' the two Poles of the Plane, and Pole of the World at p; this is to represent the Meridian of the Plane; and because it falls between the Hour-Circles of 2 and 3 in the Afternoon: This shews that the Substile of the Dial must be placed between those Hours.

3. IN the Triangle $p\,y\,z$, Right-angled at y, you have given the Complement of the Latitude $p\,z = 38° : 28′$, and Complement of the Plane's Declination $p\,z\,y = 60$ Degrees. To find,

1. THE Distance of the Substile from the Meridian —— —— $y\,z$
2. The Stiles Heighth —— —— —— —— $p\,y$
3. The Difference of Longitude, or Inclination of Meridians —— $z\,p\,y$

THESE are called the Three Requisites in all Dials of this Sort, and are found by the Proportions following.

FOR

For the Diſtance of the Subſtile from the Meridian, ſay,

As Radius	10.
Is to the Sine of the Declination 30°	9.698970
So is the Tangent Comp. of the Latitude 51 : 32′	9.900086
So the Tangent of the Subſtile's Diſtance from the Merid. 21° : 40′ = *p z*	9.599056

For the Stiles Height, ſay,

As Radius	10.
Is to the *ſ. c.* of the Latitude 51 : 32′	9.793832
So is the *ſ. c.* of the Declination 30°	9.937531
To the Sine of the Stile's Heighth 32° : 36′ = *p y*	9.731363

For the Inclination of Meridians, ſay,

As the Sine of the Latitude 51° : 32′	9.893745
Is to Radius	10.
So is the Tangent of the Declination 30°	9.761439
To the Tangent of the Inclination 36° : 24′ = *z p y*	9.867694

This reduced into Time, is 2 Hours, 25′ 36″, and ſhews that the Subſtile will fall at 25′ and 36 Seconds after 2 in the Afternoon, agreeing with the Projection.

Now by the Projection, you may ſee, that when the Sun is in the Tropic of ♑, the Shadow of the Gnomon will appear at about a Quarter after 8 in the Morning, or as ſoon as the Sun riſeth; and that when he is in the Tropic of ♋, he will leave the South Side of the Plane at about 21′ : 4″ after Seven in the Afternoon; hence it appears what Hours will be proper to put upon this Plane, which you muſt place in the firſt Column of the following Table; and in the ſecond Column, againſt 12, place the Inclination of Meridians 36° : 24′. This being done, add 15° to 36° : 24′, and the Sum is 51° : 24′ to be ſet againſt 11 o'Clock. To this Sum, add 15°, and the Sum is 66° : 24′ to be ſet againſt 10 o'Clock; and ſo by adding 15°, you will find the Sums to be 81° : 24′ and 96° : 24′; whoſe Supplement is 83° : 36′ to be ſet againſt the Hours of 9 and 8.

Again, from 36° : 24′, ſubſtract 15°, there will remain 21° : 24′ to be ſet againſt 1 o'Clock; and from 21° : 24′ ſubſtract 15°, and there will remain 6° : 24′ to be ſet againſt 2 o'Clock: Now take the Difference between 15° and 6° : 24′, and that is 8° : 36′ to be ſet againſt 3; and if you add 15° and 8° : 36′ together, the Sum will be 23° : 36′. To this add 15° continually, until you have all the Angles at the Pole, unto 7 o'Clock; where you'll have 68° : 36′.

Having thus fill'd up the ſecond Column for the Angles at the Pole, you may then fill up the third Column, by the Analogy following, *viz.*

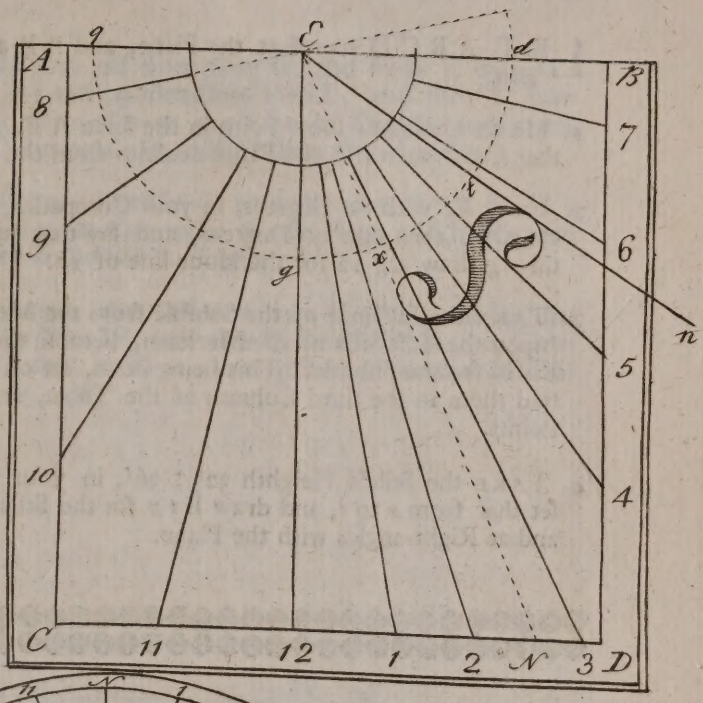
As Radius is to the Sine of the Stile's Heighth 32° : 36′	9.731404
So is the Tangent of 83° : 36′	10.950131
To the Tangent of 78° : 14′	10.681535

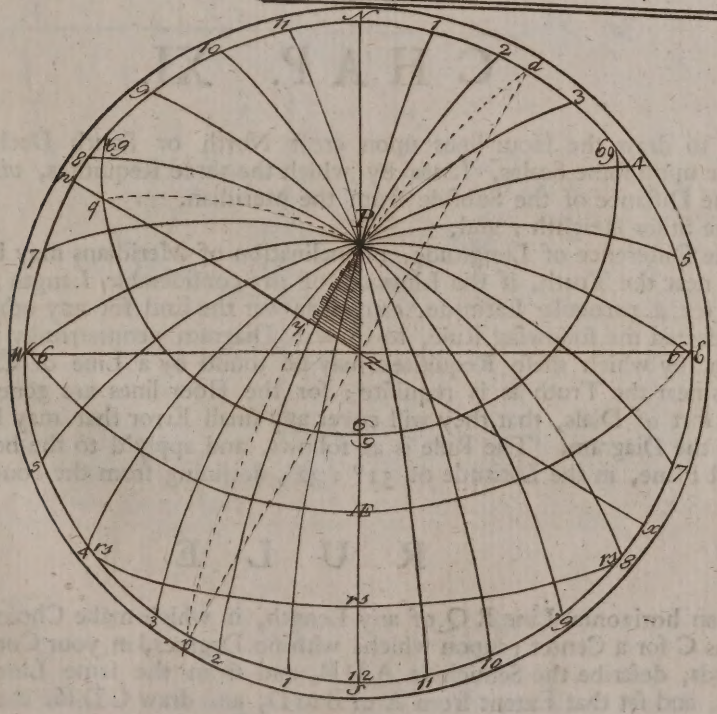
And the like for all the reſt.

The

The Table.

Ho.	Angl. Pole.	Ang. Plane.
	D : 1	D : 1
8	83 36	78 14
9	81 24	74 19
10	66 24	50 58
11	51 24	34 01
12	36 24	21 40
1	21 24	11 57
2	6 24	3 27
	Substile	
3	8 36	4 40
4	23 36	13 15
5	38 36	23 16
6	53 36	36 9
7	68 36	53 58





To draw the Hour-lines.

LET A B C D repreſent the Plane, and A B and C D Lines drawn parallel to the Horizon.

1. MAKE choice of ſome Point in the Line A B, as at E for a Center, and let it be nearer to that Coaſt to which the Plane declines from the South than the other.

2. UPON E, with 60 Degrees, in your Compaſſes, from a Line of Chords draw the Semicircle *q x d*, then take 90 Degrees, and lay that upon the Semicircle from *q* or *d* to *g*, and thro' *g* draw E *g* 12 for the Hour-line of 12.

3. TAKE the Diſtance of the Subſtile from the Meridian 21° : 40′, and lay that from *g* to *x*, (upon the Eaſt Side of the Meridian, becauſe the Declination is Weſt) then draw the Line E *x* N for the Subſtile. This being done, ſet off the ſeveral Hour Diſtances from *x*, as you find them in the third Column of the Table, and draw the Hour-lines from E thro' thoſe Points.

4. TAKE the Stile's Heighth 32° : 36′, in your Compaſſes, from the Line of Chords, and ſet that from *x* to *t*, and draw E *t n* for the Stile, which muſt ſtand upon the Subſtile E *x* N and at Right-angles with the Plane.

CHAP. XI.

IN order to draw the Hour-lines upon erect North or South Decliners inſtrumentally, there are upon ſome Scales, Lines by which the three Requiſites, *viz.*

1. The Diſtance of the Subſtile from the Meridian.
2. The Stiles Heighth; and,
3. The Difference of Longitude, or Inclination of Meridians may be readily found, and ſufficiently near the Truth, if the Lines are of any conſiderable Length; but ſuch Lines being made for a particular Latitude, cannot anſwer the End for any other. To ſupply this, we have inſerted the following Rule, to draw a Diagram geometrically for any Latitude and Declination, by which theſe Requiſites may be found by a Line of Chords of a ſufficient Length, as near the Truth as is requiſite; for the Hour-lines are generally drawn ſo bold upon theſe ſort of Dials, that they will cover any ſmall Error that may happen, either from a Scale, or the Diagram. The Rule is as follows, and apply'd to the before-going Example of an Erect Plane, in the Latitude of 51° : 32′, declining from the South Weſtward 30 Degrees.

RULE.

1. DRAW an horizontal Line R Q of any Length, in which make Choice of any convenient Point, as C for a Center; upon which, with 60 Degrees, in your Compaſſes, from a Line of Chords, deſcribe the Semicircle A D B, and from the ſame Line of Chords take 90 Degrees, and ſet that Extent from A or B to D, and draw C D for the Meridian, or Hour-line of 12.

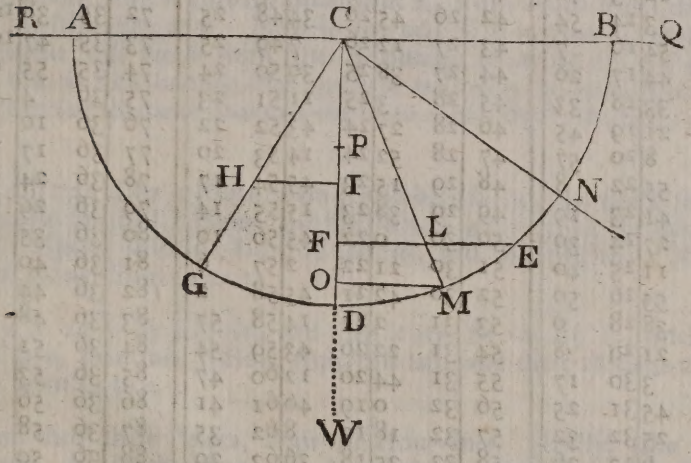
2. TAKE

2. TAKE the Complement of the Latitude 38° : 28′, and ſet that from D to E, and from E draw E F parallel to A B, then take the Declination of the Plane 30 Degrees, and lay that from D to G, and draw the Line C G.

3. TAKE EF in your Compaſſes, and ſet that off from C to H, and from H draw H I parallel to A B. This being done, take H I and lay that from F to L, and thro' L draw the Line C L M, then take D M in your Compaſſes to the Line of Chords, and it will give you about 21° : 40′, which is the Diſtance of the Subſtile from the Meridian.

4. TAKE C I in your Compaſſes, and ſetting one Foot in L, turn the other about, until it cuts the Arch B D in N, then take M N in your Compaſſes to the Line of Chords, and that Extent will reach to 32° : 36′, which is the Stile's Height.

5. FROM M draw M O parallel to E F, then take the Diſtance from O to G, and lay that upon the Meridian from O to P, or from O to W, and take M P or M W in your Compaſſes to the Line of Chords, and you will find it fall at about 36° : 24′ for the Inclination of Meridians. This reduced is 2ʰ 25′ : 36″.



A Table for the three Requisites in Dialling.

For Latitude 53° : 00′

Declin.	Subst. Dist.		Stile's Height.		Inclin. Merid.	
	°	′	°	′	°	′
1	0	45	37	0	1	15
2	1	30	36	58	2	30
3	2	16	36	56	3	45
4	3	1	36	54	4	30
5	3	45	36	50	6	15
6	4	31	36	45	7	30
7	5	15	36	41	8	44
8	5	59	36	35	9	59
9	6	43	36	28	11	13
10	7	27	36	21	12	27
11	8	11	36	12	13	41
12	8	54	36	3	14	54
13	9	37	35	54	16	7
14	10	20	35	44	17	20
15	11	2	35	33	18	33
16	11	44	35	21	19	45
17	12	25	35	8	20	57
18	13	6	34	55	22	8
19	13	47	34	41	23	19
20	14	27	34	27	24	30
21	15	7	34	11	25	40
22	15	46	33	55	26	50
23	16	24	33	38	28	0
24	17	3	33	21	29	8
25	17	40	33	3	30	17
26	18	17	32	45	31	25
27	18	53	32	25	32	32
28	19	29	32	6	33	36
29	20	4	31	46	34	46
30	20	38	31	25	35	52
31	21	13	31	3	36	57
32	21	46	30	41	38	3
33	22	15	30	18	39	7
34	22	51	29	56	40	11
35	23	23	29	32	41	14
36	23	53	29	8	42	18
37	24	23	28	44	43	21
38	24	53	28	19	44	22
39	25	22	27	53	45	24
40	25	50	27	27	46	25
41	26	18	27	1	47	26
42	26	45	26	34	48	25
43	27	12	26	7	49	25
44	27	38	25	39	50	24
45	28	3	25	11	51	23
46	28	27	24	43	52	22
47	28	52	24	14	53	20
48	29	15	23	45	54	17
49	29	38	23	15	55	14
50	30	0	22	45	56	10
51	30	21	22	2	57	6
52	30	42	21	45	58	2
53	31	2	21	14	58	57
54	31	22	20	43	59	54
55	31	44	20	12	60	47
56	32	0	19	40	61	41
57	32	18	19	8	62	35
58	32	35	18	36	63	29
59	32	52	18	4	64	22
60	33	8	17	31	65	15
61	33	23	16	58	66	7
62	33	38	16	24	67	0
63	33	53	15	52	67	51
64	34	6	15	18	68	43
65	34	20	14	44	69	34
66	34	33	14	10	70	26
67	34	45	13	36	71	16
68	34	56	13	2	72	7
69	35	8	12	27	72	57
70	35	18	11	53	73	47
71	35	26	11	18	74	37
72	35	38	10	43	75	27
73	35	47	10	8	76	17
74	35	55	9	33	77	6
75	36	4	8	58	77	55
76	36	10	8	23	78	54
77	36	17	7	47	79	19
78	36	24	7	11	80	22
79	36	29	6	35	81	11
80	36	35	6	0	81	59
81	36	40	5	24	82	49
82	36	44	4	48	83	36
83	36	48	4	12	84	24
84	36	51	3	36	85	12
85	36	53	3	0	86	0
86	36	56	2	24	86	48
87	36	58	1	48	87	36
88	36	59	1	12	88	24
89	36	59	0	36	89	12
90	37	0	0	0	90	0

CHAP.

CHAP XII.

TO draw the Hour-lines upon an erect North or South Plane, declining towards the East or West, by the Line of Latitudes and Hour Scale.

EXAMPLE.

LET A B C D be a Plane in the Latitude of 51° : 32′, declining from the South Westward 30 Degrees, as at *Chap.* 10.

1. HAVING found the Requisites, either by Calculation, as at the last Chapter, or by the Diagram, then draw E *g* 12 at Right-angles to A B; and upon E, with 60 Degrees in your Compasses, draw the Arch *g d*, upon which set off the Distance of the Substile from the Meridian 21° : 40′ from *g* to *x*, and from E thro' *x* draw the Line E *x* N for the Substile.

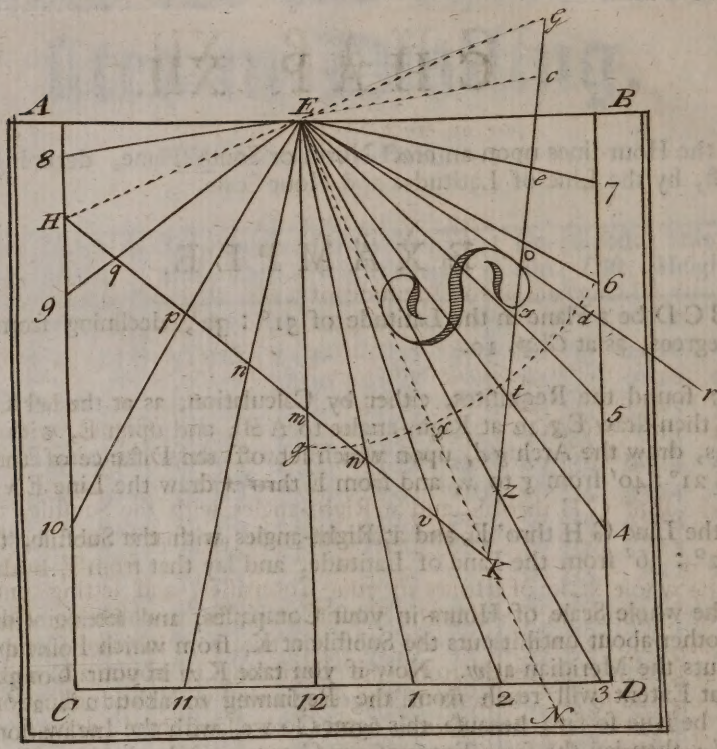
2. DRAW the Line G H thro' E, and at Right-angles with the Substile, then take the Stile's Height 32° : 36′ from the Line of Latitude, and lay that from E both Ways to G and H.

3. TAKE the whole Scale of Hours in your Compasses, and setting one Foot in G or H, turn the other about until it cuts the Substile at K, from which Point draw K G and K H; this last cuts the Meridian at *m*. Now if you take K *m* in your Compasses to the Hour-Scale, that Extent will reach from the Beginning to about 2 h. 25′½, which shews the Work to be true so far; because this agrees so well with the Inclination of Meridians 2 h. 25′ : 36″; then lay the same Extent from G to *o*, and thro' *o* draw E *o* 6 for the Hour-line of 6.

4. ENCREASE 2 h : 25′½ by 1 Hour, then it will be 3 h : 25′ ½, which set from K to *n*, and from G to *s*: To this add 1 Hour, and the Sum is 4 h : 25′ ½; set this from K to *p*, and from G to *t*. Again, add one Hour to the last, and that will make it 5 h. 25′ ½, which set from K to *q*, and from G to *z*.

5. DIMINISH 2 h. 25′ ½ by 1 Hour, and then it will be 1 h. 25′ ½, which set from K to *w*, and from G to *e*, then take 1 Hour from this last, and there remains 25′ ½, set this from K to *v*, and from G to *c*.

6. FROM E, and thro' these Points, draw the Hour-lines. Lastly, take 32° : 36′ from the Line of Chords, and set that from *x* to *d*, and thro' *d* draw E *d r* for the Stile, and it is done.



CHAP. XIII.

To draw the Hour-line upon a Plane that declines many Degrees from the North or South towards either the Eaſt or Weſt, or that falls near the Meridian.

IF a Plane declines many Degrees from the North or South, towards the Eaſt or Weſt, and that above 60 Degrees, altho' the Requiſites may be found, and the Dial made in all Reſpects as the former Dial was, yet by reaſon of the great Declination of ſuch Dials, the Gnomon will be ſo low, and the Hour-lines lie ſo very cloſe together, that they will be but of little Uſe without ſome other Help, which may be as follows,

EXAMPLE.

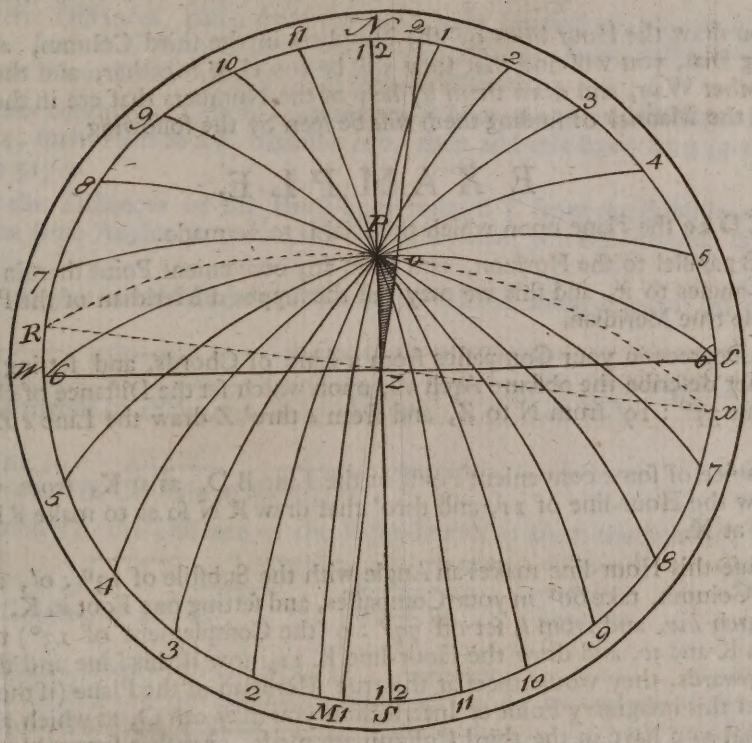
SUPPOSE a Plane in the Latitude of 52′: 20 declines from the South towards the Eaſt 81° : 00′, this may be repreſented in the following Projection by M Z Q, the two Poles where-

of

of are R and X, and its Meridian by R P X passing between the Hours of 6 and 7, and making with the Plane and Meridian of the Place N S, the little Triangle Z P O Right-angled at O, in which you have the Complement of the Plane's Declination P Z O = 9° : 0, and the Complement of the Latitude of the Place P Z = 37° : 40′ by those you will find.

1. The Distance of the Substile from the Meridian ——— Z O = 37° : 19′
2. The Stile's Height ——— ——— ——— P O = 5 : 29
3. The Inclination of Meridians ——— ——— Z P O = 82 : 51

Reduce this 82° : 51′ into Time, and it is 5^h : 31′ : 24 from the south Part of the Meridian, or 6^h : 28 : 36′ from the north Part, and shews that the Substile will fall at 28′ : 36″ past 6 in the Morning.



The three Requisites being found, you may fill up the first second and third Columns of the following Table for the Hours, the Angles at the Pole, and Hour-distances, as directed at *Chap.* X.

Hours

Hours	Ang. at the Pole	Hour Diſt.	Hours	Diſt. from X.	Diſt. from Y.
	D : ı	D : ı		Parts	Parts
4	37 : 9	4 : 9	4	31·43	37·53
5	22 : 9	2 : 13	5	16.77	20.2
6	7 : 9	0 : 41	6	5.17	6.17
—	Sub.	Stile	—	Sub.	Stile.
7	7 : 51	0 : 45	7	5.67	6.77
8	22 : 51	2 : 19	8	17·54	20.93
9	37 : 51	4 : 15	9	32.19	38.44
10	52 : 51	7 : 11	10	54.6	65.19
11	67 : 51	13 : 0	11	100.	119.4
12	82 : 51	37 : 18	12		

Now, if you draw the Hour-lines by the Numbers in the third Column, as for any other erect declining Dial, you will find that they will be too cloſe together, and therefore we muſt proceed in another Way, and draw them by help of the Numbers that are in the fifth and ſixth Columns, and the Manner of finding them will be ſeen by the following

EXAMPLE.

LET A B C D be the Plane upon which the Dial is to be made.

1. DRAW A B parallel to the Horizon, and from any convenient Point therein draw the Line *e g* at Right-angles to it, and this we may call the ſuppos'd Meridian of the Plane, as being parallel to the true Meridian.

2. TAKE 60 Degrees in your Compaſſes from a Line of Chords, and ſetting one Foot in *e*, with the other deſcribe the obſcure Arch *n o*, upon which ſet the Diſtance of the Subſtile from the Meridian 37° : 19′ from N to Z, and from *e* thro' Z draw the Line *e* Z C for the Subſtile.

3. MAKE Choice of ſome convenient Point in the Line B D, as at K, from whence you intend to draw the Hour-line of 11, and thro' that draw R N ſo as to make Right-angles with the Subſtile at X.

4. Now becauſe this Hour-line makes an Angle with the Subſtile of 13° : 0′, as you may ſee in the third Column, take 60° in your Compaſſes, and ſetting one Foot in K, with the other draw the Arch *l w*, and from *l*, ſet off 77° : 0 (the Complement of 13°) to *w*, then lay a Ruler upon K and *w*, and draw the Hour-line K 11, now if this Line and the Subſtile were produced upwards, they would meet at the true Meridian of the Plane (if the Plane was large enough) and this imaginary Point of Interſection you may call O, at which all the Angles at the Plane that you have in the third Column are made, or rather ſuppos'd to be made.

5. MEASURE X K by a Diagonal Scale, or take it in your Compaſſes, and open your Sector ſo as that Extent may be made the parallel Diſtance between 10 and 10 upon the Line of Lines, which now you may call 100 Inches, then in the imaginary Triangle K X O, Right-angled at X, you have the Side X K = 100 Inches and the Angle at O = 13, and alſo the Angle at K = 77° : 0′ to find to find the Side X O, ſay,

As the Sine of the Angle at O = 13° : 0′	*co. ar.*	0.647911
Is to X K = 100		2.
So is the Sine of the Angle X K O = 77° : 0′		9.988724
To the Diſtance between *x* and O, = 433.2 Inches		2.636635

6. To find the Diſtance of the Hour-lines upon R N from the Subſtile at *X*, you muſt make *X O* = 433.2 Radius, then ſuppoſe you would find the Point *t*, thro' which the Hour-line of

10 muſt paſs, you will find in the third Column of the Table, that the Angle at the Plane for that Hour is 7° : 11′ from the Subſtile, then ſay, as Radius ——— 10.
Is to $XO = 433.2$ ——— 2.636635
So is the Tangent of 7° : 11′ ——— 9.100487

To $xt = 54.6$ ——— 1.737122

This is to be ſet under x in the 5th Column againſt 10.

Again, to find the Diſtance from X to U for 9 a-Clock in the third Column againſt 9 you have 4° : 15′, then as Radius ——— 10.
Is to to $XO = 4332$ ——— 2.636635
So is the Tangent of 4° : 15′ ——— 8.871064

To $xu = 32.19$ ——— 1.507752

Set this in the 5th Column againſt 9 a-Clock, and ſo proceed with the reſt of the Angles in the third Column until you have done them all.

7. SET the ſeveral Diſtances, taken from the ſame Scale (or Sector) by which you meaſured X K upon the Line R N from X both Ways, and you will have the Points in that Line thro' which the Hour-lines of 4 5 6 7 8 9 and 10 muſt paſs.

8. DRAW another contingent Line S T parallel to R N at any proper Diſtance from it, as xy, ſuppoſe of 84, ſuch Parts as x K contains 100, then add this to $ox = 433.2$, and the Sum will be $yo = 517.2$.

Now to find the Diſtances of the Hour-lines upon S T from the Subſtile at y, you muſt make Uſe of the ſame Angles from the third Column that you did before, only you make yo Radius.

THUS, ſuppoſe you would find the Diſtance yb of the Hour-line of 10 from the Subſtile. Then
As Radius ——— 10.
Is to $yo = 517.2$ ——— 2.713659
So is the Tangent of 1° : 11′ ——— 9.100487

To $yb = 65.19$ ——— 1.814146

Place this in the 6th Column againſt 10.

AGAIN, to find yc, the Diſtance of the Hour-line of 9 from the Subſtile, ſay,
As Radius ——— 10.
Is to $yo = 517.2$ ——— 2.713659
So is the Tangent of 4° : 11′ ——— 8.871064

To the Diſtance from y to $c = 38.44$ ——— 1.584723

Place this in the 6th Column againſt 9. Do the like by all the reſt of the Angles, until you have compleated the 6th Column.

9. WHEN that is done, take x K in your Compaſſes, and open your Sector as directed at Art. 5, and then ſet off the ſeveral Diſtances from y both Ways, that you have in the 6th Column, and thro' theſe and their correſpondent Points upon R N, draw the Hour lines 10, 10, 9, 9, &c. and number them as in the Figure.

10. *To find the Height of the Stile above* x *as* x q.
The Stile was found to be 5° : 29′, and this is an Angle made at the ſuppos'd Point or Center o, by the Stile over the Subſtile $o\,x$, and is one Angle of the Triangle $x\,q\,o$ Right-angled at x, by which, and the Side $xo = 433.2$ you may find x thus,
As Radius ——— 10.
Is to $xq = 433.2$ ——— 2.636635
So is the Tangent of 5° : 29′ ——— 8.982251

To $xq = 41.59$ ſuch Parts as x K is to 100 ——— 1.618939

11. To

11. *To find the Height of the Stile above* y, *as* y p.

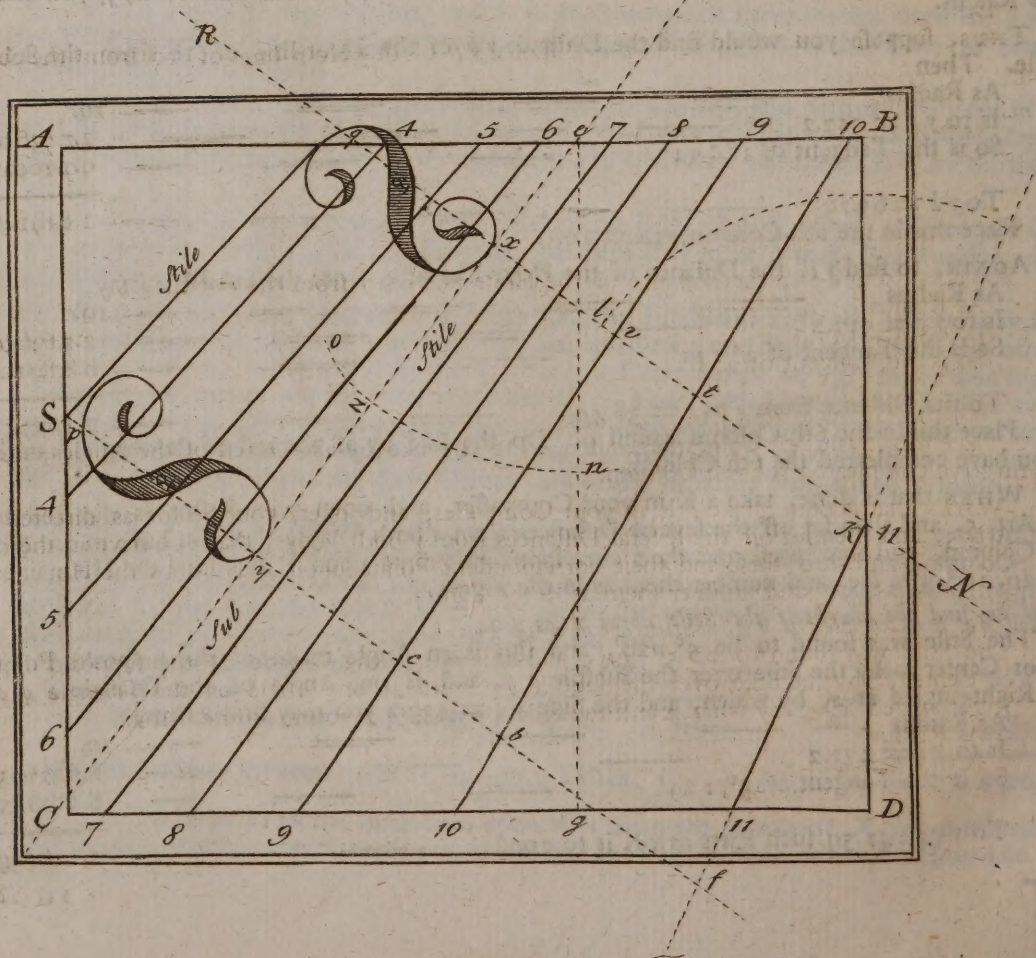
Here you muſt uſe the ſame Angle, *viz.* 5° : 29′; but make $y\,o = 517.2$ Radius; then,

As Radius	——	10.
Is to $y\,o = 517.2$	——	2.713659
So is the Tangent of 5° : 29′	——	8.982251
To the Stile's Height $y\,p = 49.65$	——	1.695910

Set theſe Diſtances from the Sector, opened as before, up from x to q, and from y to p, and draw the Line $p\,q$ for the Stile.

Note, That the Stile may be either an Iron Rod as $p\,q$, ſuppoſed by ſomething like ♀ and ♀, or an Iron Pin ſet perpendicular to the Subſtile.

Note, again, That when a Dial is truly placed, if you ſtand upon the South Side of it, with your Face towards the North, the Hours on the Left Hand of the Meridian are the Morning Hours; and thoſe on the Right Hand are the Evening Hours: But if you ſtand on the North Side, looking Southward, then the Forenoon Hours are on your Right Hand, and the Afternoon Hours on your Left.



Note, That the Trouble of finding the Numbers in the 3d, 5th and 6th Columns may be ſaved, and the Hour-lines drawn ſufficiently near the Truth, by uſing only the Angles in the 2d Column; for having found *x q* and *y p*, as directed at Art. 10 and 11, make them the Radius of the Line of Natural Tangents (upon the Sector) reſpectively; and then from *x* and *y* ſet off the Tangents of the Degrees and Minutes that are in the ſecond Column, and you will find the ſame Points upon the Lines B N and S T as before, by which you may draw the Hour-lines.

CHAP. XIV.

ANOTHER Way to draw the Hour-lines upon a North or South Dial, declining many Degrees towards the Eaſt or Weſt, and more agreeable to thoſe that are not acquainted with Trigonometry than the former.

EXAMPLE

LET the Plane upon which the Dial is to be drawn be A B C D, and the Latitnde of the Place and Declination of the Plane be the ſame as in the laſt Example, conſequently the Requiſites will be the ſame, *viz.*

1. THE Diſtance of the Subſtile from the Meridian —— 37° : 19′
2. THE Stile's Height —— 5 : 29
3. THE Difference of Longitude —— 82 : 51

Now theſe may be found. See *Chap.* 10 or 11.

1. DRAW *e g*, and upon the Center *e* deſcribe the Arch *n o*; upon which ſet the Diſtance of the Subſtile from the Meridian 37° : 19′ from *n* to *z* as before, then draw *e c* for the Subſtile, and upon the ſame Arch ſet the Stile's Height 5° : 29′ from *z* to *o*, and draw the Line *e o w* for the Stile: But this being ſo near the Subſtile, muſt be made higher, that the Hour-lines may be farther off one another than they can be, if drawn according to this Elevation of the Stile.

2. To do this, draw the Line *p q* parallel to the Stile *e w* at any convenient Diſtance from it; and this is the new or augmented Stile to ſtand over the Subſtile *e c*.

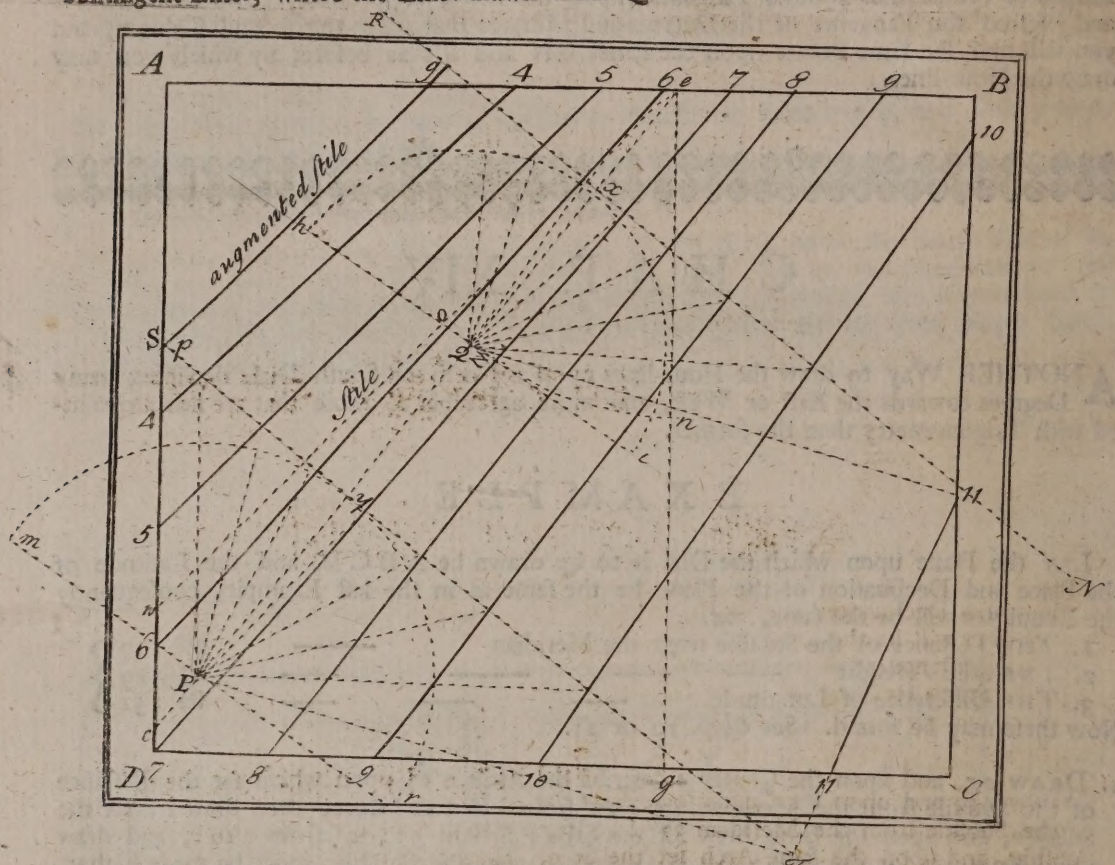
3. DRAW the two contingent Lines R N and S T thro' any aſſumed Points in the Subſtile, as *x* and *y*, and at Right-angles thereto.

4. TAKE the leaſt Diſtance from *x* to the augmented Stile, and ſet that upon the Subſtile from *x* to Q; alſo from the Point *y* take the leaſt Diſtance to the augmented Stile, and ſet that from *y* to P.

5. UPON the Points Q and P, as Centers, deſcribe the two Arches *h x i* and *m y r*, and upon theſe ſet off the Difference of Longitude 82° : 51′ from *x* to *i* and from *y* to *r*, both on the ſame Side of the Subſtile on which the Perpendicular *e g* was drawn, then draw the Diameter *i h* and *m r*.

6. DIVIDE each Semicircle into 12 equal Parts, which you may ſooneſt do by taking 15 Degrees in your Compaſſes, and run that Extent upon the Semicircles from *i* and *r* to *h* and *m*.

7. DRAW Lines from Q and P thro' the Diviſions upon the reſpective Semicircles to the contingent Lines R N and S T, then draw the Hour-lines thro' the correſpondent Points in the contingent Lines, where the Lines drawn from Q and P do direct.



CHAP. XV.

HAVING inform'd the Learner how to make the South, North, Eaſt, Weſt, and Horizontal Dials, our next Buſineſs ſhall be to ſhew the Manner of Making the North, South, Eaſt and Weſt Recliners. (See the Projection at *Chap.* 16.)

How to draw the Hour-lines upon South Reclining Planes.

Of theſe there are *Three Varities.*

VARIETY 1.

SUPPOSE a South Plane reclines leſs than the Complement of the Latitude of the Place, as W *y* E, then to find the New Latitude, in which it will be an Horizontal Dial, you muſt ſubſtract the Reclination from the Complement of the Latitude, and what remains ſhall be the New Latitude.

LET

EXAMPLE.

LET W *y* E repreſent a Plane, reclining from the Zenith 18° : 0′ in the Latitude of 51° : 32′, whoſe Complement is 38° : 28′.

From Z P	38° : 28′
Take the Plane's Reclination Z Y	18 : 00
Remains the New Latitude P Y	20 : 28

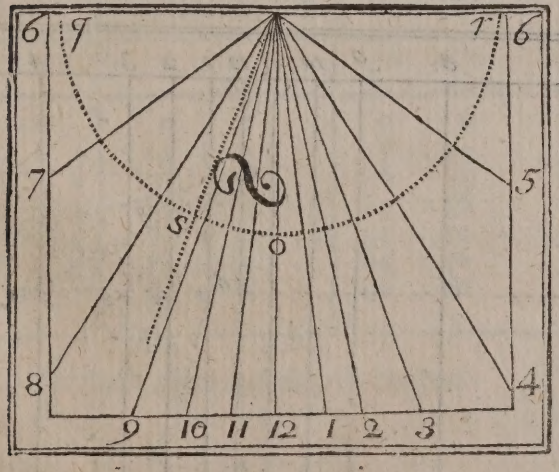
The New Latitnde being fuond, you may proceed to calculate and make a Table, and thence draw the Dial as before for the horizontal Dial at *Chap.* 5.

IN the Right-angled Triangle P Y *q* you have P Y the New Latitude = 20° : 28′, and the Angle Y P *q* = 15° : 0′ to find Y *q*, the Diſtance of the firſt Hour from the Meridian, ſay,

As Radius	10.
Is to the Sine of the New Latitude 20° : 28′ = P Y	9·543649
So is the Tangent of 15° : 0′	9·428052
To the Tangent of Y *q* = 5° : 21′	8·971701

FIND the reſt of the Angles in the 3d Column by thoſe in the 2d in like manner.

Hours.	Angles at the Pole.	Hour Diſtances.
	D : ′	D : ′
12	0 : 0	0 : 0
11 : 1	15 : 0	5 : 21
10 : 2	30 : 0	11 : 25
9 : 3	45 : 0	19 : 16
8 : 4	60 : 0	31 : 12
7 : 5	75 : 0	32 : 32
6	90 : 0	90 : 00



THE ſeveral Hour-diſtances, as you have them in the 3d Column, are ſet off from *o*, upon the Arch *q o r* and 20° : 28′ from *o* to *s* for the Stile's Height.

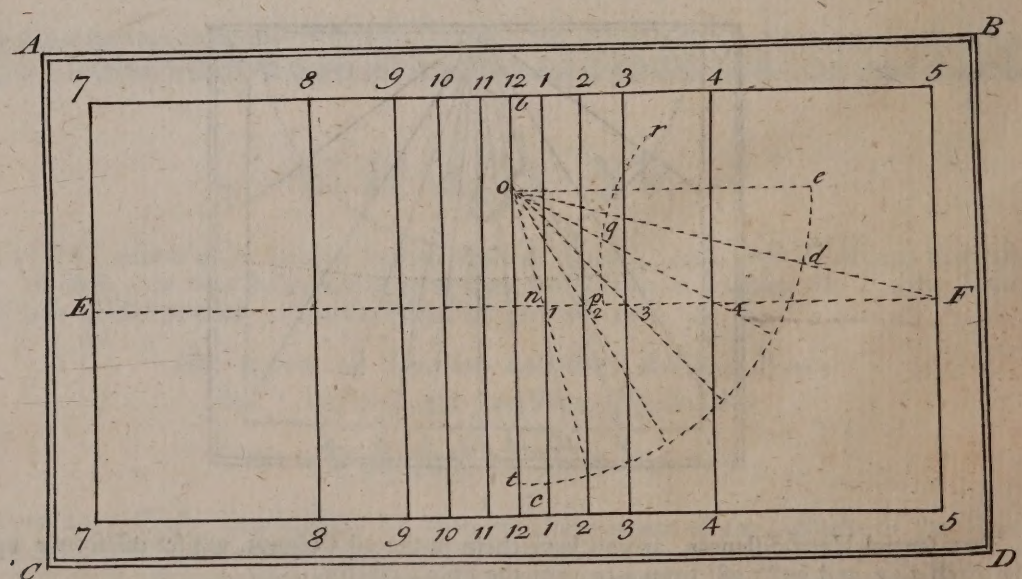
VARIETY 2.

WHEN a South Plane reclines from the Zenith northward, ſo far as that the Reclination happens to be equal to the Complement of the Latitude of the Place it is called a Polar Plane, and being parallel to the Axis of the World, can have no Center, but the Hour-lines will run parallel to each other. This may be repreſented in the Projection following, at *Chap* 16, by W P E. The Manner of making the Dial is nearly the ſame with that of an Eaſt or Weſt Plane.

EXAMPLE.

Let A B C D *be a Polar Plane.*

1. DRAW the Line E F about the Middle of the Breadth, and at Right-angles thereto draw the Line *b c* at about the Middle of the Length, for the Meridian or Hour of 12.
2. SUPPOSING the Lines 7.7 and 5.5 at an equal Diſtance from the Meridian, then take 60 Degrees in your Compaſſes from a Line of Chords, and ſetting one Foot on F, with the other draw the Arch *p r*, upon which ſet 15 Degrees from *p* to *q*, and from F thro' *q* draw a Line, which will cut the Meridian at *o*, ſo *o n* is the Height of the Stile fitted to this Plane.
3. UPON *o*, as a Center, draw the Quadrant *o e t*, which you may divide into 6 equal Parts, of which *d e* will be one; take this in your Compaſſes, and run that Extent along the Arch from *t* to *d*, ſo it will be divided into 6 equal Parts; or if from *t* you ſet off 15°, 30°, 45°, 60°, and 75°, you will have the ſame Points upon the Arch as before.
4. LAY a Ruler upon *o*, and each of thoſe Points, and it will cut the Line E F at 1, 2, 3, 4, then thro' theſe Points draw Lines parallel to the Meridian, ſo will you have the Hour-lines for 1, 2, 3, 4 and 5 in the Afternoon, and the Hour-lines for 11, 10, 9, and 8 in the Morning, muſt be drawn at the ſame Diſtance from the Meridian, toward E, as the others were from it towards F.
5. HAVING drawn the Hour-lines, ſet a Pin or Piece of Wire perpendicular to the Plane at *n* of the ſame Length with *o n*, or it may be a Piece of Braſs, in Form of a Parallelogram placed upon the Meridian, and of the ſame Breadth with the Diſtance between the Hours of 12 and 3, or 12 and 9.



N. B. That

N. B. That if you make *o n* the Radius of the Line of natural Tangents upon a Sector, and from thence take the Tangents of 15° : 30′ : 45° : 60° and 75°, and ſet them from *n* both Ways upon the Line E F, you will have Points upon that Line thro' which the Hour-lines muſt paſs, as before.

VARIETY 3.

The third Variety is of a South Plane, whoſe Reclination from the Zenith northward, is more than the Complement of the Latitude of the Place; in this Caſe you muſt ſubſtract the Complement of the Latitude from the Plane's Reclination, and the Remainder will be the new Latitude where this will be an horizontal Plane.

EXAMPLE.

Let a Plane of the Latitude 51° : 32′ recline from the Zenith 70°. This may be repreſented in the following Projection by W *h* E, and the Reclination by Z *h* = —— 70° : 00′
The Complement of the Latitude is Z P = —— —— —— 38 : 28

Remains the new Latitude P *h* = —— —— —— 31 : 32

Here the NorthPole is elevated above the Plane 31° : 32′, ſo an horizontal Dial calculated for this new Latitude will be the ſame as a Plane at *London* that reclines 70°, and to make a Table for this, you have in the Right-angled Triangle P *h* ♀, the Side P *h* = 31° : 32′, and Angle *h* P ♀ = 15° : 0′, to find *h d*, the Diſtance of the firſt Hour from the Meridian, Say,

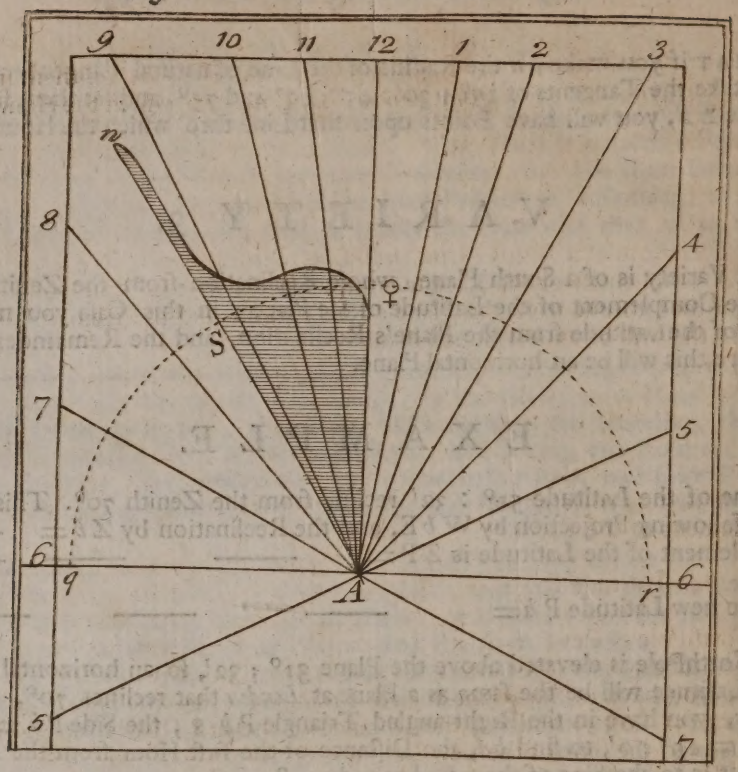
As Radius is to the Sine of the new Latitude 31° : 32′ —— —— 9.718497
So is the Tangent of 15 : 0′ —— —— —— —— 9.428052

To the Tangent of *h* ♀ = 7° : 59′ —— —— —— 9.146549

And ſo on with the reſt which you will find in the Table next following.

Hours	Angles at the Pole.	Hour Diſtances.
	° : ′	° : ′
12	0 : 0	0 : 0
11 : 1	15 : 0	7 : 59
10 : 2	30 : 0	16 : 49
9 : 3	45 : 0	27 : 36
8 : 4	60 : 0	42 : 10
7 : 5	75 : 0	62 : 50
6	90 : 0	90 : 00

A South Plane reclining 70 Degrees.



In this Figure the ſeveral Hour-Diſtances in the third Column are ſet from *o*, upon the Arch *qor*, and the Height of the Stile 31 : 32′ from *o* to *s*, thro' which draw A *n* for the Stile, which muſt point upwards to the North Pole.

CHAP. XVI.

Of Direct North Recliners.

THERE are three Varieties or Caſes of theſe Sorts of Planes, as there were of South Recliners.

Caſe

Case I.

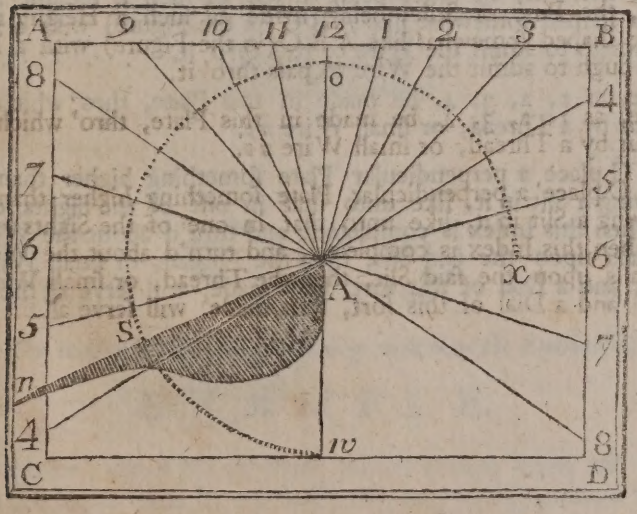
SUPPOSE a North Plane reclines from the Zenith 24° : 0′ this may be repreſented by W *d* E, which falls between the Zenith and the Equinoctial W Æ E.

Now, if the Latitude of the Place where this Dial is to ſtand be 51° : 32′, then the Complement thereof is 38° : 28′ = P *z* ; to which add the Plane's Reclination *z d* = 24° : 0′, and the Sum will be 62° : 28′ = P *d*, which is the new Latitude, where this will be an horizontal Plane ; and to find the Hour Diſtances, you have in the Right-angled Triangle P *d o*, the Side P *d* = 62° : 28′, and the Angle *d* P *o* = 15° to find the Side *d o*, ſay,

As Radius —— —— —— ——	10.
Is to the Sine of the new Latitude 62° : 28′ —— —— ——	9·947797
So is the Tangent of 15° : 0′ —— —— ——	9·428052
To the Tangent of *d o* = 13° : 22′ —— ——	9·375849

PROCEED in the like Manner for the reſt, and ſet them down as in the Table.

Hours.	Angl. at the Pole.	Angl. at the Plane.
	0 : ′	0 : ′
12	0 : 0	0 : 0
11 : 1	15 : 0	13 : 22
10 : 2	30 : 0	27 : 7
9 : 3	45 : 0	41 : 34
8 : 4	60 : 0	56 : 56
7 : 5	75 : 0	73 : 11
6	90 : 0	90 : 0



Note, That altho' we have put on the Hours of 10 and 11 in the Morning, and 1 and 2 in the Afternoon ; yet when the Sun is in the Tropick of Cancer, he continueth to ſhine upon the North Face of this Plane, only from the Time of Riſing, to about 9 Minutes before 10, and then begins to ſhine upon the South Face ; and ſo continues to do, until about

bout 9 Minutes after 2 in the Afternoon, and then begins to appear upon the North Face again, upon which he continues to ſhine until he ſets; as you may ſee by the Projection, where he leaves the North Side, and begins to ſhine upon the South Side at *k*, and appears again upon the North Side at *i*.

HERE alſo the ſeveral Hour-Diſtances from the 3d Column of the Table, are ſet upon the Arch *w s o x* from *o* both Ways, and the Stile's Height 62° : 28′ from *w* to *s*, thro' which draw A *s n* for the Stile.

Caſe 2.

THIS *Variety* or *Caſe* is of a North Plane, reclining to the Equinoctial, and therefore called an Equinoctial Plane.

EXAMPLE.

SUPPOSE at *London*, a North Plane ſhould recline from the Zenith 51° : 32′ = Z Æ, then it may be repreſented by *W Æ E*, which is 90° from the North Pole = *P Æ*, then P is the Pole of the Plane.

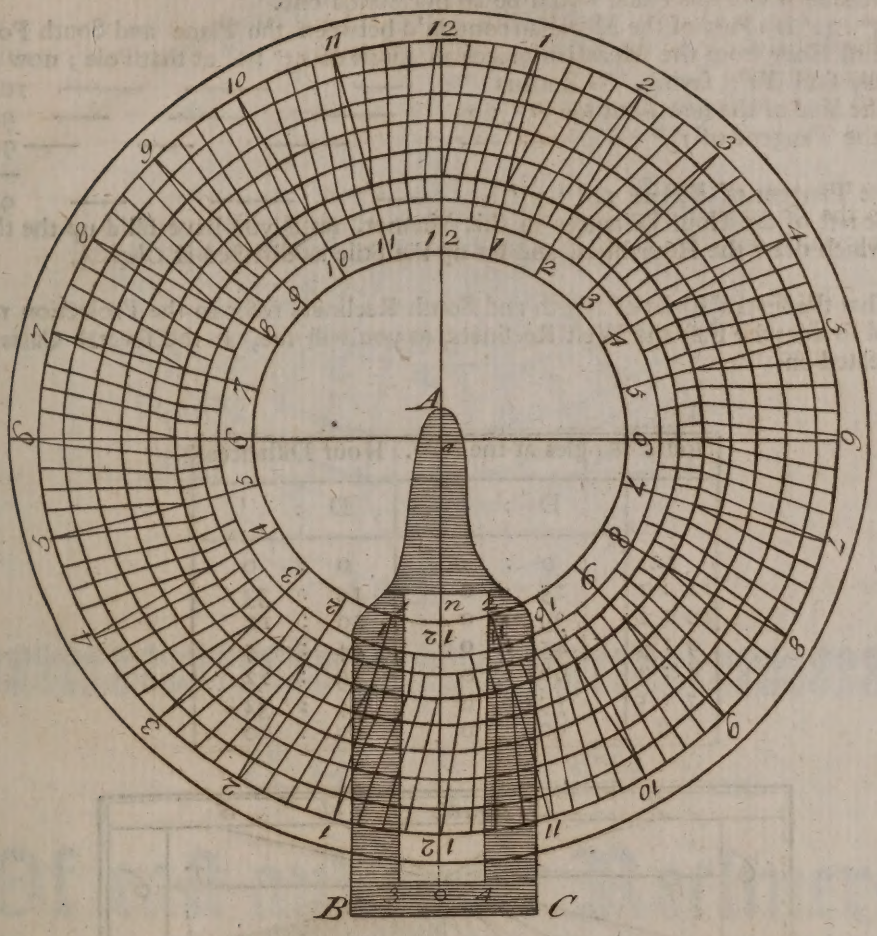
To make this Dial is very eaſy, Thus,

1. TAKE 60 Degrees in your Compaſſes from a Line of Chords, and with that Extent deſcribe a Circle, and divide the ſame into 24 equal Parts, each being one Hour, which you may again divide into Halves and Quarters, or leſſer Parts of an Hour, as you have a Mind, or as you may ſee is done in the Figure following; then ſet up a Pin or Wire in the Center at *a*, perpendicular to the Plane, and let it be ſo high above it, as that the Sun may ſhine upon ſome Part of it, when he has the greateſt South Declination, that ſo the upper Side of this Plane may ſerve the whole Year about.

1. IN order to this, the Pin, or Stile, being made of ſuch a Height as above, then let a Piece of Braſs be ſhaped ſomewhat like A B C (in the Figure) with a Hole near the End at A, juſt wide enough to admit the Wire to paſs thro' it.

2. LET another Hole, as 1, 2, 3, 4, be made in this Plate, thro' which you may ſee the Hours and Parts cut by a Thread, or ſmall Wire *n o*.

3. UPON the Part B C place a perpendicular Plate ſomething higher than the Wire that is in the Center, having a Slit in it, like unto that in one of the Sights of the Index of a Plain Table; ſo when this Index is completed, and turn'd about the Wire in the Center, until its Shadow falls upon the ſaid Slit, then the Thread, or ſmall Wire, will give the Time of the Day; and a Dial of this ſort, thus made, will ſerve all the Year.



Case 3.

This is of a North Plane reclining more than the Equinoctial.

EXAMPLE.

SUPPOSE that at *London* a direct North Plane ſhould recline from the Zenith 66 Degrees, which is more than the Equinoctial by 14° : 28′, this may be repreſented in the Projection by W X E.

The Complement of the Latitude of the Place is P Z =	38° : 28′
To which add the Plane's Reclination Z X =	66 : 00
The Sum is P X =	104 : 28

But as the Height of the Pole above a Plane can never be above 90 Degrees, then in such a Case, you must take the Supplement of the Sum to 180 Degrees, which will be 75° : 32′ the new Latitude where this Plane would be an horizontal one.

This 75° : 32′ is a Part of the Meridian contain'd between the Plane and South Pole, with which the first Hour from the Meridian makes an Angle of 15° : 0′ at that Pole; now by these two you may find *XP*, saying, As Radius —— —— —— 10.

Is to the Sine of the new Latitude 75° : 32 —— —— —— 9.986007

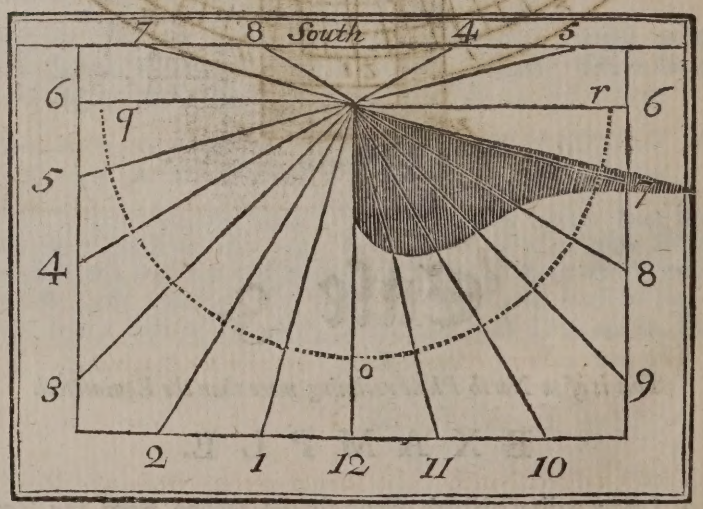
So is the Tangent of 15° : 0′ —— —— —— 9.428052

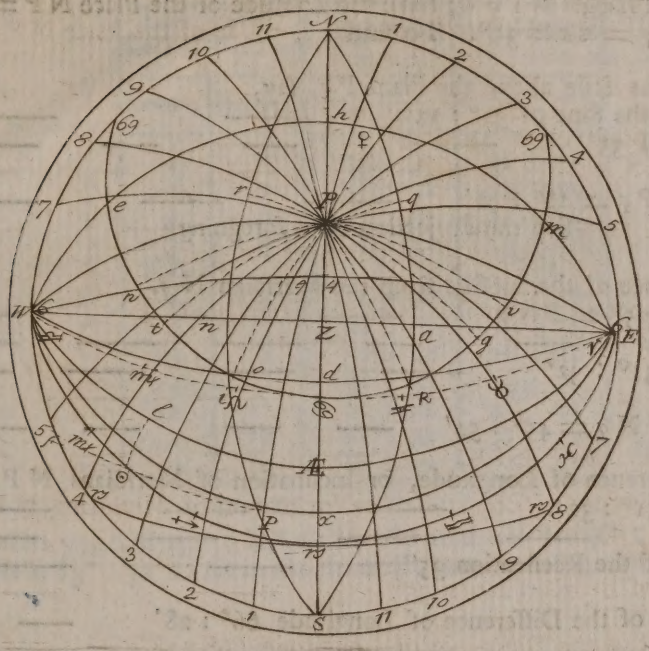
To the Tangent of $XP = 14° : 33'$ —— —— —— 9.414059

Find the rest of the Hour Distances in this Manner, until you have fill'd up the third Column, by which draw the Hour-lines, and set up the Stile as directed at *Chap.* 5.

Note, That the several Cases of North and South Recliners refer to the Projection next following, and so does the East and West Recliners, as you will see, at the several Cases, where they are treated on.

Hours.	Angles at the Pole.	Hour Distances.
	D : ′	D : ′
12	0 : 0	0 : 0
11 : 1	15 : 0	14 : 33
10 : 2	30 : 0	29 : 12
9 : 3	45 : 0	44 : 5
8 : 4	60 : 0	59 : 12
7 : 5	75 : 0	74 : 32
6	90 : 0	90 : 00





CHAP. XVII.

Of east and west Recliners.

WHEN a Plane ſtands upon the Meridian of a Place, and perpendicular to the Horizon, it is then called an Eaſt or Weſt Plane; but if ſuch a Plane falls back from the Zenith towards the Weſt, the upper Face is then upon an Eaſt Recliner, and ſo of the Weſt Plane falling from the Zenith Eaſtward, the upper Face is called a Weſt Recliner.

To make a Dial upon theſe ſort of Planes, two Things muſt be given, *viz.*

1. The Latitude of the Place, and } to find
2. The Reclination of the Plane — }
3. The Height of the Stile above the Plane — — P *q*
4. The Diſtance of the Subſtile from the Meridian — — N *q*
5. The Plane's Difference of Longitude — — N P *q*

EXAMPLE

SUPPOSE a Weſt Plane in the Latitude of 51° : 32′ reclines 35 Degrees from the Zenith, then it may be repreſented by the oblique Circle N *a s*, whoſe Pole is at *t*, its Meridian *t* P *q*, and Reclination ϰ *a* = 35° : 0′

Now in the right Triangle N P q we have the Latitude of the Place N P = 51° : 32′, and the Reclination P N q = $z a$ = 35°. To find

1. THE Height of the Stile above the Plane P q, ſay,
   As Radius is to the Sine of 51° : 32′ ——— ——— 9.893745
   So is the Sine of 35° ——— ——— ——— ——— 9.758591
   To the Sine of P q = 26° : 41′ ——— ——— ——— 9.652336
   See the Projection next foregoing.

2. To find the Diſtance of the Subſtile from the Meridian N q.
   As the S. c. of 35° = P N q ——— ——— ——— 9.913364
   Is to the Radius ——— ——— ——— ——— ——— 10.
   So is T. c. of 51° : 32′ ——— ——— ——— ——— 9.900086
   To the T. c. of N q = 45° : 54′ ——— ——— ——— ——— 9.986722

3. To find the Difference of Longitude, or Inclination of Meridians N P q.
   As the S. c. of 51° : 32′ ——— ——— ——— 9.793832
   Is to Radius ——— ——— ——— ——— 10.
   So is the T. c. of the Reclination 35° ——— ——— 10.154774
   To the Tangent of the Difference of Longitude 66° : 28′ ——— 10.360942

THIS being reduced to Time, is 4 ^{h}. 26′, and ſhews that the Subſtile will fall at 26 Minutes paſt 4 in the Afternoon in the Weſt Recliner, and at 34′ after 7 in the Morning in the Eaſt Recliner, as you may ſee in the laſt Projection or Diagram, by the Meridians of the two Planes P q and P r.

HAVING found the Three Requiſites, *viz.*
1. The Stile's Height above the Plane ——— ——— 26° : 21′
2. The Diſtance of the Subſtile from the Meridian ——— ——— 45 : 54
3. The Plane's Difference of Longitude ——— ——— 66 : 28

1. THEN make a Table to contain 4 Columns, which will ſerve for both the Eaſt and Weſt Recliners, and place the proper Hours under each, as you ſee in the firſt and ſecond Columns; then place the Difference of Longitude 66° : 28′ in the 3d Column againſt 12.

2. To 66° : 28′ add 15°, and the Sum will be 81° : 28′, the Angle at the Pole for 1 in the Eaſt Recliner, and 11 in the Weſt Recliner.

3. SUBSTRACT 15° from 66° : 28′ and there will remain 51° : 28′; from this take 15° and there will remain 36° : 28′, &c. until only 6° : 28′ remain, which you muſt take from 15°, and there will remain 8° : 32′. This you muſt ſet on the other Side of the Subſtile Line; and by a continual Addition of 15° you will have the Angles at the Pole for each Hour, as in the third Column

4. FOR the Angles at the Plane, or Hour Diſtances,
   As Radius is to the Sine of the Stile's Height 26 : 41′ ——— ——— 9.652302
   So is the Tangent of 53° : 32′ ——— ——— ——— 10.131320
   To the Tangent of 31° : 17′ ——— ——— ——— ——— 9.783623
   WORK in the ſame Manner for the reſt, to compleat the 4th Colum, as you ſee.

East Reclin.	West Reclin.	Angl. Pole.	Angl. Plane.
Hours.	Hours.	D : ′	D : ′
4	8	53 : 32	31 : 17
5	7	38 : 32	19 : 41
6	6	23 : 32	11 : 4
7	5	8 : 32	3 : 52
Sub-			ſtile
8	4	6 : 28	2 : 55
9	3	21 : 28	10 : 01
10	2	36 : 28	18 : 22
11	1	51 : 28	29 : 26
12	12	66 : 28	45 : 54
1	11	81 : 28	71 : 32
2	10	83 : 32	75 : 51

Note, That when you add 15° to 81° : 28′, the Sum will be 96° : 28′; but the Supplement of this is 83° : 32′ to ſet in the third Column againſt 2 and 10.

To draw the Dial.

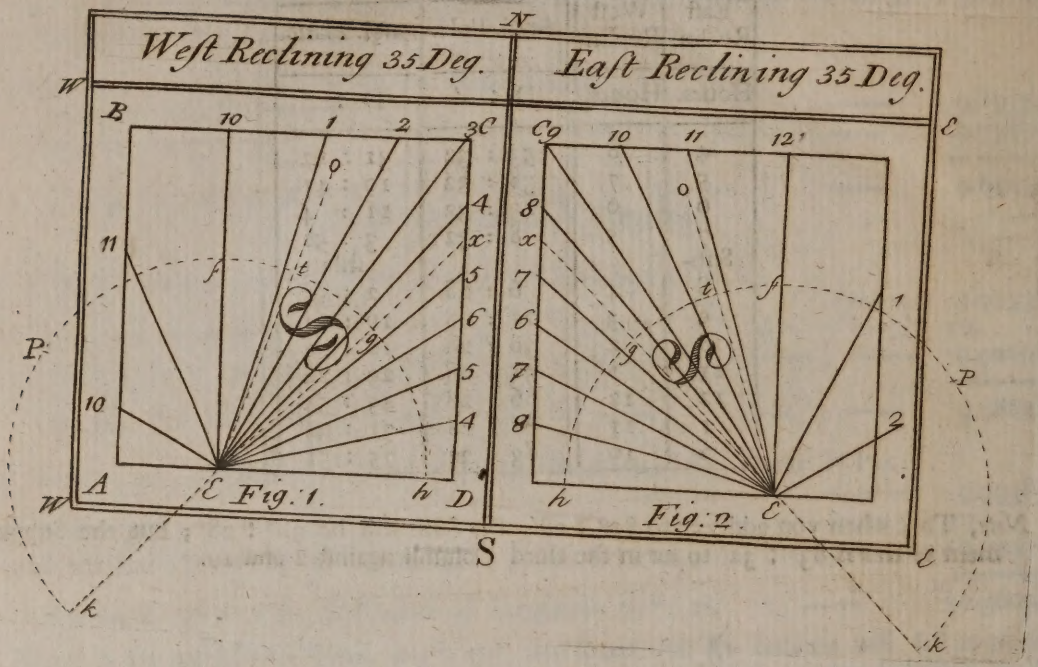
LET A B C D (*Fig.* 1.) be the Plane for a Weſt Recliner.

1. FROM any convenient Point E, in the Line A B, draw the Line E 12; then take 60 Degrees in your Compaſſes from a Line of Chords, placing one Foot in E, as a Center deſcribe the Arch *k p f g h*; upon which, ſet 45° : 54′ from *f* to *g*, and draw E *g x* for the Subſtile, and continue it to *k*.

2. TAKE the Arches, or Angles, in the fourth Column, in your Compaſſes, ſeverally from the Line of Chords, and ſet thoſe Extents upon the ſaid Arch from *g*. Except 75° : 51′ for 10 in the Morning; for that muſt be ſet from *k* to *p*, then lay a Ruler upon E, and the ſeveral Points made in the Arch; and by the Side thereof, draw the Hour-lines, as you ſee they are in *Fig.* 1. And hence you may know how to draw the Hour-line in the ſecond Figure. Theſe Directions ſerving for either.

N. B. That the Situation of theſe Dials will be Plane, if when the Figures are before you; you imagine your Face towards the North.

Weſt



Note, That in these sort of Planes, the Complement of the Latitude, where they are placed, is always the new Latitude, and the Complement of the Reclination is ever the Declination in that new Latitude, where they will become erect declining Planes. Then in this Example,

The Latitude of the Place where they would become erect Planes, is —— 38° : 28′
And the Declination in that Latitude —— —— —— 55 : 00

Now, if you would draw a Dial for this Latitude and Declination, by the Line of Latitudes and Hour Scale, you will, either by Calculation, or such a Diagram as yon were directed to make at *Chap.* 11, find the Requisites to be as before, *viz.*

1. The Stile's Height —— —— —— 26° : 41′
2. The Distance of the Substile from the Meridian —— —— 45 : 54
3. The Difference of Longitude —— —— —— 66 : 28

Which in Time is 4 h. 26′, as before.

To draw the Dial.

SUPPOSE A B C D to be the Plane upon which the Hour-lines are to be drawn.

1. DRAW the Line E 12 at a convenient Distance from the Base A B, and upon E, as a Center, with 60 Degrees from a Line of Chords in your Compasses, describe the Arch *f g h*, and from the same Line of Chords take 45° : 54′, and lay that Extent upon the Arch, from the Meridian, at *f* to *g*, then from E thro' *g* draw the Line E W for the Substile.

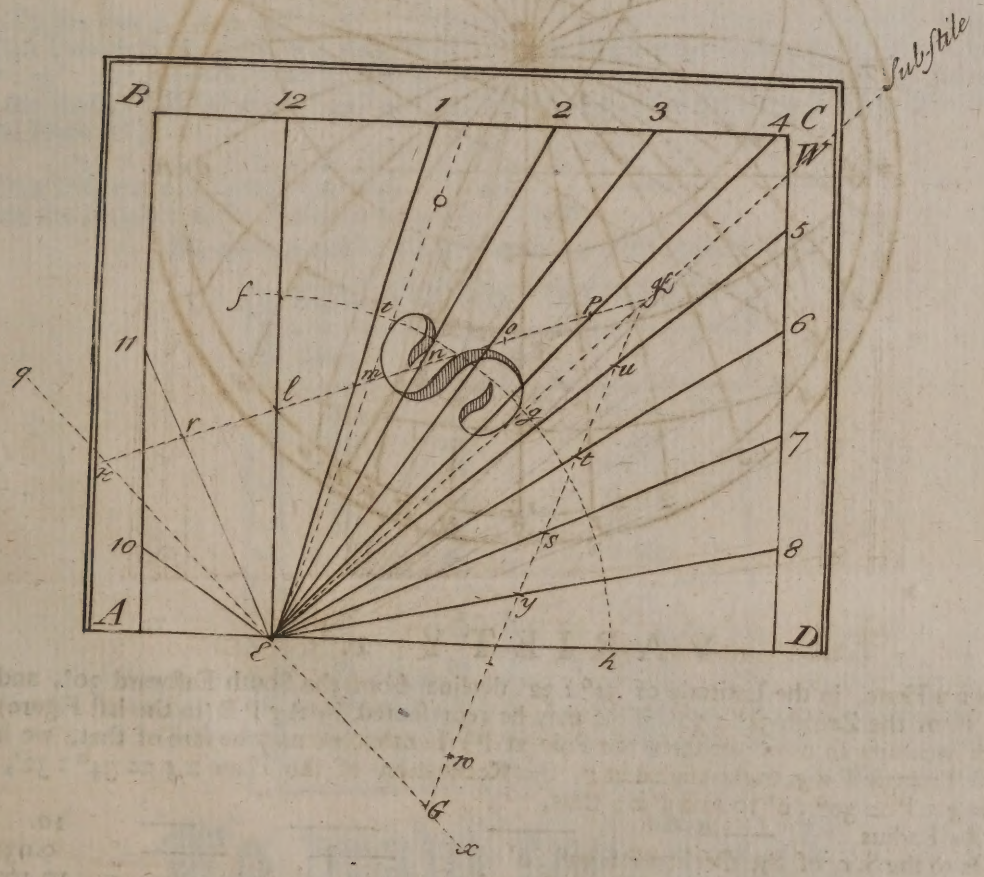
2. DRAW *q x* thro' the Center at E, and at Right-angles to the Substile E W; then from the Line of Latitudes take the Stile's Height 26° : 41′, and lay that from E to H and G.

3. TAKE

3. TAKE the whole Line of 6 Hours in your Compaſſes, and ſetting one Foot in G or H, turn the other about until it cuts the Subſtile at K, from which draw the Lines K G and K H, and this laſt cuts the Meridian at *l*. Now, if you meaſure K *l* by the Hour Scale, you will find it to be 4^h. 26′, agreeing with the Inclination of Meridians. Lay the ſame Extent, *viz.* K *l* from G to *t*, and from E thro' *t* draw a Line for the Hour of Six.

4. ADD 1 Hour to $4^h : 26'$, and the Sum is $5^h\ 26'$; take this from the Hour Scale, and ſet it from K to *r*, and from G to *u*, then ſubſtract 1 from $4^h : 26'$, and the Remainder is $3^h : 26'$. Take this from the Hour Scale, and ſet it from K to *m*, and from G to *s*. From $3^h : 26'$ take 1 Hour, and the Remainder is $2^h : 26'$, ſet this from K to *n*, and from G to *y*. Again, take 1^h from $2^h : 26'$, and the Remainder is $1^h : 26'$; ſet this from K to *o*. Laſtly, from $1^h : 26'$ take 1 Hour, and there remains 26 Minutes, ſet this from K to *p*, and from G to *w*.

5. THROUGH the ſeveral Points *r m n*, &c. draw the Hour-lines, then ſet 26° : 41′ from *g* to *t*, thro' which draw E *t o* for the Stile, which muſt be placed directly over the Subſtile E K.



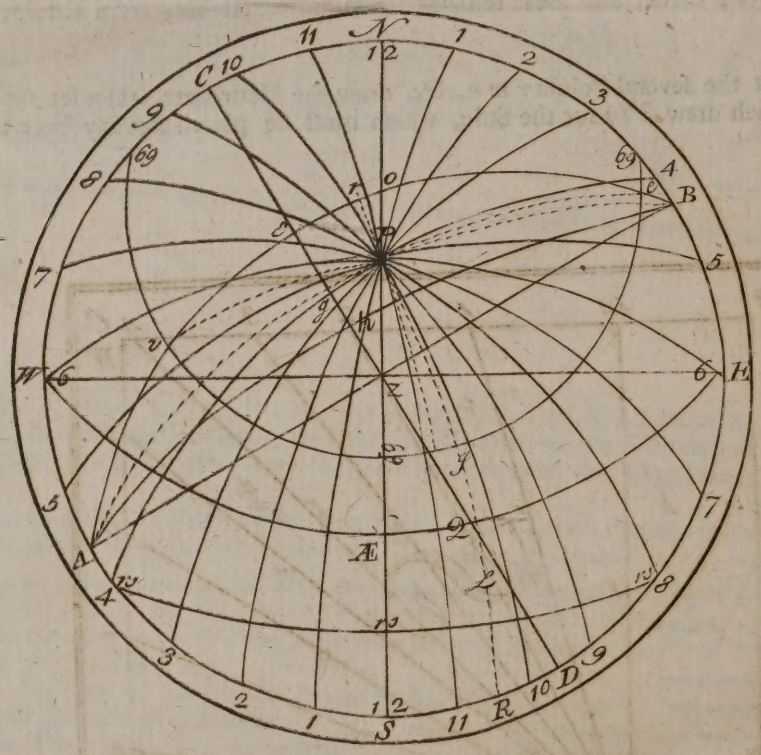
CHAP.

CHAP. XVIII.

Of South Declining Reclining Planes.

THERE are three Varieties of these Sort of Planes, for to any Declination the Plane may recline; so that,

1. It may pass thro' the Pole of the World, as ——— ——— A *g* P B
2. It may recline so, as to fall between the Pole and Zenith, as ——— A *b* B
3. Or, it may recline so, as to pass between the Pole and Horizon, as — A E *r* B



VARIETY 1.

LET a Plane, in the Latitude of 51° : 32′ decline from the South Eastward 30°, and recline from the Zenith 34° : 32′. This may be represented by A *g* P B (in the last Figure) and which we suppose does pass thro' the Pole at P; but that we may be sure of that, we have, in the Triangle P ϰ *g*, right-angled at *g*, the Reclination of the Plane ϰ *g* = 34° : 32′, and Angle *g* ϰ P = 30° : 0′ to find P ϰ; thus,

As Radius	———	———	———	10.
Is to the S. *c.* of the Declination 30° : 0′	———	———	———	9.937531
So is the T. *c.* of the Reclination 34° : 32′	———	———	———	10.162325
To the Tangent Complement of 38° : 28′ = Z P	———	———	———	10.099856

This 38° : 28′ being the Complement of the Latitude of the Place, shews, that this Plane does pass thro' the Pole, and is a Polar Plane. Now

Now before the Hour-lines can be drawn, two Things must be found, *viz.*

1. The Distance of the Meridian or Arch of the Plane contain'd between the Meridian of the Place and Horizon } P B

2. The Plane's Difference of Longitude, or Angle made by the Meridian of the Place N P Z S, and the Meridian of the Plane *r* P Q R. } Z P Q

In the Triangle Z *g* P, you have the Reclination of the Plane Z *g* = 34° : 32′, and its Declination *g* Z P = C N = 30° : 0′, to find *g* P, say,

As Radius	10.
Is to the Sine of Z *g* 34 : 32′	9·753495
So is the Tangent of *g* Z P = 30° : 0′	9·761439
To the Tangent of *g* P = 18° : 7′	9·514934

The Complement of this is P B = 71° : 53′ the Arch required.

For the Difference of Longitude Z P Q (Q being the Pole of the Plane and P Q its Meridian).

As the Sine of Z P = 38° : 28′	9·793832
Is to Radius	10.
So is the Sine of the Reclination *g* z = 34° : 32′	9·753495
To the Sine *g* P z = 65° : 41′	9·959663

The Complement of this is Z P Q = 24° : 19′ (*g* P Q being a Right-angle.) which in time is 1ʰ : 37′ : 16″, take this from 12 Hours, and there will remain 10ʰ : 22′ : 44″, hence we find that the Substile will fall at 22′ : 44″ past 10, as you may see in the Projection by the Meridian of the Plane P Q.

3. The Difference of Longitude being 24° : 19′ }
And the Distance of the Meridian from the Horizon 71 : 53 }

We may now make a Table *as for any other declining Dial.*

Hours	Angles at the Pole. Deg.	Min.	Nat. Tangents.
5	80	41	6.095
6	65	41	2.213
7	51	41	1.265
8	35	41	0.718
9	20	41	0.377
10	5	41	0.099
	Substile		
11	9	19	0.164
12	24	19	0.452
1	39	19	0.819
2	54	19	1.392
3	69	19	2.648
4	84	19	10.048

To draw the Dial.

LET A B C D be the Plane upon which the Hour-Lines are to be drawn.

1. Draw the Line M R, at a convenient Distance from the Base D C, to represent the Horizon then with 60 Degrees in your Compasses from a Line of Chords, set one Foot in M,

and with the other describe the Arch *f g h*, upon which set 71° : 53′ from *f* to *g*, and from M thro' *g* draw the Line M *g* N for the Substile.

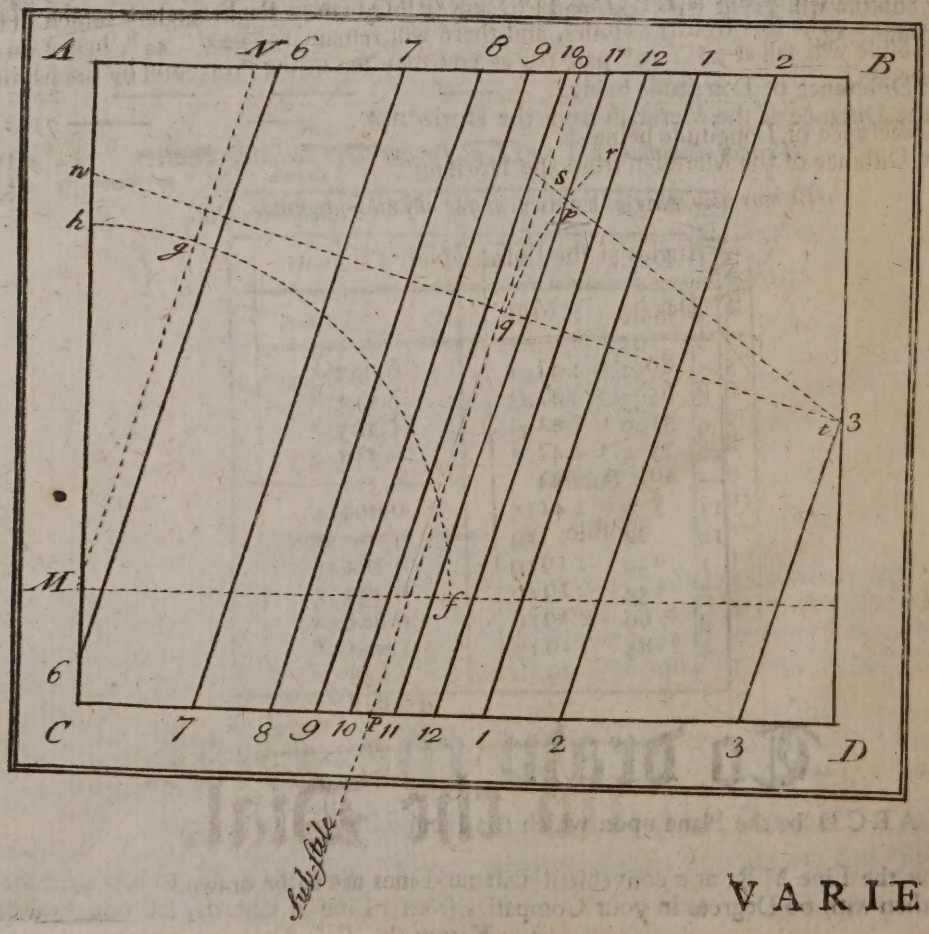
2. By the Projection or Table you see that you may have six Hours upon each Side of the Substile, therefore draw another Line about the Middle of the Plane, and parallel to M N, as P *q o*, then this shall be the Substile instead of M N, to this all the Hour-lines must be parallel.

3. If you bring in the Hour-lines of five in the Morning, and four in the Afternoon, then the Stile will be too low, so you must be content to leave these out, and accept of six and three for the extream Hours.

4. Make Choice of some convenient Point in the Line B C, as at *t* for one End of the Hour-line of 3, from whence *t w* at Right-angles to P O, then with 60 Degrees in your Compasses, and one Foot in *t*, describe the Arch *q r*, upon which set 20° : 41′ (the Complement of 69° : 19′, the Angle at the Pole for the Hour of three) from *q* to *p*, and draw out *t p*, until it cuts the Substile in S, so is S *q* the Stile's Height.

5. Make *q s* the Radius of the Line of Natural Tangents upon a Sector, and thence take of the Tangents of the several Degrees and Minutes that are in the second Column of the Table, and lay those Extents upon the Line *t w*, from *q* both Ways, as the Table and Figure will direct.

Or, Take S *q* in your Compasses, and make that a parallel Distance between 10 and 10, upon the Line of Lines, and thence take out the Numbers that you have in the third Column, and lay those Extents upon the Line *t w*, from *q*, as before.



VARIETY

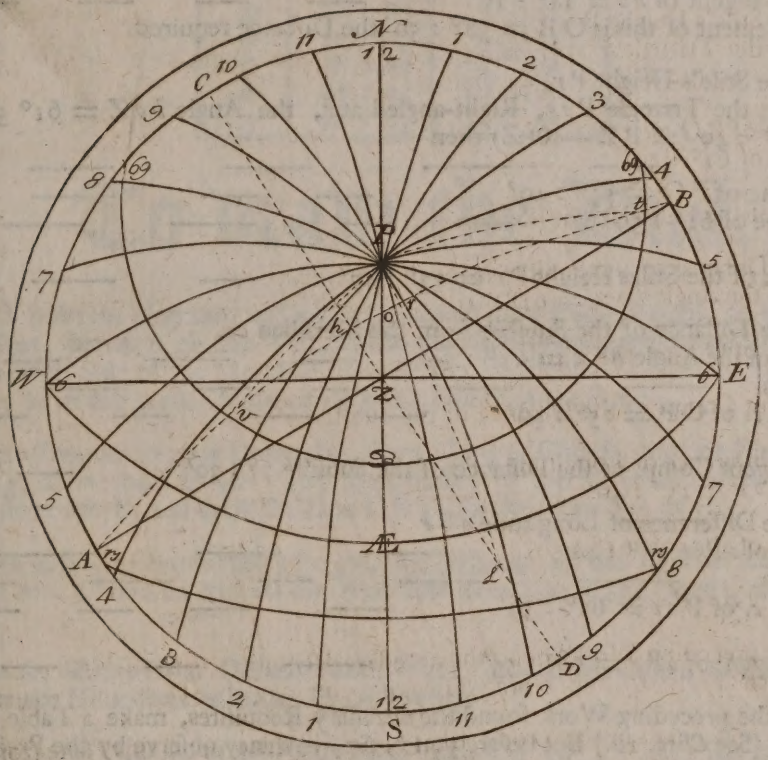
VARIETY 2.

THIS Variety is of a Plane that declines from the South towards the East or West, and reclining from the Zenith so as to pass between the Pole and the Zenith.

EXAMPLE.

SUPPOSE a Plane in the Latitude of 51° : 32′ declines from the South Eastward 30° and reclines from the Zenith 20°.

This may be represented in the *Fig.* following by A *h* O B, the Pole of which is at L, its Reclination Z *h*, and Declination N C = S D = *h* Z O.



THE best Way to make these sort of Dials is to refer them to a new Latitude, and new Declination, where they may be upright declining Planes.

Now to find the new Latitude where this Plane shall stand as an upright Decliner, you have in the Triangle Z *h o* Right-angled at *h*, the Reclination Z *h* = 20°, and Declination *h* Z O = S D = 30° to find.

1. THE new Latitude Æ *O*, say,

As Radius	——	10.
Is to the S. *c.* of the Declination *h* Z O = 30°	——	9·937531
So is the T. *c.* of the Reclination *h* Z = 20°	——	10.438934
To the T. *c.* of Z O = 22° : 48′	——	10.376465

Add this to Z Æ = 51° : 32′ and the Sum will be *O* Æ = 74° : 20′, the new Latitude required.

2. To find the Angle h O Z

As Radius	10.
Is to the S. *c.* of the Reclination 20°	9.972986
So is the Sine of the Declination 30°	9.698970
To the S. *c.* of h O z = 61° : 59′	9.671956
The Complement of this is the new Declination	28° : 01′

3. To find the Distance of the Horizon from the Meridian O B.

As Radius	10.
Is to the Sine of the Reclination 20°	9.534052
So is the Tangent of the Declination	9.761439
To the Tangent of h O = 11° : 10′	9.295491

The Complement of this is O B = 78° : 50′ the Distance required.

4. To find the Stile's Height P r.

You have in the Triangle P o r, Right-angled at r, the Angle h O Z = 61° 59′, and Side P o = 15° : 40 (= P Z — O Z) then

As Radius	10.
Is to the Sine of P O = 15° : 40′	9.431492
So is the Sine of 61° : 59′	9.945868
To the Sine of the Stiles Height P r = 13° : 47′	9.377297

5. To find the Distance of the Substile from the Meridian o r.

As the S. *c.* of the Angle h O z = 61° : 59′	9.671847
Is to Radius	10.
So is the T. *c.* of O P = 15° : 40′	10.552129
To the Tangent Comp. of the Distance of the Substile 7° : 30′	10.880282

6. To find the Difference of Longitude o P r

As the S. *c.* of o P = 15° : 40′	9.983558
Is to Radius	10.
So is the T. *c.* of P o r = 61° : 59′	9.725674
To the Tangent of o P r = 28° : 55′	9.742116

Having by the preceding Work found the neceſſary Requiſites, make a Table as for an upright Decliner (See *Chap.* 10.) But before you begin, you may obſerve by the Projection.

1. THAT when the Sun is in the Tropic of ♑, he will ſhine upon this Plane from the Time that he riſes until he ſets.

2. THAT when he is in the Equinoctial he will ſhine upon it from the Time of his Riſing until about $3^h \frac{1}{2}$ in the Afternoon.

3. THAT when he is in the Tropic of ♋, he will appear upon it between four and five in the Morning, and go off it between two and three in the Afternoon, and this will let you know what Time to put in the Table.

Hours

Hours.	Angl. Pole.	Hour Diſtances
	D : ′′′	D : ′′′
4	88 : 55	85 : 28
3	73 : 55	39 : 34
2	58 : 55	21 : 34
1	43 : 55	12 : 56
12	28 : 55	7 : 30
11	13 : 55	3 : 23
—	Sub-	ſtile
10	1 : 05	0 : 16
9	16 : 05	3 : 56
8	31 : 05	8 : 11
7	46 : 05	14 : 54
6	61 : 05	23 : 20
5	76 : 05	43 : 53

To draw the Dial.

1. DRAW A B for the Horizon, and make Choice of ſome Point therein, as C nearer to B on the Eaſt Part, than to A on the Weſt, that ſo the Hour-lines near the Subſtile (which muſt be upon the Weſt Side of the Meridian, becauſe the Declination is from the South towards the Eaſt) may be at the greater Diſtance upon the Limits of the Plane.

2. WITH 60 Degrees in your Compaſſes, from a Line of Chords, and one Foot in C draw the Arch *d* E *g*, upon which ſet 78° : 50′, the Diſtance of the Meridian and Horizon (found at Art. 3.) from *d* to E, and draw the Line C E 12 for the Hour-line of 12.

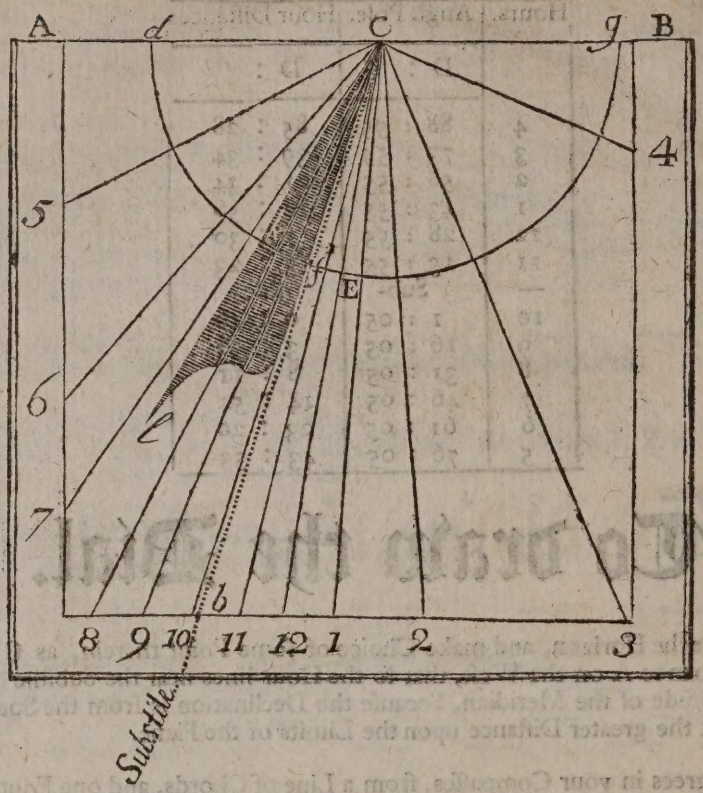
3. FROM the Line of Chords take 7° : 30′, the Diſtance of the Subſtile from the Meridian (found at Art. 5.) and ſet that on the Weſt Side from E to *f*, and from C draw C *f b* for the Subſtile.

4. FROM *f* ſet off the Hour Diſtances both Ways, as you have them in the third Column, then draw the Hour-lines as in the *Fig.* following.

5. FROM *f* ſet 13° : 47′ the Height of the Stile (found at Art. 5.) to *k*, and from C draw C *k l* for the Stile, which muſt ſtand directly over the Subſtile.

A South-

A South Dial Declining East 30° Reclining 20°.



VARIETY 3.

To draw the Hour-lines upon a South reclining Plane, declining East or West, which passeth between the Pole and the Horizon.

EXAMPLE.

Suppose a Plane in the Latitude of —— —— 51° : 32′ = NP.
Declines from the South Eastward —— —— 30 : 00 = SD.
And reclines from the Zenith —— —— 55 : 00 = ZE.

This may be represented in *Fig.* 1. of *Chap.* 18. by A E *r* O B where Z E is the Reclination, and S D = C N = E Z N, the Declination as above. By these you are to find,

1. The new Latitude where this Plane shall be an upright Decliner.
2. The Plane's new Declination in that new Latitude
3. The Substile Distance from the Meridian —— — —— *r* O
4. The Height of the Stile above the Substile —— —— *r* P
5. The Plane's Difference of Longitude —— —— *r* P O
6. The Distance of the Meridian and Horizon —— —— —— B *o*.

Note, That I is the Pole of the Plane *r* P I its Meridian, and A Z B its Base.

1. To

1. To find the New Latitude.
In the Triangle E Z O Right-angled at E, you have given the Side Z E = 55°, and the Angle E Z *o* = 30 Degrees to find O Z.

As Radius is to the S. *c.* of E Z *o* = 30°	9.937531
So is the T. *c.* of E Z = 55°	9.845227
To the T. *c* of Z O = 58° : 46′	9.782758

From this take Z P = 38° 28′ the Complement of the old Latitude, and there will remain 20° 18′ for the Complement of the new Latitude, which is 69° : 42′; or add 58° : 46′ = Z O to the old Latitude Z Æ = 51° : 32′, and the Sum is *O* Æ = 110° : 18′, the Supplement of this is 69° : 42′ the new Latitude as before.

2. To find the new Declination.

As Radius	10.
Is to the S. *c.* of E Z = 55°	9.758591
So is the Sine of the Angle E Z O = 30°	—9.698970
To the S. *c.* of the Plane's Declination in the new Latitude 73° : 20′ = *r* P O	9.457561

The Complement of this is 16° : 40′ the new Declination required.

3. To find the Distance of the Substile from the Meridian *r* O.
In the Triangle *r* O P Right-angled at *r*, you have the Angle *r* P O = 73° : 20′, and Complement of the new Latitude P O = 20° : 18′.

Then, as the S. *c.* of *r* O P = 73° : 20′	9.457584
Is to Radius	10.
So is the T. *c.* of 20° : 18′	10.431902
To the T. *c.* of *r* O = 6° : 3′	10.974318

4. To find the Height of the Stile above the Substile *r* P.

As Radius is to the Sine of P O = 20° 18′	—9.540249
So is the Sine of *r* O P = 73° : 20′	9.981361
To the Sine of the Stile's Height *r* P = 19° 25′	9.521610

5. To find the Difference of Longitud *r* P O.

As Radius is to the S. *c.* of P O = 20° : 18′	9.972151
So is the Tangent of *r* O P = 73° : 20′	—10.523777
To the T. *c.* of *r* P O = 17° : 42′	10.495928
Reduce this 17° : 42′ into Time, and it is	1ʰ : 9′ : 48″
Which take from 12 Hours, and the Remainder is	10 : 50 : 12

This shews that the Substile will fall at 50′ : 12″ after 10 in the Morning.

6. To find the Distance of the Meridian and Horizon B O.
You have in the Triangle B O N Right-angled at N the Angle N B O = 35° (the Complement of the Reclination) and N B = 60° the Complement of the Declination. Then

As Radius is to the S. *c.* of N B O = 35°	9.913364
So is the T. *c.* of N B = 60°	9.761439
To the T. *c.* of B O = 64° : 41′	9.974803

To

To know what Hour-lines must be drawn upon this Plane, you must observe by the first *Fig.* of this *Chap.* that when the Sun is in the Tropic of ♋, he will appear upon it between four and five in the Morning, and go off it between five and six in the Afternoon.

Hours.	Angles at the Pole.	Angles at the Plane.
	D : m	D : m
5	87 : 18	81 : 56
6	72 : 18	46 : 10
7	57 : 18	27 : 22
8	42 : 18	16 : 50
9	27 : 18	9 : 44
10	12 : 11	4 : 9
	Sub-	ſtile.
11	2 : 42	0 : 54
12	17 : 42	6 : 3
1	32 : 42	12 : 3
2	47 : 42	20 : 4
3	62 : 42	32 : 48
4	77 : 42	56 : 44

To draw the Dial.

1. DRAW the horizontal Line A C B to repreſent the Baſe A Z B in the Projection.

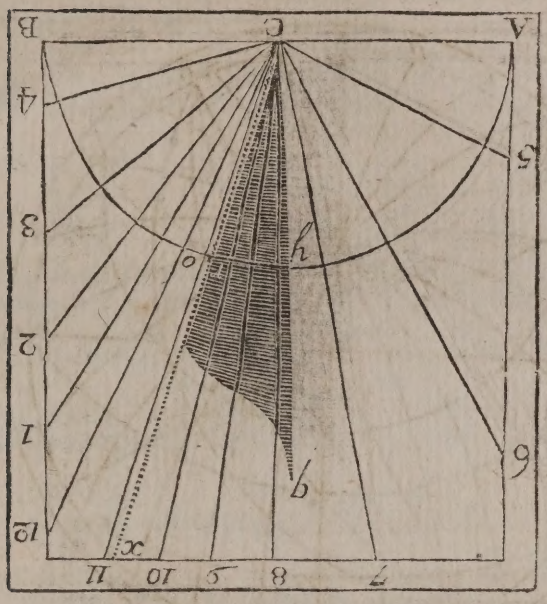
2. TAKE 60 Degrees in your Compaſſes from a Line of Chords, and ſetting one Foot in any convenient Point in the Line A B, as at C, upon that as a Center deſcribe the Arch A *r* O B, and this ſhall repreſent the reclining Plane A E *r* O B in the Projection.

3. TAKE 64° : 41′ in your Compaſſes (the Diſtance of the Meridian and Horizon, and ſet that from B to O, and from C thro' O draw the Hour-line of 12.

4. TAKE 6° : 3′, and ſet that from O to *r*, on the Weſt Side of O, and from C thro' *r*, draw C *r x* for the Subſtile, and from *r* ſet off the Angles at the Plane, or Hour Diſtances, as you have them in the third Column of the Table, and thro' the Points in the Arch ſo found, draw the Hour-lines.

5. TAKE 19° : 25′ (found at Art, 4.) and ſet that from *r* to Y, and draw C Y *q* for the Stile, which muſt be placed juſt over the Subſtile C *r* X.

CHAP.



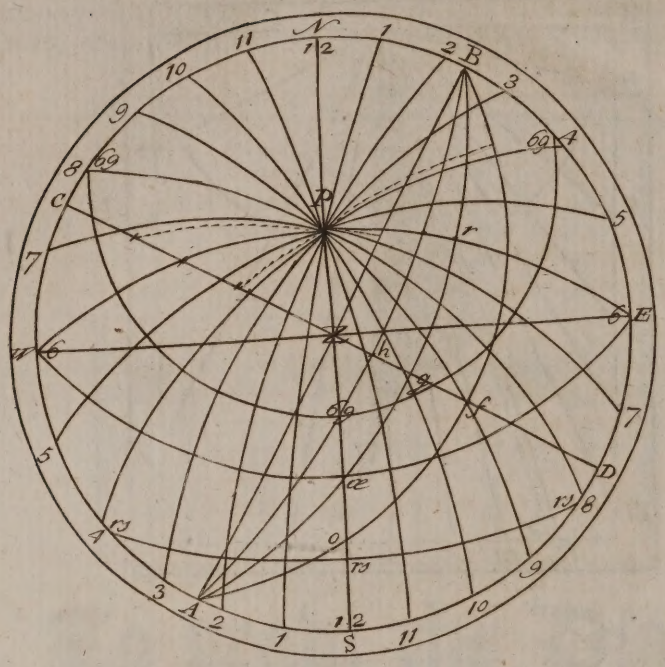
CHAP. XIX.

TO draw the Hour-lines upon a polar Plane declining Eaſt or Weſt, being the firſt Variety of North declining, reclining Planes.

THERE are three Varieties of North reclining, declining Planes, as there were of South reclining Decliners. For,

1. The Plane may paſs through the Interſection of the Meridian and Equinoctial, as A *g* B, and then it is called a declining polar Plane, and hath the Subſtile always at Right-angles with the Meridian.

2. OR elſe paſſeth above the Interſection of the Meridian and Equinoctial, as A *b* B, or under it, as A *f* B (See the following Scheme).



VARIETY 1.

In the Projection following, let A Z B be the Baſe of a Plane declining from the north Part of the Horizon at N. towards the Weſt 60° = N C. and reclining from the Zenith 32° : 11′ = Z *g*, and let the Latitude of the Place be P N = 51° : 32′, and its Complement P Z = 38° : 28′, then will the reclining Plane A *g* B paſs through the Interſection of the Meridian and Equinoctial at Œ, and be an upright Decliner in thoſe Places where the Pole lies in the Horizon, conſequently have no new Latitude.

The new Declination is the Angle *g æ* Z, and P *r* is the Height of the Stile above the Subſtile, the Subſtile being always diſtant from the Meridian 90 Degrees, the Diſtance of the Meridian from the Horizon is A Œ.

Now that you may be ſure whether this is a declining polar Plane or not, you have in the Triangle *z g æ*, Right-angled at *g*, the Reclination *z g*, and Declination *g z e*, to find Z Æ.

Say,

As Radius	10.
Is to the *S. c.* of *g z æ* = 60° : 0′	9.698970
So is the T. *c.* of Z *g* = 32° : 11′	10.201123
To the T. *c.* of Z Œ = 51° : 32′	9.900093

Which being the Latitude of the Place, ſhews that the Plane paſſeth through the Interſection of the Meridian and Equinoctial, and conſequently that it is a delining polar Plane.

Again, if you have the Latitude of the Place, ſuppoſe Z Œ = 51° : 32′, and alſo the Plane's Declination, ſuppoſe Œ Z *g* = 60°, you may find what Reclination the Plane muſt have to become a declining Polar.

Thus, As the S. *c.* of the Declination Œ Z *g* = 60°	9.698970
Is to Radius	10.
So is the Tangent Comp. of the Latitude Z Œ = 51° : 32′	9.900086
To the T. *c.* Z *g* = 32° : 11′ the Reclination required	10.201116

Or

Or if you have the Reclination given, and would find a Declination anſwering thereto, then find the Angle $g\,z\,æ$ in the ſame Triangle.

1. *To find the new Declination* $z\,æ\,g$.

In the former Triangler Z Œ g, you have the Latitude Z Æ $= 51° : 32'$, and Reclination $Zg = 32° : 11'$

Then, As the Sine of $51° : 32'$	——	9.893745
Is to Radius	——	10.
So is the Sine of $32° : 11'$	——	9.726426
To the Sine of Z Æ $g = 42° : 52'$, the new Declination	——	9.832681

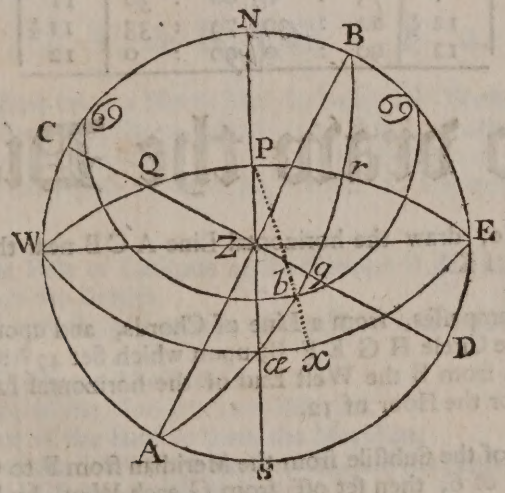
The Meaſure of this Angle is P r, the Height of the Stile above the Subſtile.

2. *To find the Diſtance of the Meridian and Horizon* A Æ.

As Radius	——	10.
Is to the Sine of the Latitu(e Z $æ = 51° : 32'$	——	9.893745
So is the Sine of the Declination Æ $Zg = 60°$	——	9.937531
To the Sine of $g\,æ = 42° : 42'$	——	9.831276

Take this from 90°, and there will remain $47° : 18' = A\,Æ$.

Note, That the Pole of the Plane A $æ\,g\,r$ B is at Q, upon the Hour-line of Six, then the Hour-line of Six, W Q P r E, is its Meridian, and makes an Angle with the Meridian of the Place of $90 = Æ\,P\,r$, which is the Difference of Longitude; hence we ſee that the Subſtile will be upon the Hour of Six, the Diſtance of which from the Meridian is Œ r, equal to 90° as above.



The Latitude of the Place is	——	51° : 32'
The Declination of the Plane S D = N C	——	60 : 00
The Reclination of the Plane $Z\,g$	——	32 : 11
The New Latitude where it would be an upright Decliner	——	0 : 0
The New Declination in that Latitude $g\,æ\,z$	——	42 : 52
The Diſtance of the Subſtile from the Meridian $æ\,r$	——	90 : 00
The Height of the Stile above the Plane P $r = g\,æ\,z$	——	42 : 52
The Plane's Difference of Longitude Æ P r.	——	90 : 00
The Diſtance of the Meridian and Horizon $A\,Æ$	——	47 : 18

The Subſtile being the Hour-line of 6, the Angles for the Hours on either Side the Hour of 6, are equal, therefore add 15° on either Side of the Subſtile, and you will make the ſecond Column of the Table. Then the third Column is made from the ſecond as for an upright Decliner, ſo for the Hours of 5 or 7, the Angle at the Pole for theſe being 15°. You may ſay,

As Radius —— —— —— ——	10.
Is to the Sine of the Stile's Height 42° : 52′ —— ——	9.832697
So is the Tangent of 15° —— —— ——	9.428052
To the Tangent of 10° : 20′ —— —— ——	9.260749

Set this in the third Column againſt 5 or 7, and in like manner find the reſt of the Numbers to fill up this Column.

Hours	Angl. Pole. D : ″′	Angle Pl. D : ″′	Hours
6	— Sub-	——	ſtile 6
5 ½	7 : 30	5 : 7	6 ½
5	15 : 0	10 : 20	7
4 ½	22 : 30	15 : 44	7 ½
4	30 : 0	21 : 27	8
3 ½	37 : 30	27 : 34	8 ½
3	45 : 0	34 : 14	9
2 ½	52 : 30	41 : 34	9 ½
2	60 : 0	49 : 41	10
1 ½	67 : 30	58 : 40	10 ½
1	75 : 0	68 : 30	11
12 ½	82 : 30	79 : 33	11 ½
12	90 : 0	90 : 0	12

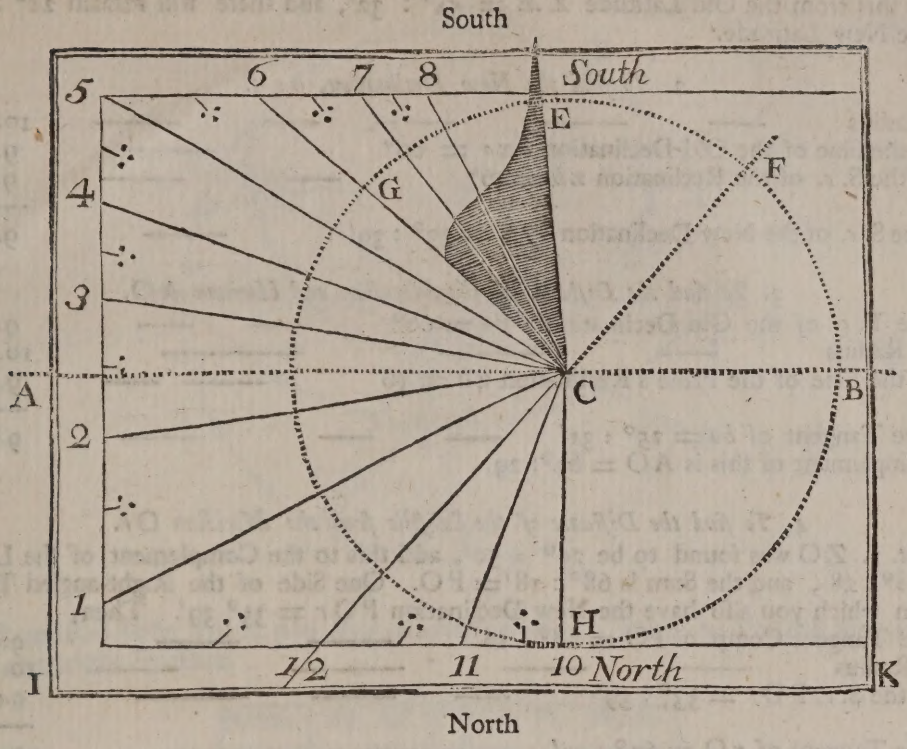
To draw the Dial.

1. THE Table being made, draw the horizontal Line A C B near the Middle of the Plane, and parallel to the Baſe I K.

2. TAKE 60° in your Compaſſes, from a Line of Chords, and upon any Point in the Line A C B, as C deſcribe the Circle H G E F B, upon which Set 47° : 18′, the Diſtance of the Meridian and Horizon from B the Weſt End of the horizontal Line to F, upwards, and draw the Line F C D for the Hour of 12.

3. SET 90° the Diſtance of the Subſtile from the Meridian from F to G, and draw C G for the Subſtile, and Hour-line of 6, then ſet off from G each Way, by help of a Line of Chords upon the Circle E G F, the Hour Diſtances, as you find them in the third Column of the Table, *viz.* For the Hours of 5 and 7, ſet off 10° : 20′; for the Hours 4 and 8, 21° : 27′, and ſo of the reſt (but you need go no further towards F than 8) ſo ſhall you have Points in the Circle through which ſtreight Lines being drawn from the Center are the true Hour-lines on the Plane.

4. SET off the Height of the Stile 42° : 52′ from G to E, and draw the Line C E for the Axis of the World, which being ſet exactly over the Subſtile will complete the Dial.



VARIETY 2.

To draw the Hour-lines upon a North Dial declining 60° Weſt, and reclining 16°, which may be repreſented in the next Scheme following by A *h* B, which cutteth the Meridian at O, between the Zenith and Equinoctial Z *h*, being the Reclination = 16°, and N C the Declination of the Plane from the North towards the Weſt = 60 Degrees.

In the Figure, Let
- The Height of the Pole or Latitude of the Place be Z Æ, which is the Diſtance of the Equinoctial from the Zenith.
- O Æ is the new Latitude, or Latitude of the Place where the Plane would ſtand as an upright Decliner.
- The Angle *h o z* is the new Declination.
- O A is the Diſtance of the Meridian and Horizon.
- O *r* is the Diſtance of the Subſtile from the Meridian.
- P *r* is the Height of the Stile above the Subſtile, and the Angle.
- O P *r*, the Plane's Difference of Longitude.

1. *To find the new Latitude* O Æ.

In the Triangle *o h z*, Right-angled at *h*, you have the Reclination z *h* = 16 Degrees, and Angle *o z h* = 60°, the Plane's Declination, Then,

As Radius	10.
Is to the S. *c.* of 60°	6.698970
So is the T. *c.* of 16°	10.542503
To the T. *c.* of Z O = 29° : 50′	10.241473

Take

Take this from the Old Latitude $Z Æ = 51° : 32'$, and there will remain $21° : 42' = O æ$ the New Latitude.

2. *To find the New Declination, h o z.*

As Radius	10.
Is to the Sine of the Old Declination $h z o = 60°$	9.937531
So is the S. *c.* of the Reclination $z h = 16°$	9.982842
To the S. *c.* of the New Declination $h o z = 33° : 39'$	9.920373

3. *To find the Distance of the Meridian and Horizon* A O.

As the T. *c.* of the Old Declination $h z o = 60°$	9.761439
Is to Radius	10.
So is the Sine of the Plane's Reclination $z h = 16$	9.440338
To the Tangent of $h o = 25° : 31'$	9.678899

The Complement of this is $A O = 64° : 29$.

4. *To find the Distance of the Substile from the Meridian* O r.

At *Art.* 1. Z O was found to be $29° : 50'$, add this to the Complement of the Latitude $P z = 38° : 28'$, and the Sum is $68° : 18' = P O$. One Side of the Right-angled Triangle O P r, in which you also have the New Declination $P O r = 33° 39'$. Then,

As the Tangent Comp. of $P O = 68° : 18'$	9.599827
Is to Radius	10.
So is the S. *c.* $P O r = 33° : 39'$	9.920352
To the Tangent of $r O = 64° : 27'$	10.320525

5. *To find the Height of the Stile above Substile* P r.

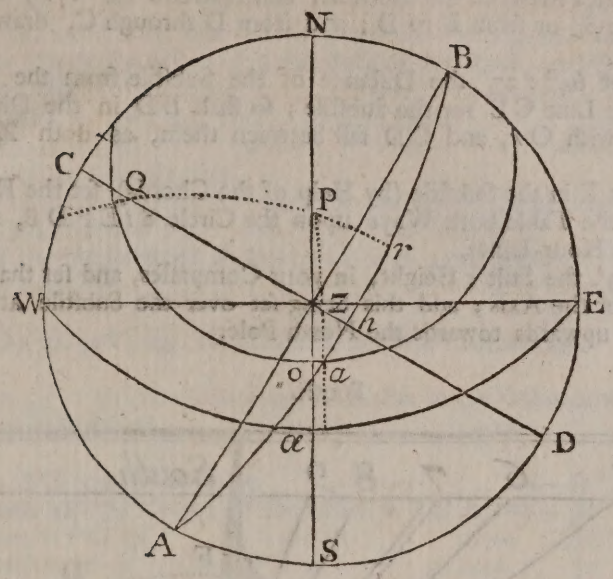
As Radius	10.
Is to the Sine of $P O = 68° : 18'$	9.968078
So is the Sine of $P O r = 33° : 39'$	9.743602
To the Sine of the Stile's Height above the Substile $P r = 30° : 59'$	9.711680

6. *To find the Plane's Difference of Longitude* O P r.

As the T. *c.* of $P O r = 33° : 29'$	10.176739
Is to Radius	10.
So is the S. *c.* $P O = 68° : 18'$	9.567904
To the T. *c.* of $O P r = 76° : 10'$	9.391165

By the preceding Work, we find the New Lat. $Æ O$ to be	$21° : 42'$
The New Declination in that Lat. $Z O h =$	$33 : 39$
The Distance of the Meridian and Horizon $A O =$	$64 : 29$
The Height of the Stile above the Substile $P r =$	$30 : 59$
The Plane's Difference of Longitude $O P r =$	$76 : 10$
And the Distance of the Substile from the Meridian	$64 : 27$

The



The Requiſites being found, you may proceed to make a Table, as directed formerly, for making an upright Decliner.

Hours.	An. Pol.		Ang. Pla.	Hours.
	d : m		d : m	
1	88 : 50		87 : 44	1
2	73 : 50		60 : 37	0
3	58 : 50		40 : 24	9
4	43 : 50		26 : 18	18
5	28 : 50		15 : 49	17
6	13 : 50		7 : 13	6
Sub.				Stile.
7	1 : 10		0 : 36	5
8	16 : 10		8 : 29	4
9	31 : 10		17 : 18	3
10	46 : 10		28 : 12	2
11	61 : 10		43 : 5	1
12	76 : 10		64 : 26	12

To draw the Dial.

1. DRAW the Horizontal Line A C B, about the Middle of the Plane, and parallel to the Baſe, to repreſent A Z B, in the laſt Figure.

2. MAKE Choice of ſome Point therein (nearer to B than to A, that Hour-Lines may have the more Room to ſpread) as C, for a Center; upon which, with 60° in your Compaſſes from a Line of Chords draw the Circle S E F D B, which may repreſent the reclining Plane A *b* B.

3. TAKE

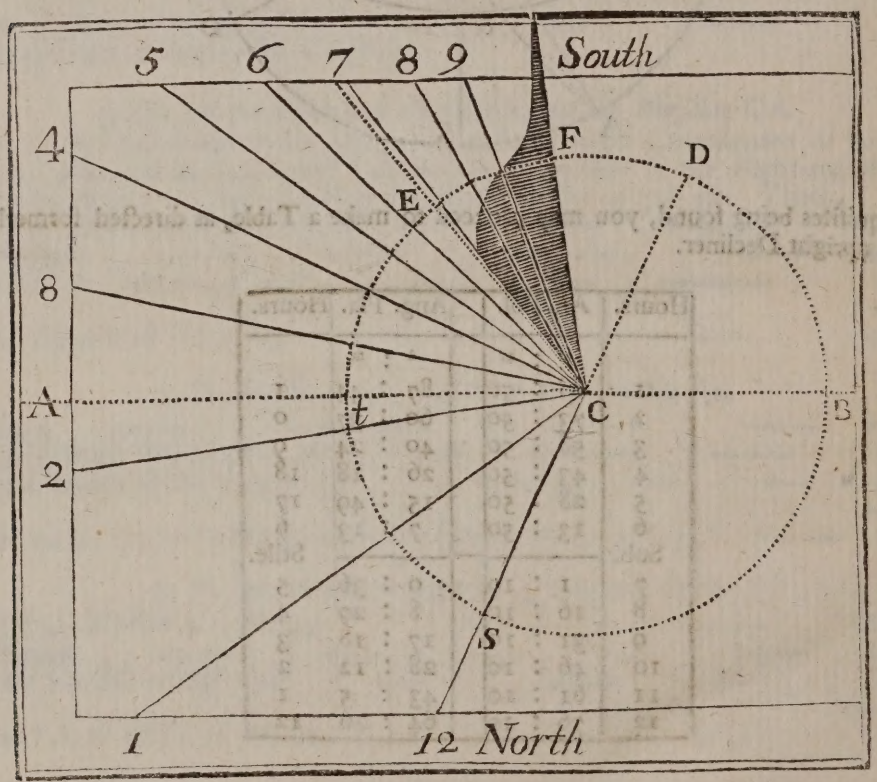
3. TAKE the Diſtance between the Meridian and Horizon 64° : 29′ in your Compaſſes, and ſet that from *t* to S, or from B to D; and from D through C, draw D C 12, for the Hour-Line of 12.

4. FROM D to E ſet 64° : 27′ the Diſtance of the Subſtile from the Meridian Eaſtward from it, and draw the Line C E for the Subſtile; ſo ſhall B D in the Dial agree with A O in the Scheme, D E with O *r*, and C D fall between them, as doth Z O S the Meridian in the Scheme.

5. FROM the Point E in the Subſtile (by Help of the Chords) ſet the Hour Diſtances from the third Column of the Table both Ways upon the Circle S *t* E F D B, and from C thro' thoſe Points draw the Hour-Lines.

6. TAKE 30° : 59′, the Stile's Height, in your Compaſſes, and ſet that from E to F, and draw the Line C F for the Axis; add this being ſet over the Subſtile at Right Angles with the Plane, will point upwards towards the North Pole.

South.



North.

VARIETY 3.

To draw the Hour-lines upon a North Dial, declining 60° Weſt, and reclining 54°, which cutteth the Meridian between the Equinoctial and the Horizon. This may be repreſented in the firſt Scheme of this Chapter, or the next following by A *f* B.

ALTHOUGH

ALTHOUH the making of this Dial differeth but a little from the Former; yet leaſt any thing ſhould be a Rub in a Learner's Way, we will ſet down the Parts given and required, as they appear in the following Figure (in which Q is the Pole of the Plane, and Q P *r* its Meridian.)

Given { The Latitude of the Place Z Œ —— —— —— 51° : 32′
The Declination of the Plane N C = O Z *f* (Weſt) —— 60 : 00
The Reclination Z *f* = —— —— —— 54 : 00 }

The Parts required may be found thus,

IN the Triangle Z *f* O Right-angled at *f*, you have the Side Z *f*, and Angle O Z *f*, to find,

1. THE Side Z O, and that will be —— 70° : 02′
From which take the old Latitude Z Æ = —— 51 : 32 } *æ o* —— 18 : 30
Remains the new Latitude

2. THE Angle Z O *f* = 59° : 24′, its Comp. is the new Declination — 30 : 36

3. The Side *f* O = 54° : 30′, the Complement of this is the Diſtance of the Meridian and Horizon A *o* = —— —— } 35 : 30

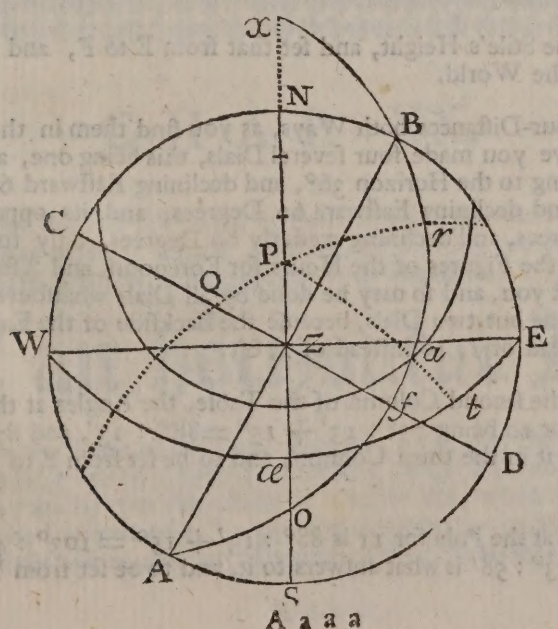
AGAIN, The Side Z O was found to be —— —— 70° : 02′
To which add the Comp. of the old Lat. P Z = —— 38 : 28
The Sum is P O = —— —— 108 : 30
The Supplement of which is P *x* = —— 71 : 30

Now in the Triangle P *x r*, Right-angled at *r*, you have P *x*, and the Angle P *x r* = Z *o f* = 59° : 24′, by theſe you will find.

4. THE Subſtile's Diſtance from the north Part of the Meridian *x r* = —— 56° : 41′
The Supplement is *r o* = the Subſtile's diſtance from the South Part of the Meridian —— —— —— 123 : 19

5. THE Stile's Height P *r* = —— —— —— —— 54 : 43

6. THE Plane's Difference of Longitude *x* P *r* = —— —— 61 : 47
Or its Supplement O P *r* —— —— —— 118 : 13



THE Difference of Longitude being found, proceed to make the TABLE.

Hours	Angl. Pole. D : ′	Angle Pl. D : ′	Hours
12	61 : 47	56 : 41	12
11	46 : 47	40 : 59	1
10	31 : 47	26 : 50	2
9	16 : 47	13 : 50	3
8	1 : 47	1 : 25	4
Sub-		ſtile	
7	13 : 13	10 : 51	5
6	28 : 13	23 : 39	6
5	43 : 13	37 : 30	7
4	58 : 13	52 : 48	8
3	73 : 13	69 : 42	9

To draw the Dial.

1. DRAW the horizontal Line A C B, *&c.* as directed for drawing the Laſt, and upon C as a Center, deſcribe the Circle S *t* E F D B, then from B to D, or from *t* to S ſet 35° : 30′, the Diſtance of the Meridian and Horizon, and draw the Line D C S 12 for the Hour of 12.

2. FROM S ſet 56° : 41′ (the Diſtance of the Subſtile from the Meridian) to E, and draw C E for the Subſtile.

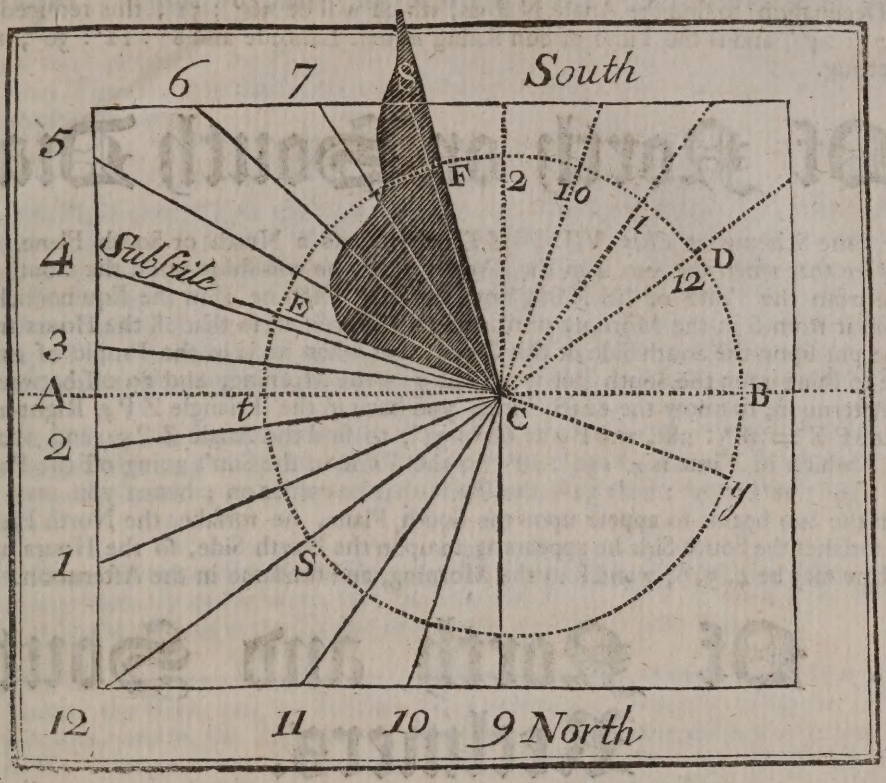
3. TAKE 54° : 43′, the Stile's Height, and ſet that from E to F, and draw C F out to repreſent the Axis of the World.

4. FROM E, ſet the Hour-Diſtances both Ways, as you find them in the third Column of the Table; and thus have you made four ſeveral Dials, this being one, and its Underſide being a South Plane inclining to the Horizon 36°, and declining Eaſtward 60°. As alſo the North Plane reclining 54° and declining Eaſtward 60 Degrees, and its oppoſite South inclining to the Horizon 36 Degrees, and declining weſterly 60 Degrees, only turning the Dials upſide down, and changing the Figures of the Hours for Forenoon, and Afternoon, as the Nature of the Dial will direct you, and ſo may be done by all Dials whatſoever, except the Eaſt and Weſt; for there can be but two Dials, becauſe the Backſide of the Eaſt is a Weſt Dial, only here you put 11 inſtead of 1, 10 inſtead of 2, *&c.*

Note, That in making the ſecond Column of the Table, the Angles at the Pole for 10, and 11, were omitted, that for 10 being 73° : 13′ + 15° = 88° : 13′, and 87° : 53′, is what ſhould have been ſet againſt it in the third Column, and to be ſet from E to that 9 that is between F and D.

AGAIN, The Angle at the Pole for 11 is 88° : 13′ + 15° = 103° : 47′, the Supplement of this is 76° : 47′, and 73° : 58′ is what anſwers to it, and to be ſet from Y, up to 11, then from theſe

theſe Points in the Arch F D draw Lines thro' the Center at C, and theſe ſhall be the Hour-lines for 9, 10, and 11.



CHAP. XX.

TO find at what Time the Sun will begin, or ceaſe to ſhine upon any Plane, that ſo by knowing this, the Maker may not appear ignorant, by drawing more, or not ſo many Hour-lines or Parts upon a Dial, than what is agreeable to its Situation.

Of an horizontal Dial.

THIS Plane will give the Time of the Day, from the Time that the Sun riſeth until he ſets; but the Time of his Riſing will be different, according to the Latitude of the Place for which it is made, as you may ſee by the Projection at *Chapter* VII, where W N E S repreſents the Horizon of *London* in the Latitude of 51° : 32′, and is cut by the Tropic of ♋ (the Parallel of the longeſt Day) at a little before 4 in the Morning, and at the ſame Time after 8 in the After-noon.

If you would be more exact, then in the Triangle N P ♋ Right-angled at N, you have the Latitude of the Place N P = 51° 32′, and P ♋ = 66° 31′ (the Complement of the Sun's greateſt Declination) to find the Angle N P ♋, which will be 56° : 51′, this reduced to Time is 3ʰ : 47′ : 24″, and is the Time of Sun Riſing in that Latitude and 8ʰ : 12′ : 36″, the Time of his Setting.

2. Of North or South Dials.

In the ſame Scheme at *Chap.* VII. W Z E, repreſents a North or South Plane, in which you may ſee that when the Sun is in the Tropic of ♑, he will ſhine upon the South Side of this Plane from the Time of his Riſing until he ſets, when he is in the Equinoctial, he will ſhine upon it from 6 in the Morning until 6 in the Afternoon, ſo that all the Hours from 6 to 6, may be put upon the South Side of this Plane; but when he is in the Tropic of ♋, he will not begin to ſhine upon the South Side until paſt 7 in the Morning, and go off between 4 and 5 in the Afternoon, to know the exact Time, you have in the Triangle Z P *q*, Right-angled at Z, the Side P Z = 38° : 28′, and P *q* = 66° : 31′, to find the Angle Z P *q*, and that will be 69° : 52′, which in Time is 4ʰ : 39′ : 28″ for the Time of the Sun's going off the Plane, and 12ʰ − 4ʰ : 39′ : 28″ = 7ʰ : 20′ : 32″, the Time that he comes on; hence you may obſerve, that when the Sun begins to appear upon the South Plane, he forſakes the North Plane; and when he forſakes the South Side he appears again upon the North Side, ſo the Hours upon the North Plane may be 4, 5, 6, 7 and 8 in the Morning, and the ſame in the Afternoon.

3. Of North and South Recliners.

Given the Latitude of the Place, and Declination of the Sun, to find what Time he ſhall part from one Side of any erect North or South Plane declining Eaſt or Weſt, to the other.

EXAMPLE.

Let the Plane be that at *Chap.* X, the Declination of which is from the South weſtward —— —— —— — 30° : 0′
The Stile's Height is P Y = —— —— —— 32 : 36
The Difference of Longitude, or Inclination of Meridian is Z P Y = —— 36 : 24

This is repreſented in the Projection by *w z x*, in this Scheme you may obſerve, that when the Sun is in the Tropic of ♑, he will ſhine upon the Plane as ſoon as he riſes, which is a little after 8 a-Clock, and when he is in the Tropic of ♋ he will depart from the South Side of the Plane at 9, where this Tropic cuts it, and then begin to ſhine upon the North Side: Now to find the Time when the Sun comes to 9, you have in the Triangle *q* P Y, Right-angled at Y, the Side P *q* = 66° : 31′, and the Stile's Height P Y = 32° : 36′, by theſe you will find the Angle *q* P Y, to be 73° : 52′, to which add the Difference of Meridians 36° : 24′, and the Sum is 110° : 16′, which in Time is 7ʰ : 21′ : 4″ in the Afternoon; hence we may know that this Plane will admit of all Hours from 8 in the Morning to 7 in the Afternoon.

4. Of South Recliners.

GIVEN the Latitude of the Place, the Declination of the Sun, and Reclination of a Plane, to find what Time the Sun ſhall forſake a North inclining Plane, to ſhine upon a South Recliner oppoſite thereto.

EXAMPLE.

1. LET the Plane propos'd be the ſame with the firſt Variety of *Chap.* XV, repreſented in the Projection at *Chap.* XVI, by W Y E, which is cut by the Tropic of ♋ in two Points U and *w*, to either of which draw an Arch, as P U, or P *w*, and you will have the Triangle P Y *w* Right-angled at Y, in this Triangle you have the Elevation of the Pole, or new Latitude of the Plane P Y = 20° : 28′, and Side P *w* = 66° : 31′ given by theſe you will find the Angle Y P *w* = 80° : 40′, which in Time is 5^h : 21′ : 40″ in the Afternoon, when the Sun will ceaſe to ſhine upon the South Side of this Plane, and begin to ſhine upon the North Side.

Then if from 12 Hours, you take 5^h : 21′ : 40″, there will remain 6^h : 38′ : 23″, the Time in the Morning when he will ceaſe to ſhine upon the North, or inclining Side, and begin upon the South or reclining Side, and the like you may do for any other Declination of the Sun.

2. IF the Plane reclines to the North Pole, as does W P E (See the ſecond Variety of *Chap.* XV) then it is evident by the Scheme, that the Sun will ſhine upon it from 6 in the Morning until 6 at Night; when in the Tropic of Cancer, not before nor after.

3. LET the Plane recline more than the Complement of the Latitude of the Place, as W *h* E, (See Variety the third, and the Scheme above-mentioned) which is cut by the Tropic of ♋, in two Points *e* and *m*, the Elevation of the Pole, or new Latitude is P *h* = 31° : 32′, one Side of the Triangle P *h m*, Right-angled at *h*, and P *m* = 66° : 31′ is alſo given, by theſe you will find the Angle *h* P *m* to be 74° : 32′ which reduc'd to Time is 4^h : 58′ : 2″, when the Sun in the Tropic of ♋ will ceaſe to ſhine upon the North or inclining Side of this Plane, and begin upon the South or reclining Side, upon which it will continue to 7^h : 1′ : 58″ in the Afternoon.

5. North reclining Planes.

EXAMPLE 1.

1. Suppoſe a north Plane reclines from the Zenith 24° (See Caſe the firſt of *Chap.* XVI) this is repreſented in the Projection of that *Chap.* by W *d* E, which falls between the Zenith Z, and Equinoctial W Æ E, and is cut by the Tropic of ♋ at the Points *i* and *k*.

From P draw the Meridian P *i* = 66° : 31′ with which, and the new Latitude P *d* = 62° : 28′, you will find the Angle *d* P *i* (of the Triangle *d* P *i*) to be 36° : 55, which in Time is 2^h : 27′ 40″, when the Sun in the Afternoon at *i*, will ceaſe to ſhine upon the South or inclining Side of this Plane, and then begin upon the north Side.

Take 2^h : 27′ : 40″ from 12 Hours, and there will remain 9^h : 32′ : 20″, the Time in the Morning, at *k*, when he will forſake the upper, or north Side, and begin to ſhine upon the South or inclining Side again.

N. B. That when the Sun is in the ſouthern Signs, he ſhines only upon the South or inclining Side of this Plane; but when his Declination is north, as it is from about the 10th of *March* to about the 12th of *September*, he ſhines ſome Part of the Day upon the reclining Side, and ſome Part upon the inclining, as is evident by the Projection.

EXAMPLE 2.

LET a North Plane recline from the Zenith 51° : 32′ at *London*, this Reclination being equal to the Latitude of the Place ſhews it to be an equinoctial Plane, and is repreſented in the Projection at *Chap.* XVI, by W Æ E. See Caſe the 2d of that Chapter.

EXAMPLE 3.

SUPPOSE a North Plane in the Latitude of 51° : 32′ reclines from the Zenith ſouthward 66° : 00′ (See Caſe the 3d of *Chap.* XVI) which is more than the Equinoctial by 14° : 28′, this is repreſented in the Projection by W X E.

It is evident by this Scheme, that the Sun will only illuminate the Reclining or upper Side of this Plane whilſt he is in the northern Signs, the reſt of the Year he ſhineth upon both, until his Declination South be more than 14° : 28′; but 14° : 28′ is the Declination of the Parallel, when the Sun ceaſeth to ſhine upon the reclining Side, and only ſhines upon the inclining Side of the Plane.

Now when the Sun's Declination is 14° : 28′ South, his Amplitude for the Latitude of 51° : 32′ will be 23° : 41′, ſet this from W to *f*, and draw the Parallel *f x*, which will cut the Ecliptic at ☉, from which Point draw Part of a Meridian ☉ *l* to cut the Equinoctial at *l*, and that will form the Triangle W ☉ *l*, Right-angled at *l*, in which you have the preſent Declination ☉ *l* = 14° 28′, and greateſt Declination *l* W ☉, to find W ☉, thus,

As the Sine of the greateſt Declination 23° : 29′ —— ——	9.600409
Is to the Sine of the preſent Declination 14 : 28 —— ——	9.397621
So is Radius —— —— —— ——	10.
To the Sine of W ☉ = 38° : 50′ —— —— ——	9.797212
From this take —— 30 : 00 for Libra	

And there remains —— 8 : 50 in Scorpio, and this is the Place of the Sun, when he forſakes the North, or reclining Side, and begins to ſhine upon the South, or inclining Side, and if you look for this in an Ephemeris, you will find it anſwer to the 21ſt Day of *October*.

AGAIN, when the Sun aſcends to 21° : 10′ of ♒ to be in the ſame Parallel, *viz.* of 14° : 28′ he will forſake the inclining Side, and begin to ſhine upon the reclining Side, and that will be upon the 28th Day of *January*.

6. Of Eaſt or Weſt Recliners.

THE Elevation of the Pole, and Declination of the Sun being given, to find at what Hour the Sun paſſeth, from one Side of an Eaſt or Weſt Recliner to the other.

EXAMPLE.

SUPPOSE a Weſt Plane in the Latitude of 51° : 32′ reclines 35 Degrees from the Zenith, then it may be repreſented by the oblique Circle N *a* S, whoſe Pole is at *t*, its Meridian *t* P *q*, and Reclination Z *a* = 35°. See *Chap.* XVII, where the Stile's Height was found to be 26° : 41′, and Difference of Longitude N P *q* = 66° : 28′.

THE

THE Tropic of ♋ cuts this Plane at *k*, to which Point draw the Meridian P *k*, and that will form the Triangle P *q k*, Right-angled at *q*, in which you have given the Height of the Stile P *q* = 26° : 41′, and Side P *k* = 66° : 31′, by theſe you will find the Angle *q* P *k* to be 77° : 23′, to this add the Difference of Longitude N P *q* = 66° : 28′, and the Sum is N P *k* = 143° : 51′, this reduced to Time is 9ʰ : 35′ : 24″ in the Morning, when the Sun (in the Tropic of ♋.) will ceaſe to ſhine npon the Eaſt inclining Plane, and then begin to ſhine upon the Weſt Recliner, and take 9ʰ : 35′ : 24″ from 12 Hours, and there will remain 2ʰ : 24′ : 36″ in the Afternoon, when the Sun will ceaſe to ſhine upon the Eaſt Recliner N *n* S, and begin to ſhine upon the oppoſite Side, or Weſt Recliner, and the like for any other Reclination and Declination of the Sun.

7. Of South declining Recliners.

EXAMPLE 1.

LET the Plane propos'd be the ſame with the firſt Variety of *Chapter* XVIII, *viz.* a declining polar Plane, as A P B in the firſt Figure of that Chapter.

EVERY ſuch Plane is coincident with ſome Hour-Cirlce or Part, and therefore the Sun in what Parallel ſoever paſſeth from one Side to the other, at the ſame Hour and Minute, which you may find by the Triangle N P B, in which you have N P = 51° : 32′, the Latitude of the Place, and N B = 60 Degrees, the Complement of the Plane's Declination, to find the Angle N P B, which will be 65° : 41′, or 4ʰ : 23′ from Midnight, when the Sun paſſeth off the inclining to the reclining Plane ; but *g* P *z* is equal to N P B, or the Time from Noon, when he forſaketh the Reclining, and ſhineth upon the inclining Side again ; and thus it continueth till the northern Amplitude of the Sun be equal to E B = 30, the Declination of the Plane, from henceforth he ſhineth no more upon the inclining Side in the Morning ; and when the ſouthern Amplitude of the Sun is equal to W A = 30 Degrees, the Plane's Declination, then it ceaſeth to ſhine upon the inclining Side in the Evening alſo, and the reſt of the Year it only ſhines upon the reclining Side of the Plane.

EXAMPLE 2.

GIVEN the Latitude of the Place, and Declination of the Sun, together with the Declination and Reclination of the Plane to find what Time the Sun ſhall ceaſe to ſhine upon the north inclining Side to ſhine upon the ſouth Recliner oppoſite thereto.

1. LET this Example be of the declining reclining Plane, propos'd at Variety the 2d, of *Chap.* XVIII, of which the Height of the Stile above the Plane was found to be P *r* = 13° : 47′
And Difference of Longitude O P *r* = ——— ——— 28 : 55

See the Projection at this 2d Variety, where this Plane is cut by the Tropic of ♋ at *t*, between 4 and 5 in the Morning. and again at *v*, between 3 and 4 in the Afternoon.

2. DRAW the Meridian P *t*, and that will complete the Triangle P *r t*, Right-angled at *r*, in which is given P *t* = P *v* = 66° : 31′, and the Stile's Height P *r* = 13° 47′, to find the Angle *r* P *t*, and that will be 83° : 53′, to which add the Difference of Longitude *o* P *r* = 28° 55′, and the Sum will be 112° : 48′ = *o* P *t*, the Supplement of which is

N P *t*

N P t = 67° : 12′ = 4ʰ : 29′ the Time in the Morning when the Sun (in ♋) at t, will ceaſe to illuminate the inclining Side, and begin to ſhine upon the reclining Side.

3. FROM 83° : 53′ take 28° : 55′ and there will remain 54° : 58′ equal to 3ʰ : 40′, the Time in the Afternoon, when the Sun at v will forſake the Reclining, and begin to illuminate the inclining or oppoſite Side.

EXAMPLE 3.

1.) LET there be given the Latitude of the Place, the Declination of the Sun, *&c.* as at the laſt Example, and let the Plane be the ſame as that at the 3d Variety, at *Chap.* XVIII, and repreſented in *Fig.* 1ſt of that *Chap.* by A E r o B, where the Height of the Stile was found to be P r = —— —— —— —— —— 19° : 25′
And Difference of Longitude r P o = —— —— —— 17 : 42
Here the Tropic of ♋ cuts the Plane at e, between 4 and 5 in the Morning, and at v, between 5 and 6 in the Afternoon.

2.) FROM P draw the Meridians P e, and P v, and theſe will form two equal Triangles r P v, and r P e, Right-angled at r, in one of which, *viz.* r P e is given the Side P e = 66° 31′, and P r = 19° 25′, to find the Angle r P e, which will be 81° : 12′, from which ſubſtract the Difference of Longitude r P o = 17° : 42′ and there will remain N P e = 63° : 30′, or 4ʰ : 14′, the Time in the Morning when the Sun at e will begin to ſhine upon the upper, or reclining Side.

3.) To 81° : 12′ add 17° : 42′, and the Sum is o P v = 98° : 54′, whoſe Supplement is 81° : 6′ = 5ʰ : 24′ : 24″, when the Sun (in ♋) at v, will forſake the upper or reclining Side, and begin to illuminate the lower or inclining Side.

8. Of North declining Recliners.

THERE are three Varieties of theſe Sort of Planes, as at *Chap.* XIX, and for Examples we will take the ſame Declination and Reclinations that you have at the 1ſt, 2d and 3d Varieties of that *Chap.* and by what is there given and found in each Variety, together with the Declination of the Sun, to find what Time he will begin, and ceaſe to ſhine upon the Reclining and Inclining Sides of any of them.

EXAMPLE 1.

LET the Plane A æ b B in the *Fig.* at Variety the 1ſt of *Chap.* XIX be that propos'd, which is cut by the Equinoctial at æ, and the Tropic of ♋ at b, then from P thro' b draw the Meridian P b x, which will from the Triangle P b r, Right-angled at r, (the Pole of the Plane being at Q) in which you have the Side P b = 66° : 31′, and Height of the Stile P r = 42° : 52′, by theſe you will find the Angle b P r, to be 66° : 13′, to which add the Angle N P r = 90°, and the Sum is 156° : 13′ which reduc'd to Time is 10ʰ : 24′ : 52″ in the Morning, when the Sun being in the Tropic of ♋ ſhall ceaſe to ſhine upon the Inclining, and begin to ſhine upon the Reclining Side of this Plane.

EXAMPLE

EXAMPLE 2.

LET it be required to find (the Sun being in the Tropc of ♋) the Time when he will ceaſe to ſhine upon the inclining Side of the Plane A *b* B, whoſe Pole is at Q) See the *Fig.* for Variety the 2d of *Chap.* XIX.) and begin to ſhine upon its reclining Side.

DRAW the Meridian P *a* from P to the Point *a*, where this Plane interſects the Tropic of ♋, and that will form the Triangle P *r a* Right-angled at *r*, in which you have the Side P *a*=66°:31′, and Height of the Stile P *r* = 30° : 59′, by theſe you will find the Angle *a* P *r* to be 74° : 53′ which take from 76° : 10′ (found at Art. 6th of Variety 2d, *Chap.* XIX.) and there will remain 1° : 17′, the Supplement of which is 178° : 43′, this reduc'd is 11^{h} : 54′ : 52″ in the Morning, when the Sun will forſake the Inclining, and begin to ſhine upon the Reclining Side.

EXAMPLE 3.

To find the Time (the Sun being in the Tropic of ♋) when he will forſake the inclining Side of the Plane A *f* B, whoſe Pole is at Q (See the *Figure* at the 3d Variety of *Chap.* XIX,) and begin to ſhine upon the Reclining Side.

DRAW the Meridian P *a t*, which will complete the Triangle P *r a* Right-angled at *r*, in which you have P *a* = 66° : 31′, and the Stile's Height P *r* = 54° : 43′, to find the Angle *a* P *r*, and that will be 52° : 7′, to which add ✗P *r* = 61° : 47′, and the Sum is the Angle ✗ P *a* = 113° : 54′, which in Time is 7^{h} : 35′ : 36″ in the Morning, when the Sun will forſake the Inclining and begin to ſhine upon the Reclining Side, and theſe are all the Varieties that can happen, and tho' we have given Examples of the Sun's ceaſing, and Beginning to ſhine upon the ſeveral Planes here mentioned; when he is in the Tropic of ♋, yet theſe will be ſufficient Inſtructions to ſhew how to proceed when he is in any other Parallel of Declination.

F I N I S.

In Works of this Nature, especially the tabular Part, some Errors will be almost unavoidable; and if the following List be enlarged by some Variations between the transcrib'd and the original Copy, the Author hopes his great Distance from the Press, will by his candid Readers be thought a sufficient Excuse.

ERRATA.

Page		For	Read
		s : o : ′ ″	*s* : o : ′ ″
205	Against 2 deg. and under ♈ ♎	1 : 40	0 : 40
207	Against 801	9 : 13 : 15 : 50	9 : 13 : 55 : 50
207	Against 101	9 : 8 : 58 : 30	9 : 8 : 38 : 30
207	Against 1621, under Anom. ☉	6 : 13 : 54 : 24	6 : 13 : 54 : 4
207	Against 1641	6 : 13 : 42 : 28	6 : 13 : 42 : 8
208	In the Column, Years	5776	1776
208	Against 1784	6 : 11 : 32 : 7	6 : 11 : 32 : 27
209	Against 1830	9 : 21 : 28 : 32	9 : 21 : 27 : 32
209	Against 1846	9 : 21 : 35 : 47	9 : 21 : 34 : 47
209	Against 1846, under Anom ☉	6 : 11 : 24 : 2	6 : 11 : 25 : 2
209	Against 1847	9 : 21 : 21 : 27	9 : 21 : 20 : 27
211	Against *April* 3	3 : 1 : 39 : 53	3 : 1 : 39 : 55
212	Against *May* 2, under Anom. ☉	3 : 0 : 14 : 36	4 : 0 : 14 : 36
212	Against *May* 13	4 : 17 : 5 : 27	4 : 11 : 5 : 27
213	Against *July* 2	6 : 0 : 22 : 14	6 : 0 : 22 : 24
213	Against *August* 4	7 : 2 : 53 : 32	7 : 2 : 53 : 22
213	Against *August* 5	7 : 3 : 52 : 3	7 : 3 : 52 : 30
213	Against *August* 23	1 : 21 : 37 : 37	7 : 21 : 37 : 37
213	Against *August* 30	7 : 28 : 30 : 33	7 : 28 : 30 : 53
217	Against 7 deg. and under 2 : ♊	1 : 46 : 0	1 : 46 : 11
217	Against 17 deg. and under 1 : ♉	1 : 23 : 51	1 : 24 : 51
217	Against 18 deg. and under 1 : ♉	1 : 25 : 14	1 : 25 : : 54
217	Against 28 deg. and under 4 : ♌	1 : 2 : 47	1 : 3 : 47
218	Against 1st Year, and under Apog. ☽	0 : 12 : 8 : 45	9 : 12 : 8 : 45
218	Against 1101 under Long. ☽	8 : 8 : 17 : 30	8 : 28 : 17 : 30
218	Against 1201 under Long. ☽	7 : 16 : 7 : 55	7 : 6 : 7 : 55
218	Against 1661 under Long. ☽	1 : 8 : 11 : 50	1 : 18 : 11 : 50
218	Against 1701, under Node ☽	4 : 27 : 23 : 2	4 , 27 : 23 : 20
219	Against 1742, under Node ☽	3 : 14 : 23 : 7	2 : 14 : 23 : 07
219	Against 1744, under Long. ☽	8 : 30 : 37 : 20	8 : 10 : 37 : 20
219	Against 1761, under Long. ☽	4 : 26 : 2 : 15	11 : 26 : 2 : 15
220	Against 1793, under Long. ☽	0 : 11 : 44 : 46	9 : 11 : 44 : 46
220	Against 1806, under Apog. ☽	9 : 20 : 15 : 8	9 - 20 - 57 : 8
221	Against 1844, under Long. ☽	6 : 18 : 21 : 45	6 - 18 - 27 - 45
221	Against 1849, under Long. ☽	5 : 18 : 44 : 13	5 : 1 - 44 - 13
222	Against 13 Years, under Node ☋	8 : 15 : 25 : 52	8 : 11 - 25 - 52
222	Against 40, under Apog. ☾	6 : 7 : 4 : 30	6 : 7 - 40 - 30
223	Against *January* 6, under Long. ☽	2 : 19 : 3 : 30	2 - 19 3 : 30

ERRATA.

Page		For	Read
		∫ : o : ′ ″	∫ : o : ′ : ″
224	Againſt *Feb.* 11, under Node ☽	—— o : 2 : 13 : 17	o : 2 - 13 i 27
225	Againſt *March* 5, under Long. ☽	— 3 : 3 : 17 : 21	4 : 3 : 17 : 21
226	Againſt *April* 28, under Long. ☽	—— 3 : 24 : 48 : 58	3 : 24 - 48 - 52
227	Againſt *May* 16, under Long. ☾	—— 11 : 21 : 59 : 34	11 : 21 - 59 : 23
227	Againſt *May* 30, under Apog. ☽	—— o : 16 : 22 : 40	o : 16 - 42 : 40
229	Againſt *July* 22, not 12 under Apog. ☽	—— o : 22 : 55 : 56	o : 22 - 36 : 56
230	Againſt *Auguſt* 3, under Long. ☽	—— 10 : 12 : 45 : 29	10 - 12 - 55 : 29
231	Againſt *September* 2, under Node ☽	—— o : 13 : 58 : 27	o - 12 : 58 : 27
232	Againſt *October* 21, under Agog. ☾	—— 1 : 2 : 41 : 14	1 : 2 : 45 : 14
232	Againſt *October* 22, under Apog. ☽	—— 1 : 2 : 55 : 55	1 : 2 : 51 : 55
232	Againſt *October* 24, under Apog. ☾	—— 1 : 3 : 5 : 7	1 : 3 : 5 : 17
233	Againſt *November* 4, under Long. ☽	—— 3 : 8 : 19 : 36	3 : 8 : 19 : 46
233	Againſt *November* 30, under Long. ☽	—— 2 : 2 : 54 : 57	2 : 20 : 55 : 57
235	Againſt 31′, under Long. ☾ ——	— 17 : 28 : 14	17 : 1 : 10
235	Againſt 40, under Long. ☾ ——	—— 21 : 27 : 38	21 : 57 : 38
238	Againſt 6 deg. under Sig. 4. and Ap. ☽	—— 17 : 55	17 : 58
238	Againſt 7 deg. under Sig. 4. and Ap. ☾	—— 17 : 48	17 : 45
	∫ ∫		
239	Againſt 6 deg. under 2 : 8	—— 10 : 28 : 28	10 : 28 : 18
239	Againſt 12 deg. under 1 : 7	—— 11 : 43 : 3	11 : 43 : 8
239	Againſt 28 deg. under o : 6	—— 8 : 58 : 40	8 : 58 : 30
239	Againſt 29 deg. under o : 6	—— 29 : 13 : 41	9 : 13 : : 41
239	Againſt 30 deg. under o : 6	—— 29 : 28 : 24	9 : 28 : : 24
241	Againſt 12 deg. under 1 : 7	—— 9.050025	9.950025
241	Againſt 24 deg. under 2 : 8	—— 9.662064	6.962064
242	Againſt 16 deg. and under Sig. o under 9.952	—— o′ : 43″	o′ : 42″
244	Againſt 26 deg. and under 9.947	— o : 38	o : 28
246	Againſt 14 deg. and under 9.947	—— 1 : 33	1 : 30
246	Againſt 30 deg. and under 9.942	— 1 : 16	1 : 36
279	Againſt 2201, under Mot. Lat.	—— 8 : 17 : 59 : 16	8 : 17 : 57 : 16
280	Againſt 1781, under Mot. Lat.	—— 7 : 24 : 44 : 17	7 : 24 : 24 : 17
281	Againſt 200, under Mot Lat.	—— 8 : 8 : 12 : 52	8 : 8 : 13 : 52
282	Againſt 6 —— ——	— 3 : 2 : 54	3 : 2 : 52

The SUPPLEMENT.

THE Two Tables next following are intended as a Supplement to thoſe that begin at Page 279, from whence they were extracted, as thoſe were from the Tables of the mean Motions of the Sun and Moon in this Book; and it is preſumed to be the moſt expeditious Way of any that has hitherto been publiſhed, for finding the true mean Times of the Fulls and Changes of the Moon, her Quadratures with the Sun, and alſo when an Eclipſe of either the Sun or Moon may be expected. Calculated for 142 Years, beginning with 1748, and ending 1890, and may by the ſaid Tables be calculated in like manner for any Time paſt, or to come.

	Epacts.				Mot.		Lat.			Epacts.				Mot.		Lat.	
Years	D.	H.	′	″	ſ	o	′	″	Years	D.	H.	′	″	ſ	o	′	″
1748	18	18	51	15	11	21	29	50	1782	2	15	12	31	9	3	7	17
1749	7	3	39	51	11	29	32	37	1783	21	12	45	9	10	11	50	19
1750	26	1	12	31	1	8	15	37	1784	10	21	33	47	10	19	53	6
1751	15	10	1	7	1	16	18	24	1785	28	19	6	25	11	28	36	8
1752	4	18	49	45	1	24	21	11	1786	18	3	45	3	0	6	38	55
1753	22	16	22	25	3	3	4	14	1787	7	12	43	37	0	14	41	41
1754	12	1	10	59	3	11	7	0	1788	2	6	10	16	1	23	24	43
1755	1	9	59	39	3	19	9	47	1789	14	19	4	53	2	1	27	30
1756	20	7	32	17	4	27	52	49	1790	4	3	53	31	2	9	30	16
1757	8	16	20	51	5	5	55	37	1791	23	1	26	9	3	18	13	17
1758	27	15	53	31	6	14	38	35	1792	12	10	14	47	3	26	16	4
1759	16	22	42	5	6	22	45	22	1793	0	19	3	25	4	4	18	53
1760	6	7	30	45	7	0	44	12	1794	19	16	36	1	5	13	1	53
1761	24	5	3	23	8	9	27	11	1795	9	1	24	41	5	21	4	40
1762	13	13	52	0	8	17	29	57	1796	27	22	57	19	6	29	47	42
1763	2	20	40	36	8	25	31	43	1797	16	7	45	53	7	7	50	30
1764	21	20	13	15	10	4	15	44	1798	5	16	34	31	7	15	53	14
1765	10	5	1	51	10	12	18	32	1799	24	14	7	7	8	24	40	15
1766	29	2	34	31	11	21	1	33	1800	13	22	56	32	9	2	39	5
1767	18	11	23	5	11	25	4	19	1801	2	7	44	20	9	10	41	48
1768	7	20	11	43	0	7	7	7	1802	21	5	16	59	10	19	24	48
1769	25	17	44	21	1	15	50	8	1803	10	14	5	35	10	27	27	36
1770	15	2	32	59	1	23	52	54	1804	29	11	38	15	0	6	10	37
1771	4	11	21	35	2	1	55	41	1805	17	20	26	51	0	14	13	25
1772	23	8	54	15	3	10	38	42	1806	7	5	15	29	0	22	16	12
1773	11	17	42	53	3	18	41	31	1807	26	2	48	5	2	0	59	12
1774	1	2	31	27	3	26	44	17	1808	15	11	36	43	2	9	2	0
1775	20	0	4	9	5	5	27	18	1809	3	20	25	19	2	17	4	47
1776	9	8	52	45	5	13	30	6	1810	22	17	57	59	3	25	46	47
1777	27	6	25	21	6	22	13	8	1811	12	2	46	35	4	3	30	34
1778	16	15	13	59	7	0	15	52	1812	1	11	35	13	4	11	53	21
1779	6	0	2	33	7	8	22	39	1813	20	9	7	53	5	20	36	24
1780	24	21	35	15	8	17	1	43	1814	8	17	56	27	5	28	39	10
1781	13	6	23	51	8	25	4	28	1815	27	15	29	9	7	7	22	11

Years	Epacts. D.	H.	′	″	Mot. Lat. ☽	°	′	″
1816	17	0	17	45	7	15	24	59
1817	5	9	6	19	7	23	27	47
1818	24	6	38	59	9	2	10	45
1819	13	15	27	33	9	10	17	32
1820	3	0	16	13	9	18	16	22
1821	20	21	48	51	10	26	59	21
1822	10	6	37	51	11	5	2	9
1823	29	4	10	9	0	13	45	11
1824	18	12	58	47	0	21	47	58
1825	6	21	47	23	0	29	50	46
1826	25	19	20	3	2	8	33	47
1827	15	4	8	37	2	16	36	33
1828	4	12	57	15	2	24	39	21
1829	22	10	29	53	4	3	22	22
1830	11	19	18	31	4	11	25	8
1831	1	4	7	7	4	19	27	55
1832	20	1	39	47	5	28	10	56
1833	8	10	28	25	6	6	13	45
1834	27	8	1	1	7	14	56	45
1835	16	16	49	41	7	22	59	32
1836	6	1	38	17	8	1	2	20
1837	23	23	10	53	9	9	45	22
1838	13	7	59	31	9	17	48	6
1839	2	16	48	5	9	25	54	53
1840	21	14	20	47	11	4	33	57
1841	9	23	9	20	11	12	36	41
1842	28	20	41	59	0	21	19	41
1843	18	5	30	35	0	29	22	29
1844	7	14	19	13	1	7	25	16
1845	25	11	51	51	2	16	8	18
1846	14	20	40	29	2	24	11	5
1847	4	5	29	3	3	2	13	51
1848	23	3	1	43	4	10	56	53
1849	11	11	50	19	4	18	59	40
1850	0	21	38	57	4	27	2	26
1851	19	18	11	35	6	5	45	27
1852	9	3	0	13	6	13	48	14
1853	27	0	32	53	7	22	31	17
1854	16	9	21	27	8	0	34	3
1855	5	18	10	7	8	8	36	50
1856	24	15	42	45	9	17	19	52
1857	13	0	31	19	9	25	22	40
1858	2	9	19	57	10	3	25	24
1859	21	6	52	33	11	12	12	25
1860	10	15	41	13	11	20	11	15
1861	28	13	13	52	0	28	54	25
1862	17	22	2	29	1	6	57	11
1863	7	6	51	5	1	14	59	59
1864	26	4	23	45	2	23	43	0
1865	14	13	12	21	3	1	45	48
1866	3	22	0	59	3	9	48	35
1867	22	19	33	35	4	18	31	35
1868	12	4	22	13	4	26	34	23
1869	0	13	10	49	5	4	37	10
1870	19	10	43	29	6	13	20	10
1871	8	19	32	5	6	21	22	57
1872	27	17	4	45	8	0	5	58
1873	16	1	53	23	8	8	8	47
1874	5	10	41	57	8	16	11	33
1875	24	8	14	39	9	24	54	34
1876	13	17	3	15	10	2	57	22
1877	2	1	51	49	10	11	0	10
1878	20	23	24	29	11	19	43	8
1879	10	8	13	3	11	27	49	55
1880	29	5	45	45	1	6	28	59
1881	17	14	34	21	1	14	31	44
1882	6	23	22	57	1	22	34	19
1883	25	4	55	35	3	1	17	21
1884	15	5	44	13	3	9	20	8
1885	3	14	32	49	2	17	22	56
1886	22	12	5	29	4	26	5	57
1887	11	20	54	3	5	4	8	43
1888	1	5	42	41	5	12	11	31
1889	19	3	15	19	6	20	54	32
1890	8	12	3	57	6	28	57	18

A TABLE

A TABLE of Months.

	Epacts.				Mot. Lat.			
	D.	H.	′	″	♑	°	′	″
January	29	12	44	3	1	0	40	14
February	28	1	28	6	2	1	20	28
March	29	14	12	9	3	2	0	42
April	28	2	56	12	4	2	40	56
May	27	15	40	15	5	3	21	10
June	26	4	24	18	6	4	1	24
July	25	17	8	21	7	4	41	38
August	24	5	52	24	8	5	21	52
September	22	18	36	27	9	6	2	6
October	22	7	20	30	10	6	42	20
November	20	20	4	33	11	7	22	34
December	20	8	48	36	0	8	2	48
First Quarter	7	9	11	01	0	7	40	4
Full Moon	14	18	22	2	0	15	20	8
Laſt Quarter	22	3	33	3	0	23	0	12

In the Firſt and Fourth Columns are placed the Years of our Lord; and under the Word Epacts, and againſt the given Year, you have the true mean Time of the firſt Conjunction, or New Moon that happens in *January*, for that Year; and in the ſame Line, under the Motion of Latitude, you have the Motion of Latitude for that Time. And to know whether the Sun may be eclipſed at thoſe Times, obſerve what is ſaid at Page 284; and when you have found the mean Time of a Full Moon, you will ſee in the ſame Page whether ſhe is likely to ſuffer an Eclipſe at that Time or not.

Now, when you would find the true mean Time of a New Moon in any other Month (except *February*) you muſt firſt write down the Epact and Motion of Latitude that you find againſt that Year; to theſe add the Epact, and Motion of Latitude belonging to the Month next before that which was given: If the Sum be leſs than the Number of Days in that preceding Month, that is the Time of the mean Conjunction in that Month; to which add a Lunar Month, *viz.* $29^d : 12^h : 44' : 3''$ for the Epact, and $1^s : 0° : 40' : 14''$ for the Motion of Latitude; from the Sum, take the Days of the preceding Month, and the Remainder will be the Time of the New Moon, and Motion of Latitude for the Month required, which being found, you may find the Time of the Full Moon that ſhall happen next before that Change, or that next after, by ſubſtracting $14^d : 18^h : 22' : 2''$ from the Epact, and $0^s : 15° : 20' : 8''$ from the Motion of the Latitude, or adding theſe for the next Full Moon after that Change.

Here obſerve, that if the true mean Time of the New Moon be required in *February*, for any of theſe Years, then write down the given Year, and after that the Epact and Motion of Latitude, and to thoſe add the Epact and Motion of Latitude for a Lunar Month, and the Sum will be the true mean Time of the Conjunction, and alſo the Motion of Latitude for *February*: And having thus found the mean Time of a Conjunction, you may eaſily find the Firſt Quarter, Full Moon, and Laſt Quarter, by the Numbers at the Bottom of the Table of Months.

Note, That if the given Year be a Leap-Year, and given Month after *February*, ſet down one Day leſs for the preceding Month than what you find in the Table of Months.

EXAMPLE

EXAMPLE 1.

LET it be required to find the true mean Time of the New Moon that will happen in *June*, 1748.

	Epact.				Mot. Lat.			
	D.	H.	′	″	☌	o	′	″
1748	18	18	51	15	11	21	29	50
May	26	15	40	15	5	3	21	10
New ☽ *June*	14	10	31	30	2	24	31	0

EXAMPLE 2.

LET it be required to find the mean Time of the New Moon that will happen in *February* 1756.

	Epact.				Mot. Lat.			
	D.	H.	′	″	☌	o	′	″
1756	20	7	32	17	4	27	52	49
Add a Lunar Month	29	12	44	3	1	0	40	14
New ☽ and ☉ Eclip.	18	20	16	20	5	28	33	03

EXAMPLE 3.

LET it be required to find the mean Time of the New Moon that will happen in *September* 1755.

	Epact.				Mot. Lat.			
	D.	H.	′	″	☌	o	′	″
1755	1	9	59	39	3	19	9	47
August	24	5	52	24	8	5	21	52
New ☽ and ☉ Eclip. *August*	25	15	52	03	11	24	31	39
To this add	29	12	44	3	0	0	40	14
New ☽ *September*	24	4	36	6	0	25	11	53